Intern. J. Fuzzy Mathematical Archive
Vol. 3, 2013, 42-49
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 24 December 2013
www.researchmathsci.org

International Journal of
Fuzzy Mathematical Archive

Construction of PBIBD (2) Designs Using MOLS

***R.Jaisankar*[1] and *M.Pachamuthu*[2]**

[1]Department of Statistics, Bharathiar University, Coimbatore-641046, India
Email: r_jaisankarstat@gmail.com
[2]Department of Mathematics and Statistics, KSR College of Arts and Science, Tiruchengode-637 215, India
Email: kpmstat@gmail.com

Received 25 November 2013; accepted 6 December 2013

Abstract. In this paper, a construction methodology of Partially Balanced Incomplete Block Design with two associate classes using Mutually Orthogonal Latin Squares (MOLS) is discussed. The MOLS have been constructed using the theory of Galois Field (GF). The application of this method is explained through an example.

Keywords: LSD, MOLS, Orthogonality, Galois Field, PBIBD(2)

***AMS Mathematics Subject Classification (2010)*:** 62K10, 62K99

1. Introduction

Heterogeneity in the experimental material is one of the important problems to be reckoned with the designing of scientific experiments. When the heterogeneity present in the experimental material, we can employ the principle of local control, that is, dividing the experimental materials into homogenous blocks in such a way that the variation within each block is minimum and the variation between the blocks is maximum and then allot the treatments at random within each block. The designs involving the principles of blocking are known as a block designs. These block designs may be broadly classified into two types, namely, (i) Complete Block Designs and (ii) Incomplete Block Designs. In complete block designs, all the treatments are allocated in every block. This is possible only when we have limited number of treatments. However, when the number of treatments is sufficiently large it leads to the increase the block size, hence the homogeneity within the block is in question, and hence the precision of the design gets affected. To overcome this problem a special type of designs are evolved in which the block size is less than the number of treatments. Such designs are called ‘Incomplete block’ designs. Since the block size ’k’,(say) is less, in an incomplete block design at least one treatment is not placed in each block. Consequently, the number of replication ’r’(say) of the each treatment may not be uniquely defined. To rectify this problem a concept called ’Balancing’ is introduced. The technique of adjusting of random allocation of treatment in order to get equal number of replication in the incomplete block designs known as ‘Balancing the incomplete Block Design’. An incomplete block design having balanced is known as Balanced Incomplete Block Design (BIBD). But, BIBD’s are not available for all number of treatments. To overcome this drawback to some extent

Partially Balanced Incomplete Block Designs are evolved. A set of 'ϑ' treatments distributed in 'b' blocks each containing 'k' distinct treatments (k < ϑ), is said to form a PBIBD with 'm' associate classes when

(i). Every treatment occurs in 'r' blocks.

(ii). Two treatments occur together in $\lambda_1, \lambda_2, \lambda_3, ..., \lambda_m$ blocks.

(iii). Given a treatment Θ each of n_i treatments occurs with Θ in λ_i blocks (i=1,2,3,...m) so that $\sum n_i \lambda_i = \vartheta$-1 and $\sum n_i \lambda_i$ = r(k-1).

(iv). For a given treatment θ each of n_i treatments occurs with θ in λ_i blocks (i=1,2,3,...,m) satisfying the condition $\sum n_i = (\vartheta - 1)$ and $\sum n_i \lambda_i$=r(k-1). The PBIBD with 2 associated classes is the simplest design for the analysis, since a general solution of the treatment effect t_i can be obtained by solving only two equations.

1.1. Parameter relationship of PBIBD(2)

1. The relation $p^i_{ik} = p^i_{ik}$ always holds because of symmetry, that is, from the fact if $\oplus$ is an ith associate ϕ, then ϕ is an ith associate of $\oplus$.

