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The Semi inverse of Max-Min Product of Fuzzy Matrices

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Abstract. We introduce the concept of sandwich set of two fuzzy matrices.By using the new concept of sandwich set, we provide a method for finding a semi inverse of max-min product of fuzzy matrices.

Keywords: fuzzy matrix, sandwich set, semi inverse

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1. Introduction

Generalized inverse of matrices and its applications are studied by Rao and Mitra[3].

Ren et.al[4] introduced the concept of sandwich sets of matrices. By using the sandwich sets of matrices, they provided an effective method for finding a semi inverse of product of two matrices.

The Fuzzy matrices are successfully used when fuzzy uncertainty occurs in a problem. Fuzzy matrices become popular for last two decades.

In 1977, Thomasan [5] initiated the study on convergence of powers of fuzzy matrices. Kim and Roush [1] gave a systematic development to fuzzy matrix theory. They introduced various inverses of a fuzzy matrix. Also, they gave algorithms to find the inverse and generalized inverse of a given fuzzy matrix. It is well known that, semi inverse exist for a complex matrix [2],whereas for a fuzzy matrix,this need not be true. On this line, we establish that, if A has a semi inverse and B has a semi inverse, then AB has a semi inverse with some conditions.

2. Prelimineries

A fuzzy algebra is a mathematical system $(F,+,.)$ with two binary operations '+' and '.' defined on the set F by $a+b = max(a,b)$ and $a.b = min(a,b)$. By a fuzzy matrix, we mean a matrix over a fuzzy algebra.Here we confine with matrices over a fuzzy algebra $F = [0,1]$ under the max-min operation and with the usual ordering on real numbers.

Definition 2.1. *[2] Let $\mathcal{F}_{mn}$ denote the set of all $m\times n$ fuzzy matrices over F. If $m=n$, in short we write $\mathcal{F}_n$.*

Definition 2.2. *[2] Let $A = (a_{ij})\in\mathcal{F}_{mn}$ and $B = (b_{ij})\in\mathcal{F}_{mn}$. Then the matrix $A+B=(max\{a_{ij},b_{ij}\})\in\mathcal{F}_{mn}$ is called the sum of A and B.*

Definition 2.3. *[2] For* $A=(a_{ij})\in\mathcal{F}_{mp}$ *and* $B=(b_{ij})\in\mathcal{F}_{pn}$*, the max-min product* $AB=(\max_k\{min\{a_{ik},b_{kj}\}\})\in\mathcal{F}_{mn}$.

Definition 2.4. *[2] A matrix* $A\in\mathcal{F}_n$ *is said to be invertible if and only if there exists* $B\in\mathcal{F}_n$ *such that* $AB=BA=I_n$.

Definition 2.5. *[2] A square matrix is called a permutation matrix if every row and every column contains exactly one* 1 *and all the other entries are* 0.

Definition 2.6. *[2] A fuzzy matrix* A *which satisfies the relation* $A^2=A$ *is called idempotent fuzzy matrix. Let* $\mathcal{E}$ *be the set of all* $n\times n$ *idempotent fuzzy matrices, that is* $\mathcal{E}=\{E/E^2=E, E\in\mathcal{F}_n\}$.

Proposition 2.7. *(pp32[2]) Let* $A\in\mathcal{F}_n$. A *is invertible if and only if* A *is a permutation matrix.In this case, the transpose* A^T *is the unique inverse of* A.

Definition 2.8. *[2] For* $A\in\mathcal{F}_{mn}$ *if there exists* $X\in\mathcal{F}_{nm}$ *such that* $AXA=A$ and $XAX=X$ (2.1), then X is called a semi inverse or $\{1,2\}$ inverse of A, denoted by A^-. We denote the set of all $\{1,2\}$ inverses A^- of a fuzzy matrix A by $\mathcal{F}(A)$.

