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Multi Objective Fuzzy Inventory Model With Demand Dependent Unit Cost, Storage Space and Lead Time Constraints–A Karush Kuhn Tucker Conditions Approach

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Abstract. A multi-objective inventory model with demand dependent unit cost and leading time has been formulated with storage space, number of orders and production cost as constraints. In most of the real world situations the cost parameters the objective function and constraints of the decision makers are imprecise in nature. A demand dependent unit cost is assumed and solved using Karush Kuhn Tucker conditions. Here the unit production cost is considered under fuzzy environment. The model has been solved with demand, lot size and leading time as decision variables. The model is illustrated for a single item.

Keywords: Inventory, Membership Function, Karush -Kuhn-Tucker Condition, demand dependent, lead time

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1. Introduction

In most of the existing literature, inventory related costs are assume to be deterministic and represented as real numbers. But, in real situation the inventory costs are usually imprecise in nature due to the influence of various uncontrollable factors. For example, costs may depend on some foreign monetary unit. In such a case, due to exchange rates, the costs are often not known precisely. Inventory carrying cost may also dependent on some parameters like interest rate and stock keeping unit's market price, which are not precise. Also the shortage cost is often difficult to determine precisely in the case when it reflects not just 'lost sale' but also 'a loss of customers will'. Therefore, these cost parameters are described as "approximately equal some certain amount" and so it is more reasonable to characterize these parameters as fuzzy.

Since the development of EOQ model by Harris [1], lot of research works have been carried out in inventory control system. In the existing literature, inventory models are generally developed under the assumption of constant or stochastic lead-time. A number of research papers have already been published in this direction Das.C [2], and Foote et.al [3] etc). Recently, Kalpakam and Sapan [4] studied a perishable inventory model with stochastic lead-time. But in real life situations, the lead-time is normally vague and imprecise i.e. uncertain in non-stochastic sense. It will be more realistic to consider the lead time as fuzzy in nature.

In general the classical inventory problems are designed by considering that the demand rate of an item is constant and deterministic and that the unit price of an item is considered to be constant and independent in nature. But in practical situation, unit price and demand rate of an items may be related to each other. When the demand of an item is high, an item is produced in large numbers and fixed cost of production are spread over a large number of items. Hence the unit cost of the item decreases. i.e., the unit price of an item inversely relates to the demand of that item. So demand rate of an item may be considered as a decision variable.

In multi-objective mathematical programming problems, a decision maker is required to maximize/minimize two or more objectives simultaneously over a given set of possible situations.

In the crisp environment, all parameters in the total inventory cost such as holding cost, ordering cost, set-up cost, purchasing cost, deterioration rate, demand rate and production rate etc. are known and have definite value without ambiguity [15]. Some of the business situations fit such conditions, but in most of the situations and in the day-by-day changing market scenario the parameters and variables are highly uncertain or imprecise.

This paper develops an inventory problem with demand dependent unit rate for a prescribed finite time horizon allowing imprecise lead-time. A demand dependent unit cost had been treated by some researchers in the problem of EOQ model. Chang [5] studied an EOQ model with demand – dependent unit cost of single item. Ben-Daya and Abdul Raouf [6] described the problem of inventory models involving lead time as a decision variable. Abou-et-Ata and Kotb[7] developed a crisp inventory model under two restricitions. Also Teng and Yang [8] examined deterministic inventory lot size model with time invarying demand and cost under generalized holding costs. Other related studies were written by Jang & Klein [9].The concept of fuzzy set theory was first introduced by Zadeh [10]. Later on Bellman and Zadeh [11] used the fuzzy set theory to the decision making problems. Hence Fuzzy set theory has made on entry into the inventory control system. Many researchers solved fuzzy multi item multi objective inventory especially using geometric programming method [12,13,14]. Here we solve the model using Karush-Kuhn-Tucker Conditions with unit production cost under fuzzy environment.

2. Notations and assumptions

To construct the model, we define the following notations

D = Annual demand rate (decision variable)

p_i = unit purchase(production) cost

S_i = ordering cost

H_i = unit holding(Inventory carrying)cost per unit item
Ss= Kσ √Li = Safety stock
n = number of different items (carried in inventory)
L_i = Leading rate time (decision variable)
Q_i = Production (order) quantity taken (decision variable)
TC(D_i, Q_i, L_i) = Average annual total cost for the ith item
w_i = storage space per item
W = Floor or Shelf Space available
B = Total investment cost for replenishment
t = number of orders

Assumptions

The following basic assumptions are made in the model

(1) Time horizon is finite
(2) No shortages are allowed
(3) Unit production cost is inversely related to the demand rate. (i.e) $p_i = A_i D_i^{-\beta_i}$ i = 1,2,3……n , A > 0 , β ≥ 1
Where A,β are real constants ,selected to provide the best fit of the estimated cost function.
(4) Lead time crashing cost is related to the lead time by a function of the form R(L_i)= αL_i^{-b} , i=1,2,.......n , α >0 , 0 < b ≤ 0.5
(5) Objective is to minimize the annual relevant total cost.

