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Distance Degree Sequence and Distance Neighborhood Degree Sequence of Vertices of Drastic Sum of two Fuzzy Graphs on Stars

***K. Radha*[1*] and *S. R. Gayathri*[2]**

[1,2]PG and Research Department of Mathematics,
Thanthai Periyar Government Arts and Science College
(Autonomous and Affiliated to Bharathidasan University),
Trichy-23, Tamilnadu, India.
[1]Email: radhagac@yahoo.com; [2]Email: srgayathrimaths@gmail.com
*Corresponding author.

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Abstract. The distance degree sequence and distance neighborhood degree sequence are two sequences in fuzzy graphs. In graph theory, the distance degree sequence of a vertex involves the concept of the number of vertices at various distances from the given vertex of a graph. The concept of distance degree sequence of a vertex in fuzzy graph is developed by taking into account the membership values of the edges which are incident on the vertices at various distances from the given vertex of fuzzy graphs, and the distance neighborhood degree sequence considers the membership values of the vertices at various distances. The drastic sum of fuzzy graphs is an operation in fuzzy graph theory, which is the extension of the standard operations on graphs using the drastic sum of fuzzy sets. The drastic sum provides a way to combine two fuzzy graphs into a new fuzzy graph by using a specific rule for combining the membership values of edges and vertices between two fuzzy graphs. In this paper, the distance degree sequence and distance neighborhood degree sequence of the vertices in the drastic sum of two fuzzy graphs on star graphs are obtained in terms of the parameters of the given graphs by considering all possible cases of drastic sum in detail.

Keywords: Fuzzy graphs, distance degree sequence, distance neighborhood degree sequence, drastic sum, Star graph.

AMS Mathematics Subject Classification (2010): 03E72, 05C72, 05C12

1. Introduction

The concept of fuzzy graphs originates from the broader field of fuzzy set theory, which was introduced byRosenfiled in 1975 [11]. In graph theory, the degree sequence of a graph is a fundamental concept that refers to the sequence of vertex degrees (i.e., the number of edges incident to each vertex) in a non-decreasing order. The distance degree sequence (DDS) is an extension of this idea, introduced by Huilgol in the year 2019 [12]. The drastic sum and drastic product of fuzzy sets were introduced by Dubois in the year 1980 [14].

Masaharu Mizumoto, in the year 1981 [13], studied the algebraic properties of these two operations. The distance degree sequence (DDS), distance neighborhood degree sequence (DnDS) and the drastic operations are introduced in fuzzy graphs by Radha and Gayathri [10].

2. Methodology

In this section, the basic definitions used in this paper are given.

Definition 2.1. Consider a fuzzy graph G:(σ, μ) where G*(V, E) is the underlying crisp graph and V = $\{v_1, v_2, \cdots, v_n\}$. The DDS of a vertex v_i in G is given by the sequence $(d_{i_0}, d_{i_1}, d_{i_2} \dots, d_{i_j} \dots, d_{i_{e(v_i)}})$ where d_{i_0} is given by σ(v_i), d_{i_1} is given by $\sum_{u \in N_1(v_i)} \mu(uv_i)$, d_{i_j} is given by $\sum_{u \in N_j(v_i)} \mu(wu)$ where wu is the edge adjacent to u in the $v_i - u$ path of length *j* in G* for every *j*≠ 0, *j*≠ 1. Here $e(v_i)$ is the eccentricity of the vertex $v_i$in G*.

The DDS(G) with n vertices is given by the n-tuple of DDS of the n vertices of G and is denoted by $\text{DDS}_{\text{G}}(v_1, v_2, v_3, \cdots, v_n)$.[2]

Definition 2.2. Let G:(σ,μ) denote a fuzzy graph with the underlying crisp graph G* having vertex set V = $\{v_1, v_2, \cdots, v_n\}$. The DnDS of vertex $v_i$in G, denoted by DnDS(v_i), is given by the sequence $(nd_{i_0}, nd_{i_1}, nd_{i_2}, \cdots, nd_{i_j} \cdots, nd_{i_{e(v_i)}})$, where nd_{i_0} is given by σ(v_i), nd_{i_j} is given by $\sum_{u \in N_j(v_i)} \sigma(u)$, where i = 1, 2, 3, ⋯, n, *j* = 2, 3, ⋯, e(v_i), e(v_i) is the eccentricity of v_i and $N_j(v_i)$ is the j^{th} neighborhood of v_i in G*.