3. Main Results

In order to obtain a semi inverse for product of fuzzy matrices , we now introduce the following:

Definition 3.1. *Suppose that* E, F *are idempotent fuzzy matrices of order n. Then we call* $S(E,F)=\{G\in\mathcal{E}\mid GE=FG=G \text{ and } EGF=EF\}$*,the sandwich set of fuzzy matrices* E *and* F.

The sandwich set of fuzzy matrices have the following properties.

Proposition 3.2. *(i)* $S(E,F)$ *defined above is non empty.*

(ii) $|S(E,F)|=1$ if and only if $GH=HG$ for any $G,H\in S(E,F)$.

(iii) For any $E\in\mathcal{E}$, $S(E,E)$ contains a unique idempotent fuzzy matrix E, i.e $S(E,E)=\{E\}$.

(iv) Suppose that I is the usual identity fuzzy matrix,then $S(I,I)$ contains a unique identity fuzzy matrix I.

Proof. (i) It is clear that, for any two idempotent fuzzy matrices $E, F \in \mathcal{F}_n$,its product EF is also a fuzzy matrix of order n. Assume that a $\{1,2\}$ inverse of EF exists. Let it be P, i.e $P \in \mathcal{F}(EF)$, $G = FPE$. Then we have

$G^2 = FPE.FPE = F(PEFP)E = FPE = G.$

So that, G is an idempotent fuzzy matrix. Also, by Definition 3.1 and formula (2.1) we can see that,

$GE = (FPE).E = (FP)(EE) = FPE^2 = FPE = G.$

$FG = F(FPE) = (FF)PE = FPE = G$

and $EGF = E(FPE)F = (EF)P(EF) = EF$ since $P \in \mathcal{F}(EF)$.

This proves that $G \in S(E,F)$

and hence the proof is complete.

(ii) First, let $|S(E,F)| = 1$. Let $G \in S(E,F)$ which implies G is idempotent.

$\Rightarrow G.G = G^2 = G$

$\Rightarrow GH = HG$ for any $G, H \in S(E,F)$.

On the other hand, let $GH = HG$ for any $G, H \in S(E,F)$. Then by the Definition 3.1,it is evident that $G, H \in \mathcal{E}$ such that $GE = FG = G$ and $EGF = EF$. Also $HE = FH = H$ and $EHF = EF$. This implies,

$GHG = (GE)H(FG) = G(EHF)G = G(EF)G = (GE)(FG) = G.G = G^2 = G.$

By the same way

$HGH = H$

Therefore

$G = GHG = G(HG) = G(GH) = G^2H = GH = GH^2 = (GH)H = (HG)H = H.$

This proves that $|S(E,F)| = 1$.

(iii) Suppose that, $G \in S(E,E)$. Then by the Definition 3.1, we have $EGE = E^2 = E, EG = GE = G$. Hence $E = (EG)E = GE = G$.

Thus, $S(E,E)$ contains a unique idempotent fuzzy matrix.

(iv) Suppose that, I is the usual identity fuzzy matrix, from (iii), $S(I,I)$ contains a unique identity fuzzy matrix I.

Theorem 3.3. *Suppose that* $A \in \mathcal{F}_{mn}$ *and* $B \in \mathcal{F}_{np}$ *such that* $A^- \in \mathcal{F}(A)$ *and* $B^- \in \mathcal{F}(B)$ *. Then* $B^-GA^- \in \mathcal{F}(AB)$ *for any* $G \in S(A^-A, BB^-)$.

Proof. A^-A and BB^- are both idempotent fuzzy matrices. Taking $A^-A = EBB^- = F$. Then by using the Definition 3.1, for any $G \in S(E,F)$, we have

$$(AB)(B^-GA^-)(AB) = A(BB^-)G(A^-A)B = A(FG)EB = A(GE)B$$
$$= AGB = (AA^-A)G(BB^-B) = A(A^-A)G(BB^-)B = A(EGF)B$$
$$= AEFB = A(A^-A)(BB^-)B = AB.$$

On the other hand,

$(B^-GA^-)(AB)(B^-GA^-) = B^-G(A^-A)(BB^-)GA^- = B^-(GE)(FG)A^-$

$= B^-G^2A^- = B^-GA^-.$

Thus by the semi inverse of a fuzzy matrix, we can see that $B^-GA^- \in \mathcal{F}(AB)$.