2. $\sum_{k-1}^{m} = n_j$ if i$\neq$ j. $= n_j - 1$ i =j. 3. $n_i p^i_{ik} = n_j p^j_{ik}$

1.2. Construction of Mutually Orthogonal Latin Squares

Let ϑ = $S_1, S_2, ..., S_p$, where S_i (i = 1,2,..., p) is either a prime or a prime power. The S_i elements of GF(S_i) are used for forming combinations of elements of the 'p' different fields. Combine the elements from 'p'different fields taking one from each field in all possible ways. There is evidently 'ϑ' such combinations. If 'ϑ' is a prime or prime power then p=1 and each such combinations is just an element of its field. Such combinations of the 'p' field element are used as symbols for writing the Latin squares. Let 'ϑ' combinations be written in a row and again in a column so as to obtain the summation table of all possible sums, two by two, of the row column combinations by using MOLS. This column will be called the principal column and the row, the principal row. By addition or multiplications of two combination that is by addition or multiplication of each pair of corresponding element in two combinations in the respective field. It can be easily seen that the summation table gives a Latin square. Next, each combination in the principal column is multiplier, say, $a_1, a_2, ..., a_p$, where,$a_i \neq 0$ 1 (i=1,2,..., p) or the resultant column is the second principle column. Again another summation table is formed by using this second principal row. This table gives the second Latin square which is orthogonal to the one obtained earlier. Again a third principal column is obtained by multiplying the different elements by the first principal column by another multiplier, say,($b_1, b_2, ..., b_p$) where,$b_i \neq 0$ or 1 (i=1,2,..., p). That is the multiplier is so chosen that no element in any field is repeated in the different multiplier. The third Latin square is obtained by adding the third principal column and first principal row. This square is orthogonal to previous two and this process is continued till suitable multipliers are available. As each time, a new element is introduced in each field in the multiplier combinations, we can not get more than (s-1) multipliers, where 's' is the minimum factor of ϑ. Each of these multipliers contains a different non zero element of the field of 's' and so not more than (s-1) can be taken with out repeating an element in the field. If ϑ is prime or prime power, each multiplier combination consists of only one element. We can therefore get ($\vartheta - 2$) multipliers which are the different non zero elements in its field other than unity.

2. Main Results

2.1. Construction of MOLS of order (3^2)

The elements of GF (3^2) with primitive roots 0, 1, 2, α, α^2 (α^2 = α+1), $\alpha+2$, 2α, $2\alpha+1$, $2\alpha+2$.

2.2. Table of first summation of the elements of G.F.(3^2)

+	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
0	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
1	1	2	0	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α
2	2	0	1	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$
α	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$	0	1	2
$\alpha+1$	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α	1	2	0
$\alpha+2$	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1
2α	2α	$2\alpha+1$	$2\alpha+2$	0	1	2	α	$\alpha+1$	$\alpha+2$
$2\alpha+1$	$2\alpha+1$	$2\alpha+2$	2α	1	2	0	$\alpha+1$	$\alpha+2$	α
$2\alpha+2$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1	$\alpha+2$	α	$\alpha+1$

Substituting $\alpha = 3$ and Reduce it to Mod 3, we get first Latin square as,

0	1	2	3	4	5	6	7	8
1	2	0	4	5	3	7	8	6
2	0	1	5	3	4	8	6	7
3	4	5	6	7	8	0	1	2
4	5	3	7	8	6	1	2	0
5	3	4	8	6	7	2	0	1
6	7	8	0	1	2	3	4	5
7	8	6	1	2	0	4	5	3
8	6	7	2	0	1	5	3	4

Next, construct the table of second summation. The second summation Table 3.3 of the element of G.F.(3^2) can be obtained by multiplying the principal column of the first summation Table 3.2 by α.