The DnDS(G) with n vertices is given by n-tuple of DnDS of the n vertices of G and is denoted by DnDS$(v_1, v_2, v_3, \cdots, v_n)$.[2]

Definition 2.3. Let $G_1 : (\sigma_1, \mu_1)$ and $G_2 : (\sigma_2, \mu_2)$ be two fuzzy graphs with underlying crisp graphs $G_1^* : (V_1, E_1)$ and $G_2^* : (V_1, E_2)$ respectively. The drastic sum of $G_1$and G_2 is given by $(G_1 \hat{\cup} G_2)$:($\sigma_1 \hat{\cup} \sigma_2$, $\mu_1 \hat{\cup} \mu_2$),

$$(\sigma_1 \hat{\cup} \sigma_2)(u) = \begin{cases} \sigma_1(u), & if\ u \epsilon V_1 - V_2 \\ \sigma_2(u), & if\ u \epsilon V_2 - V_1 \\ 1, & if\ u \in V_1 \cap V_2 \end{cases}$$

$$(\mu_1 \hat{\cup} \mu_2)(uv) = \begin{cases} \mu_1(uv), & if\ uv \epsilon E_1 - E_2 \\ \mu_2(uv), & if\ uv \epsilon E_2 - E_1 \text{"[9]} \\ 1, & if\ uv \in E_1 \cap E_2 \end{cases}$$

$(G_1 \hat{\cup} G_2)$:($\sigma_1 \hat{\cup} \sigma_2$, $\mu_1 \hat{\cup} \mu_2$) is also denoted as $(G_1 \hat{\cup} G_2)$:($\sigma_{G_1 \hat{\cup} G_2}$, $\mu_{G_1 \hat{\cup} G_2}$).

3. Results and discussions

In this section, the DDS and DnDS of Drastic sum of fuzzy graphs on stars in all possible cases are discussed.

3.1. DDS of drastic sum of fuzzy graphs on star graphs

In this section, the DDS of the vertices in drastic sum of two fuzzy graphs on star graphs are obtained in terms of the parameters of the given graphs by considering all possible combinations of star graphs for the drastic sum.

Theorem 3.1.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ have vertex sets $\{\varepsilon,\varepsilon_1,\varepsilon_2,\dots,\varepsilon_n\}$ and $\{\varepsilon,\vartheta_1,\vartheta_2,\dots,\vartheta_m\}$ respectively, where ε is the apex vertex of both G_1 and G_2 and $\varepsilon_i \neq \vartheta_j$ for any i and j. Then the DDS of vertices in the drastic sum are as follows:
DDS(ε)= $(1, d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon))$,
For $i=1,2,\dots,n$, DDS(ε_i)= $\left(\sigma_1(\varepsilon_i), d_{G_1}(\varepsilon_i), d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)-\mu_1(\varepsilon\varepsilon_i)\right)$,
For $j=1,2,\dots,m$, DDS(ϑ_j)= $\left(\sigma_2(\vartheta_j), d_{G_2}(\vartheta_j), d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)-\mu_2(\varepsilon\vartheta_j)\right)$.