Corollary 3.4. *Suppose that* $A \in \mathcal{F}_{mn}$ *and* $B \in \mathcal{F}_{np}$. *If* $A^- \in \mathcal{F}(A)$ *and* $B^- \in \mathcal{F}(B)$ *such that* $A^-A = BB^- = E$. *Then* $B^-A^- \in \mathcal{F}(AB)$.

Proof. $(AB)(B^-A^-)(AB) = A(BB^-)(A^-A)B$

$= AE^2B$

$= AEB$ since A^-A and BB^- are fuzzy idempotent matrices.

$= A(BB^-)B$

$= AB.$

Also $(B^-A^-)(AB)(B^-A^-) = B^-(A^-A)(BB^-)A^-$

$= B^-E^2A^-$

$= B^-EA^-$

$= B^-(A^-A)A^-$

$= B^-A^-.$

Hence, by the definition of the semi inverse of a fuzzy matrix, we have

$B^-A^- \in \mathcal{F}(AB)$.

Corollary 3.5. *Suppose that* $A \in \mathcal{F}_{mn}$ *and* $B \in \mathcal{F}_{np}$. *If* $A^- \in \mathcal{F}(A)$ *and* $B^- \in \mathcal{F}(B)$ *such that* $A^-A = BB^- = I$. *Then* B^-A^- *is a semi inverse of* AB.

Proof. $(AB)(B^-A^-)(AB) = A(BB^-)(A^-A)B = AB.$

Also, $(B^-A^-)(AB)(B^-A^-) = B^-(A^-A)(BB^-)A^- = B^-A^-.$

Hence, by the definition of the semi inverse of a fuzzy matrix, we have B^-A^- is a semi inverse of AB.

Corollary 3.6. *(i) If* A *is an invertible fuzzy matrix of order* n *with the inverse* A^T *and* $B \in \mathcal{F}_{np}$*, then for any* $B^- \in \mathcal{F}(B)$, $B^-A^T \in \mathcal{F}(AB)$.

(ii) If $A \in F_{mn}$ and B is an invertible fuzzy matrix of order n with the inverse B^T, then for any $A^- \in \mathcal{F}(A)$, $B^TA^- \in \mathcal{F}(AB)$.

Proof. (i) Consider,

$$\begin{aligned} AB(B^-A^T)AB &= A(BB^-)(A^TA)B \\ &= A(BB^-)B \\ &= AB. \end{aligned}$$

Also,

$$\begin{aligned}(B^{-}A^{T})(AB)(B^{-}A^{T}) &= B^{-}(A^{T}A)(BB^{-})A^{T} \\ &= B^{-}(BB^{-})A^{T} \\ &= B^{-}A^{T}.\end{aligned}$$

Hence, $B^{-}A^{T} \in \mathcal{F}(AB)$.

(ii) It is similar to (i).

Corollary 3.7. *Suppose that* $A \in \mathcal{F}_{m1}$ *and* $B \in \mathcal{F}_{1p}$. *Then for any* $A^{-} \in \mathcal{F}(A)$ *and* $B^{-} \in \mathcal{F}(B)$, *the product* $B^{-}A^{-}$ *is a semi inverse for* AB.

Proof. It is obvious that, the fuzzy sandwich set $S(A^{-}A, BB^{-})$ contains a unique element 1.

4. Conclution

We extended the concept of sandwich set of two real matrices to the concept of sandwich set of two fuzzy matrices. By using the new concept of sandwich set, we provided a method for finding a semi inverse of max-min product of fuzzy matrices.

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