2.3. Table of second summation of the elements of G.F.(3^2)

+	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
0	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
α	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$	0	1	2
2α	2α	$2\alpha+1$	$2\alpha+2$	0	1	2	α	$\alpha+1$	$\alpha+2$
$\alpha+1$	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α	1	2	0
$2\alpha+1$	$2\alpha+1$	$2\alpha+2$	2α	1	2	0	$\alpha+1$	$\alpha+2$	α
1	1	2	0	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α
$2\alpha+2$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1	$\alpha+2$	α	$\alpha+1$
2	2	0	1	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$
$\alpha+2$	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1

Substituting $\alpha=3$ and Reduce it to Mod 3, we get the second Latin square as,

0	1	2	3	4	5	6	7	8
3	4	5	6	7	8	0	1	2
6	7	8	0	1	2	3	4	5
4	5	3	7	8	6	1	2	0
7	8	6	1	2	0	4	5	3
1	2	0	4	5	3	7	8	6
8	6	7	2	0	1	5	3	4
2	0	1	5	3	4	8	6	7
5	3	4	8	6	7	2	0	1

Next, construct the table of third summation. The second summation table (3.3)of the element of G.F.(3^2) can be obtained by multiplying the principal column the first summation table (3.2) by 2α.

2.4. Table of third summation of the elements of G.F.(3^2)

+	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
0	0	1	2	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$
2α	2α	$2\alpha+1$	$2\alpha+2$	0	1	2	α	$\alpha+1$	$\alpha+2$
α	α	$\alpha+1$	$\alpha+2$	2α	$2\alpha+1$	$2\alpha+2$	0	1	2
$2\alpha+2$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1	$\alpha+2$	α	$\alpha+1$
$\alpha+2$	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$	2	0	1
2	2	0	1	$\alpha+2$	α	$\alpha+1$	$2\alpha+2$	2α	$2\alpha+1$
$\alpha+1$	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α	1	2	0
1	1	2	0	$\alpha+1$	$\alpha+2$	α	$2\alpha+1$	$2\alpha+2$	2α
$2\alpha+1$	$2\alpha+1$	$2\alpha+2$	2α	1	2	0	$\alpha+1$	$\alpha+2$	α

Substituting $\alpha=3$ and Reduce it to Mod 3, we get sixth Latin square as,

0	1	2	3	4	5	6	7	8
6	7	8	0	1	2	3	4	5
3	4	5	6	7	8	0	1	2
8	6	7	2	0	1	5	3	4
5	3	4	8	6	7	2	0	1
2	0	1	5	3	4	8	6	7
4	5	3	7	8	6	1	2	0
1	2	0	4	5	3	7	8	6
7	8	6	1	2	0	4	5	3

Similarly, we can multiply remaining five primitive elements in principal columns(3.2) get all Latin squares. When n=9, we have 8 MOLS. A method of construction of PBIBD(2) using MOLS is explained in the next section.

3. Construction of PBIBD(2)

Let the Latin square design be of order 9 and to construct PBIBD with order $\vartheta=9$ treatments with block size k=3. We have 8 MOLS of order 9. These MOLS are merged corresponding to the first row in which all the elements are same. Omitting the first row and notify the treatments in 72 cell, each cell consists of 8 treatments in order. Taking the first three treatments in all 72 cells, we can form a PBIBD(2), which is presented below.

Blk	*Treatments*			*Blk*	*Treatments*			*Blk*	Treatments			*Blk*	Treatments		
1	1	3	6	19	3	4	8	37	5	1	2	55	7	2	1
2	2	4	7	20	4	5	6	38	3	2	0	56	8	0	2
3	0	5	8	21	5	3	7	39	4	0	1	57	6	1	0
4	4	6	0	22	6	7	2	40	8	4	5	58	1	5	4
5	5	7	1	23	7	8	0	41	6	5	3	59	2	3	5
6	3	8	2	24	8	6	1	42	7	3	4	60	0	4	3
7	7	0	3	25	0	1	5	43	2	7	8	61	4	8	7
8	8	1	4	26	1	2	3	44	0	8	6	62	5	6	8
9	6	2	5	27	2	0	4	45	1	6	7	63	3	7	6
10	2	6	3	28	4	7	5	46	6	8	4	64	8	5	7
11	0	7	4	29	5	8	3	47	7	6	5	65	6	3	8
12	1	8	5	30	3	6	4	48	8	7	3	66	7	4	6
13	5	0	6	31	7	1	8	49	0	2	7	67	2	8	1
14	3	1	7	32	8	2	6	50	1	0	8	68	0	6	2
15	4	2	8	33	6	0	7	51	2	1	6	69	1	7	0
16	8	3	0	34	1	4	2	52	3	5	1	70	5	2	4
17	6	4	1	35	2	5	0	53	4	3	2	71	3	0	5
18	7	5	2	36	0	3	1	54	5	4	0	72	4	1	3