Proof: Since the apex vertex ε is the only common vertex of G_1 and G_2, the drastic sum of G_1 and G_2 is also a star with apex vertex ε and pendant vertices $\varepsilon_1,\varepsilon_2,\dots,\varepsilon_n,\vartheta_1,\vartheta_2,\dots,\vartheta_m$.
Also, there is no edges in $E_1\cap E_2$. By the definition of the drastic sum,
$(\sigma_{G_1\hat{\cup}G_2})(\varepsilon)=1$, $(\sigma_{G_1\hat{\cup}G_2})(\varepsilon_i)=\sigma_1(\varepsilon_i)$, for all i and $(\sigma_{G_1\hat{\cup}G_2})(\vartheta_j)=\sigma_2(\vartheta_j)$, for all j.
$(\mu_{G_1\hat{\cup}G_2})(\varepsilon\varepsilon_i)=\mu_1(\varepsilon\varepsilon_i)$ for all i and $(\mu_{G_1\hat{\cup}G_2})(\varepsilon\vartheta_j)=\mu_2(\varepsilon\vartheta_j)$ for all j.
Let the vertices of $G_1\hat{\cup}G_2$ be arranged as $\varepsilon,\varepsilon_1,\dots,\varepsilon_n,\vartheta_1,\vartheta_2,\dots,\vartheta_m$.
Consider the vertex ε. The eccentricity of ε in $(G_1\hat{\cup}G_2)^*$ is 1.

$$d_{i_0}(\varepsilon)=(\sigma_{G_1\hat{\cup}G_2})(\varepsilon)=1$$

$d_{i_1}(\varepsilon)=\sum_{\varepsilon w\in E_1\cup E_2}(\mu_{G_1\hat{\cup}G_2})(\varepsilon w)=\sum_{i=1}^{n}\mu_1(\varepsilon\varepsilon_i)+\sum_{j=1}^{m}\mu_2(\varepsilon\vartheta_j)=d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)$.
Therefore, DDS(ε)= $(1,\ d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon))$.
For any $i=1,2,\dots,n$, the eccentricity of ε_i is 2.
$d_{(i+1)_0}(\varepsilon_i)=(\sigma_{G_1\hat{\cup}G_2})(\varepsilon_i)=\sigma_1(\varepsilon_i)$, $\quad d_{(i+1)_1}(\varepsilon_i)=(\mu_{G_1\hat{\cup}G_2})(\varepsilon\varepsilon_i)=\mu_1(\varepsilon\varepsilon_i)=d_{G_1}(\varepsilon_i)$
The paths of length 2 are $\varepsilon_i\varepsilon\varepsilon_k, k=1,2,\dots,n$, k$\neq i$ and $\varepsilon_i\varepsilon\vartheta_j, j=1,2,\dots,m$. Therefore
$d_{(i+1)_2}(\varepsilon_i)=\sum_{\substack{k=1\\k\neq i}}^{n}(\mu_{G_1\hat{\cup}G_2}))(\varepsilon\varepsilon_k)+\sum_{j=1}^{m}(\mu_{G_1\hat{\cup}G_2}))(\varepsilon\vartheta_j)=\sum_{\substack{k=1\\k\neq i}}^{n}\mu_1(\varepsilon\varepsilon_k)+\sum_{j=1}^{m}\mu_2(\varepsilon\vartheta_j)$
$=\sum_{k=1}^{n}\mu_1(\varepsilon\varepsilon_k)-\mu_1(\varepsilon\varepsilon_i)+\sum_{j=1}^{m}\mu_2(\varepsilon\vartheta_j)=d_{G_1}(\varepsilon)-\mu_1(\varepsilon\varepsilon_i)+d_{G_2}(\varepsilon)$
Therefore, DDS(ε_i)= $\left(\sigma_1(\varepsilon_i), d_{G_1}(\varepsilon_i), d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)-\mu_1(\varepsilon\varepsilon_i)\right)$
For any $j=1,2,\dots,m$, the eccentricity of ϑ_j is 2.
$d_{(j+n+1)_0}(\vartheta_j)=(\sigma_{G_1\hat{\cup}G_2})(\vartheta_j)=\sigma_2(\vartheta_j)$,
$d_{(j+n+1)_1}(\vartheta_j)=(\mu_{G_1\hat{\cup}G_2})(\varepsilon\vartheta_j)=\mu_2(\varepsilon\vartheta_j)=d_{G_2}(\vartheta_j)$.
The paths of length 2 are $\vartheta_j\varepsilon\varepsilon_i, i=1,2,\dots,n,$ and $\vartheta_j\varepsilon\vartheta_k, k=1,2,\dots,m, k\neq j$.
$d_{(j+n+1)_2}(\vartheta_j)=\sum_{i=1}^{n}(\mu_{G_1\hat{\cup}G_2})(\varepsilon\varepsilon_i)+\sum_{\substack{k=1\\k\neq j}}^{m}(\mu_{G_1\hat{\cup}G_2})(\varepsilon\vartheta_k)=\sum_{i=1}^{n}\mu_1(\varepsilon\varepsilon_i)+$
$\sum_{\substack{k=1\\k\neq j}}^{m}\mu_2(\varepsilon\vartheta_k)$
$=\sum_{i=1}^{n}\mu_1(\varepsilon\varepsilon_i)+\sum_{k=1}^{m}\mu_2(\varepsilon\vartheta_k)-\mu_2(\varepsilon\vartheta_j)=d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)-\mu_2(\varepsilon\vartheta_j)$
Therefore, DDS(ϑ_j)= $\left(\sigma_2(\vartheta_j), d_{G_2}(\vartheta_j), d_{G_1}(\varepsilon)+d_{G_2}(\varepsilon)-\mu_2(\varepsilon\vartheta_j)\right)$.