3. Mathematical formulation

The annual relevant total cost [sum of production, order, inventory carrying and lead time crashing costs]which according to the basic assumptions of the EOQ model is:

$$TC(D_i, Q_i, L_i) = \sum_{i=1}^{n} \left\{ p_i D_i + \frac{S_i D_i}{Q_i} + \left[\frac{Q_i}{2} + K\sigma\sqrt{L_i} \right] H_i + \frac{D_i}{Q_i} R(L_i) \right\} \qquad (1)$$

Substituting p_i and R(L_i) in (1) gives

$$TC(D_i, Q_i, L_i) = \sum_{i=1}^{n} \left\{ AD_i^{1-\beta_i} + \frac{S_i D_i}{Q_i} + \left[\frac{Q_i}{2} + K\sigma\sqrt{L_i} \right] H_i + \frac{D_i}{Q_i} \alpha L_i^{-b_i} \right\}$$

To derive the optimal total cost in an inventory problems ,there are some restrictions on available resourses.

(i) There is a limitation on the available warehouse floor space where the items are to be stored. i.e. $\sum_{i=1}^{n} w_i Q_i \le W$

(ii) Investment amount on total production cost cannot be infinite , it may have an upper limit on the maximum investment

i.e. $\sum_{i=1}^{n} p_i Q_i \le B$

or $\sum_{i=1}^{n} AD_i^{-\beta_i} Q_i \le B$

(iii) An upper limit on the number of orders that can be made in a time cycle on the system, i.e. $\sum_{i=1}^{n} \frac{D_i}{Q_i} \le t$.

4. Fuzzy inventory model

The above crisp model under fuzzy environment with p_i's as fuzzy decision variable, reduces to

$$MinTC(D_i, Q_i, L_i, \tilde{p}_i) = \sum_{i=1}^{n} \left\{ AD_i^{1-\beta_i} + \frac{D_i}{Q_i} S_i + \left[\frac{Q_i}{2} + K\sigma\sqrt{L_i} \right] H_i + \frac{D_i}{Q_i} \alpha L_i^{-b_i} \right\}$$

Subject to the constraints

$$\sum_{i=1}^{n} w_i Q_i \le W$$

$$\sum_{i=1}^{n} AD_i^{-\beta_i} Q_i \le B$$

$$\sum_{i=1}^{n} \frac{D_i}{Q_i} \le t$$

[Here cap '$\sim$' denotes the fuzzification of the parameter]

5. Membership function

A membership function for the fuzzy variable p_i is defined as follows

$$\mu_{p_i}(x) = \begin{Bmatrix} 1, p_i \le L_{L_i} \\ \frac{U_{L_i} - p_i}{U_{L_i} - L_{L_i}}, L_{L_i} \le p_i \le U_{L_i} \\ 0, p_i \ge U_{L_i} \end{Bmatrix}$$

Here U_{L_i} and L_{L_i} are upper limit and lower limit of p_i respectively.

6. Numerical example

The decision variables namely the optimal order quantity Q_i, optimal demand rate D_i and optimal lead time L_i whose values determine the minimum annual relevant total cost are computed for different values of β.

The parameters of the model are shown in Table -1

Assume the standard deviation σ =6 unit/year, K=2 , $3 \le p_i \le 8$

The optimal values of the production batch Q_i ,demand rate D_i, lead time L_i and minimum total cost are given in Table – 2.

n	S_i	A	H_i	α_i	w_i	W	B	T
1	200$	15	0.8$	1	2	100	300	1

Table 1:

The optimal solution of Q_i , D_i and L_i as a function of β

a	β	b	D_i	Qi	L_i	Min TC	p_i	μ_{pi}
1	2	0.1	1.399	26.47	$1.2x10^{-5}$	32.078	7.664	0.0672
1	2.2	0.1	1.503	27.63	$1.25x10^{-5}$	31.333	6.120	0.3760
1	2.4	0.1	1.562	28.12	$1.25x10^{-5}$	30.597	5.143	0.5714
1	2.6	0.1	1.594	28.46	$1.29x10^{-5}$	29.907	4.463	0.7074
1	2.8	0.1	1.612	28.58	$1.31x10^{-5}$	29.272	3.939	0.8122

Table 2:

The optimal solution is D_i = 1.612, Q_i = 28.58, L_i = $1.31x10^{-5}$ and Min TC = $29.272 which corresponds to maximum membership function 0.8122. It has been seen that as β value increases, the lot size Q, the demand D, lead time L increases whereas the minimum total cost decreases.

7. Conclusion

In this paper we have proposed a concept of the optimal solution of the inventory problem with fuzzy cost price per unit item. An inventory model with demand dependent unit cost and lead time dependent of on leading time crashing cost with limited lot size, warehouse and investment is solved using Karush-Kuhn-Tucker Conditions. Here the optimal solution is calculated with fuzzy unit price per item. The result revels the minimum expected annual total cost of the inventory model. The model can be extended for more than one item. Also it can be solved for various constraints like limited budgetary, set up cost etc.

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