Here, the number of treatments $\vartheta = 9$, Number of Blocks size $= 72$, Number of replications r= 24 and Size of the Block k= 3. As k< ϑ $(3 < 9)$ and the above arrangement is incomplete design.

3.1. Parametric Relations of PBIBD(2)

Let us check the parameters relations of PBIBD(2) constructed above,

$\vartheta = k^2$, b=mk, r=m, k=k, n_1=m(k-1), n_1=(k-1)(k+1-m) and

$$\lambda = \frac{r(k-1)}{(\vartheta - 1)}.$$

$\vartheta = 3^2 = 9$ treatments, k=r= 3, r=m= 3, $n_1 = 6$, $n_2 = 2$, $n_1 + n_2 = 8$,

$\lambda = 0.75$, $\lambda_1 = 1$. and p_{ij} matrix is given below

$$\begin{pmatrix} 4 & 2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

As the relationships are satisfied the above arrangement is a PBIBD(2). Now, by taking any three treatments of sequential order in the merged cells or MOLS, we can form 72 PBIBD(2) arrangement which may have the following associateship.

3.2. Associates of PBIBD (2)

Thus for treatments θ = $(0, 1, ..., 8)$ the remaining (ϑ -1) treatments are grouped into 'm' groups (m= 2). The i^{th} group contains blocks (i= 0 ,1). These n_i treatments can be said as i^{th} associate class of θ.The parametric relations of PBIBD(2) and associates of PBIBD (2) are satisfied.

θ	I Association class	II Association class	θ	I Association class	II Association class
0	5,8,4,6,7,3	1,2	0	5,8,4,6,7,3	1,2
1	3,6,5,7,8,4	0,2	1	3,6,5,7,8,4	0,2
2	4,7,3,8,6,5	0,1	2	4,7,3,8,6,5	0,1
3	1,6,8,2,7,0	4,5	3	1,6,8,2,7,0	4,5
4	2,7,6,0,8,1	3,5	4	2,7,6,0,8,1	3,5
5	0,8,7,1,6,2	3,4	5	0,8,7,1,6,2	3,4
6	1,3,4,0,2,5	7,8	6	1,3,4,0,2,5	7,8
7	2,4,5,1,0,3	6,8	7	2,4,5,1,0,3	6,8
8	0,5,3,2,1,4	6,7	8	0,5,3,2,1,4	6,7

θ	I Association class	II Association class	θ	IAssociation class	II Association class
0	7,8,1,5,2,4	3,6	0	6,7,2,5,3,1	4,8
1	8,6,0,5,2,3	4,7	1	7,8,4,2,0,3	5,6
2	6,7,1,3,0,4	5,8	2	8,6,1,4,5,0	3,7
3	4,8,5,7,1,2	0,6	3	5,8,6,4,0,1	2,7
4	3,8,5,6,2,0	1,7	4	7,5,3,6,1,2	0,8
5	4,6,3,7,0,1	2,8	5	8,3,4,7,2,0	1,6
6	4,5,7,2,8,1	0,3	6	3,4,8,2,0,7	1,5
7	5,3,6,2,8,0	1,4	7	4,5,1,8,6,0	2,3
8	3,4,7,0,6,1	2,5	8	5,3,7,1,2,6	0,4