Theorem 3.1.2. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ have κ common end vertices with vertex sets $\{\varepsilon,w_1,w_2,\dots,w_\kappa,\varepsilon_{\kappa+1},\dots,\varepsilon_n\}$ and $\{\vartheta,w_1,w_2,\dots,w_\kappa,\vartheta_{\kappa+1},\dots,\vartheta_m\}$, where ε and ϑ are the apex vertices of G_1 and G_2 respectively. Then the DDS of drastic sum are as follows:
DDS(ε)= $\left(\sigma_1(\varepsilon), d_{G_1}(\varepsilon), \sum_{j=1}^{\kappa}d_{G_2}(w_j), \sum_{j=\kappa+1}^{m}d_{G_2}(\vartheta_j)\right)$
DDS(ϑ)= $\left(\sigma_2(\vartheta), d_{G_2}(\vartheta), \sum_{i=1}^{\kappa}d_{G_1}(w_i), \sum_{i=\kappa+1}^{n}d_{G_1}(\varepsilon_i)\right)$
For $i=1,2,\dots,\kappa$, DDS(w_i)= $\left(1, d_{G_1}(w_i)+d_{G_2}(w_i), \sum_{l=\kappa+1}^{n}d_{G_1}(\varepsilon_l), \sum_{j=\kappa+1}^{m}d_{G_2}(\vartheta_j)\right)$
For $i=\kappa+1,\kappa+2,\dots,n$,

DDS(ε_i)= $\left(\sigma_1(\varepsilon_i), d_{G_1}(\varepsilon_i), d_{G_1}(\varepsilon) - d_{G_1}(\varepsilon_i), \sum_{j=1}^{\kappa} d_{G_2}(w_j), \sum_{j=\kappa+1}^{m} d_{G_2}(\vartheta_j)\right)$

For $j = \kappa+1, \kappa+2, \dots, m$,

DDS(ϑ_j)= $\left(\sigma_2(\vartheta_j), d_{G_2}(\vartheta_j), d_{G_2}(\vartheta) - d_{G_2}(\vartheta_j), \sum_{i=1}^{\kappa} d_{G_1}(w_i), \sum_{i=\kappa+1}^{m} d_{G_1}(\varepsilon_i)\right)$.

Proof: Since $w_1, w_2, \dots, w_\kappa$ are the only κ common vertices of G_1 and G_2, and no two w_i's are adjacent, there is no common edge between G_1 and G_2.

Therefore, by the definition of the drastic sum,

$$(\sigma_{G_1 \hat{\cup} G_2})(w_i) = 1, i = 1,2,\dots,\kappa$$

$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = \sigma_1(\varepsilon)$, $(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i), i = \kappa+1, \kappa+2, \dots, n$,

$(\sigma_{G_1 \hat{\cup} G_2})(\vartheta) = \sigma_2(\vartheta)$, $(\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sigma_2(\vartheta_j), j = \kappa+1, \kappa+2, \dots, m$,

$\left(\mu_{G_1 \hat{\cup} G_2}\right)(e) = \mu_1(e)$ for every edge e in G_1

and $\left(\mu_{G_1 \hat{\cup} G_2}\right)(e) = \mu_2(e)$ for every edge e in G_2.

Let the vertices be arranged in the order $\varepsilon, \vartheta, w_1, \dots, w_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n, \vartheta_{\kappa+1}, \dots, \vartheta_m$.

Consider the vertex ε. The eccentricity of ε in$(G_1 \hat{\cup} G_2)^*$ is 4.

$d_{i_0}(\varepsilon) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = \sigma_1(\varepsilon)$.

$d_{i_1}(\varepsilon) = \sum_{\varepsilon x \epsilon E_1 \cup E_2} \left(\mu_{G_1 \hat{\cup} G_2}\right)(\varepsilon x) = \sum_{\varepsilon x \in E_1} \mu_1(\varepsilon x) = d_{G_1}(\varepsilon)$.

$\varepsilon w_j \vartheta$ is the path of length 2 for $j = 1,2,\dots,\kappa$. Therefore

$d_{i_2}(\varepsilon) = \sum_{j=1}^{\kappa} \left(\mu_{G_1 \hat{\cup} G_2}\right)(w_j \vartheta)$

$= \sum_{j=1}^{\kappa} \mu_1(w_j \vartheta)$

$= \sum_{j=1}^{\kappa} d_{G_2}(w_j)$

$\varepsilon w_i \vartheta \vartheta_j$ is the path of length 3 for $i = 1,2,\dots,\kappa$ and $j = \kappa+1, \kappa+2, \dots, m$.

Therefore,

$$d_{i_3}(\varepsilon) = \sum_{j=\kappa+1}^{m} \left(\mu_{G_1 \hat{\cup} G_2}\right)(\vartheta \vartheta_j) = \sum_{j=\kappa+1}^{m} \mu_2(\vartheta \vartheta_j) = \sum_{j=\kappa+1}^{m} d_{G_2}(\vartheta_j)$$

Therefore, DDS(ε)= $\left(\sigma_1(\varepsilon), d_{G_1}(\varepsilon), \sum_{j=1}^{\kappa} d_{G_2}(w_j), \sum_{j=\kappa+1}^{m} d_{G_2}(\vartheta_j)\right)$.

Similarly, DDS(ϑ)= $\left(\sigma_2(\vartheta), d_{G_2}(\vartheta), \sum_{i=1}^{\kappa} d_{G_1}(w_i), \sum_{i=\kappa+1}^{m} d_{G_1}(\varepsilon_i)\right)$.

Consider the vertex $w_i, i = 1,2,\dots,\kappa$. The eccentricity of w_i in the underlying graph $(G_1 \hat{\cup} G_2)^*$ of the drastic sum is 2.

For $i = 1,2,\dots,\kappa$,

$$d_{(i+2)_0}(w_i) = (\sigma_{G_1 \hat{\cup} G_2})(w_i) = 1$$

$d_{(i+2)_1}(w_i) = \sum_{w_i x \epsilon E_1 \cup E_2} \left(\mu_{G_1 \hat{\cup} G_2}\right)(w_i x) = \mu_1(\varepsilon w_i) + \mu_2(\vartheta w_i) = d_{G_1}(w_i) + d_{G_2}(w_i)$.

The paths of length 2 are $w_i \varepsilon \varepsilon_l, l = \kappa+1, \kappa+2, \dots, n$ and $w_i \vartheta \vartheta_j, j = \kappa+1, \kappa+2, \dots, m$.