θ	I Association class	II Association class	θ	I Association class	II Association class
0	3,2,4,1,8,6	5,7	0	2,7,1,8,5,4	3,6
1	5,2,4,0,6,7	3,8	1	0,8,2,6,3,5	4,7
2	5,1,3,0,7,8	4,6	2	0,7,1,6,4,3	5,8
3	2,0,6,5,7,4	1,8	3	8,7,5,1,4,2	0,6
4	0,1,8,5,7,3	2,6	4	6,8,3,2,5,0	1,7
5	1,2,8,4,6,3	0,7	5	7,6,3,1,4,0	2,8
6	5,3,0,8,1,7	2,4	6	8,4,7,5,2,1	0,3
7	3,4,2,8,1,6	0,5	7	6,5,8,3,0,2	1,4
8	4,5,2,7,0,6	1,3	8	6,4,7,3,1,0	2,5

θ	I Association class	II Association class	θ	I Association class	II Association class
0	8,2,6,1,4,3	5,7	0	6,2,1,7,3,5	4,8
1	7,2,6,0,5,4	3,8	1	7,0,2,8,4,3	5,6
2	7,1,8,0,3,5	4,6	2	8,1,0,6,5,4	3,7
3	2,5,0,4,7,6	1,8	3	6,8,0,5,4,1	2,7
4	1,5,0,3,8,7	2,6	4	7,6,5,2,1,2	0,8
5	1,4,2,3,6,8	0,7	5	8,7,2,4,3,0	1,6
6	1,0,5,8,3,7	2,4	6	3,8,7,4,0,2	1,5
7	2,1,4,8,3,6	0,5	7	8,5,4,6,1,0	2,3
8	0,2,4,7,5,6	1,3	8	5,7,6,3,2,1	0,4

This analogy can be extended even when the block size is more than 3 (<9) .

4. Conclusion

In this paper, the construction of PBIBD(2) using MOLS has been discussed with the basis of Galois Field primitive element 3^2. It can be noted that a complete set of MOLS exists only when the number of treatments is in prime or prime power. This methodology are can also be extended to the case of primitive elements 2^2, 2^5, 7^3, 11^2, 13^2, 17^3, 19^2 and 23^5.

REFERENCES

1. R.C.Bose, On the construction of balanced incomplete block designs, *Ann. Eugen.*, 9 (1939) 353-99.
2. R.C.Bose and Nair, Partially balanced incomplete designs, *Sankhya*, 4 (1939) 337-72.
3. R.C.Bose, S.S.Shrikhande and E.T.Parker, Further results on the construction of mutually orthogonal latin squares and falsity of Euluer's conjecture, *Can. J.math*, 12 (1960) 189.
4. R.C.Bose, W.H.Clatworthy and E.T.Shrikhande, Tables of partially balanced designs with two associate classes, *North carolina Agricultrual Experiments Station*, (1954) 107.
5. W.S.J.R. Connor, On the consturction of balanced incomplete block designs, *Ann. Math. Stat.*, 23(1952) 57-71.
6. M.N.Das and Giri, Design and analysis of experiments, Second Edition,Wiley Eastern Publications Limited, New Delhi, 1986.
7. M.Dharmalingam,Construction of partial triallel crosses based on trojan square design, *Journal of Applied Statistics*, 5(2002) 695-702.
8. N.C.Giri, On reinforced PBIBD designs, *Journ. Ind. Soc. Agri. Stat.*, 12(1958) 41-51.
9. H.B.Mann, Constructions of orthogonal latin squares, *Ann. Math. Sstat*, 13 (1942) 418.
10. P.K.Menon, Method of constructing two mutually orthogonal latin squares of order 3n+1, *Sankhya*, 23 (1961) 281-82.
11. R.Jaisankar and M.Pachamuthu, Construction of circular neighbour designs based on MOLS, *International Journal of Statistics and Analysis*, 2(4) (2012) 437 – 446.
12. M.Pachamuthu, Construction of (12x12) GLSD based on MOLS, *Proceeding of the ICETMCA 2010*, 239-242.
13. M.Pachamuthu, Construction of 3^2 MOLS and check parameter relations of BIBD, *IJMSA*, 2 (2011) 911-922.