Therefore, for $i = 1,2,\dots,\kappa$,

$d_{(i+1)_2}(w_i) = \sum_{l=\kappa+1}^{n} \left(\mu_{G_1 \hat{\cup} G_2}\right))(\varepsilon \varepsilon_l) + \sum_{j=\kappa+1}^{m} \left(\mu_{G_1 \hat{\cup} G_2}\right))(\vartheta \vartheta_j)$

$= \sum_{l=\kappa+1}^{n} \mu_1(\varepsilon \varepsilon_l) + \sum_{j=\kappa+1}^{m} \mu_2(\vartheta \vartheta_j) = \sum_{l=\kappa+1}^{n} d_{G_1}(\varepsilon_l) + \sum_{j=\kappa+1}^{m} d_{G_1}(\vartheta_j)$.

Therefore, $i = 1, 2, \dots, \kappa$,

DDS(w_i)= $\left(1, d_{G_1}(w_i) + d_{G_2}(w_i), \sum_{l=\kappa+1}^{n} d_{G_1}(\varepsilon_l) + \sum_{j=\kappa+1}^{m} d_{G_1}(\vartheta_j)\right)$

Consider the vertex ε_i, $i = \kappa+1, \kappa+2, \dots, n$.

The eccentricity of ε_i in the drastic sum is 4.

$d_{(i+\kappa+2)_0}(\varepsilon_i) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$.

$d_{(i+\kappa+2)_1}(\varepsilon_i) = \sum_{\varepsilon_i x \epsilon E_1 \cup E_2} \left(\mu_{G_1 \hat{\cup} G_2}\right)(\varepsilon_i x) = \left(\mu_{G_1 \hat{\cup} G_2}\right)(\varepsilon_i \varepsilon) = \mu_1(\varepsilon \varepsilon_i) = d_{G_1}(\varepsilon_i)$.

The paths of length 2 are $\varepsilon_i \varepsilon w_j, j = 1,2, \dots, \kappa$ and $\varepsilon_i \varepsilon \varepsilon_j, j = \kappa+1, \kappa+2, \dots, n, j \neq i$. Therefore,

$$d_{(i+\kappa+1)_2}(\varepsilon_i) = \sum_{j=1}^{\kappa} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon w_j) + \sum_{j=\kappa+1}^{m} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_j)$$

$= \sum_{j=1}^{\kappa} \mu_1(\varepsilon w_j) + \sum_{\substack{j=\kappa+1 \\ j \neq i}}^{m} \mu_1(\varepsilon \varepsilon_j) = d_{G_1}(\varepsilon) - \mu(\varepsilon \varepsilon_i) = d_{G_1}(\varepsilon) - d_{G_1}(\varepsilon_i)$.

The paths of length 3 from ε_i in the drastic sum are $\varepsilon_i \varepsilon w_j \vartheta, j = 1,2, \dots, \kappa$. Therefore,
$d_{(i+\kappa+1)_3}(\varepsilon_i) = \sum_{j=1}^{\kappa} (\mu_{G_1 \hat{\cup} G_2})(w_j \vartheta) = \sum_{j=1}^{\kappa} \mu_2(w_j \vartheta) = \sum_{j=1}^{\kappa} d_{G_2}(w_j)$.
The paths of length 4 from ε_i in the drastic sum are $\varepsilon_i \varepsilon w_l \vartheta \vartheta_j, l = 1,2, \dots, \kappa$, $j = \kappa+1, \kappa+2, \dots, m$. Therefore,
$d_{(i+\kappa+1)_4}(\varepsilon_i) = \sum_{j=\kappa+1}^{m} (\mu_{G_1 \hat{\cup} G_2})(\vartheta_j \vartheta) = \sum_{j=\kappa+1}^{m} \mu_2(\vartheta_j \vartheta) = \sum_{j=\kappa+1}^{m} d_{G_2}(\vartheta_j)$.
Hence, for $i = \kappa+1, \kappa+2, \dots, n$,
DDS(ε_i)=$\left(\sigma_1(\varepsilon_i), d_{G_1}(\varepsilon_i), d_{G_1}(\varepsilon) - d_{G_1}(\varepsilon_i), \sum_{j=1}^{\kappa} d_{G_2}(w_j), \sum_{j=\kappa+1}^{m} d_{G_2}(\vartheta_j)\right)$
Similarly, for $j = \kappa+1, \kappa+2, \dots, m$,
DDS(ϑ_j)=$\left(\sigma_2(\vartheta_j), d_{G_2}(\vartheta_j), d_{G_2}(\vartheta) - d_{G_2}(\vartheta_j), \sum_{i=1}^{\kappa} d_{G_1}(w_i), \sum_{i=\kappa+1}^{n} d_{G_1}(\varepsilon_i)\right)$.

Theorem 3.1.3. Let $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ have $n+1$ and $\kappa+1$ vertices respectively ($n > \kappa$) such that G_2 is a fuzzy subgraph of G_1.Then the DDS of drastic sum are as follows:
If ε is the apex vertex of G_1, DDS(ε)= $\left(1, \kappa, \sum_{\vartheta \in V_1 - V_2} d_{G_1}(\vartheta)\right)$.
For $\vartheta \in V_1 \cap V_2$, DDS($\vartheta$)= $\left(1, 1, \kappa - 1 + \sum_{w \in V_1 - V_2} d_{G_1}(w)\right)$.
For $\vartheta \in V_1 - V_2$, DDS(ϑ)= $\left(\sigma_1(\vartheta), d_{G_1}(\vartheta), \kappa + \sum_{\substack{w \neq \vartheta \\ w \in V_1 - V_2}} d_{G_1}(w)\right)$.
Proof: Let $\{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ be the vertex set of G_1. Since both G_1 and G_2 are fuzzy graphs on star graphs and G_2 is a fuzzy subgraph of G_1, ε is also the apex vertex of G_2.
Without loss of generality, let $V_2 = \{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa\}$ be the vertex set of G_2 and let $V_1 = \{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n\}$ be the vertex set of G_1. (This can be done by relabeling the vertices if necessary.)
Then, $\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa \in V_1 \cap V_2$ and $\varepsilon_{\kappa+1}, \varepsilon_{\kappa+2}, \dots, \varepsilon_n \in V_1 - V_2$.
By the definition of the drastic sum,

$$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1, (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1, i = 1,2, \dots, \kappa$$

$$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i), i = \kappa+1, \kappa+2, \dots, n$$

$$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_i) = 1, i = 1,2, \dots, \kappa$$

$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_i) = \mu_1(\varepsilon \varepsilon_i), i = \kappa+1, \kappa+2, \dots, n$.
Here, the drastic sum $G_1 \hat{\cup} G_2$ is a fuzzy graph on a star graph with vertices $\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n$ having ε as the apex vertex.
Consider the vertex ε. The eccentricity of ε in the drastic sum is 1.
$d_{i_0}(\varepsilon) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
$d_{i_1}(\varepsilon) = \sum_{i=1}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_i) = \sum_{i=1}^{\kappa} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_i) + \sum_{i=\kappa+1}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon \varepsilon_i)$
$= \sum_{i=1}^{\kappa} 1 + \sum_{i=\kappa+1}^{n} \mu_1(\varepsilon \varepsilon_i) = \kappa + \sum_{i=\kappa+1}^{n} d_{G_1}(\varepsilon_i)$

Therefore, DDS(ε)= $(1, \kappa + \sum_{i=\kappa+1}^{n} d_{G_1}(\varepsilon_i))$.
Consider the vertex $\varepsilon_i, i = 1,2, \dots, \kappa$. The eccentricity of ε_i in the drastic sum is 2.
$d_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1$; $\quad d_{(i+1)_1}(\varepsilon_i)$=$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_i) = 1$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are paths of length 2. Therefore,
$d_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j\neq i}}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j) = \sum_{\substack{j=1 \\ j\neq i}}^{\kappa} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j) + \sum_{j=\kappa+1}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j)$
$=\sum_{\substack{j=1 \\ j\neq i}}^{\kappa} 1 + \sum_{j=\kappa+1}^{n} \mu_1(\varepsilon\varepsilon_j) = \kappa - 1 + \sum_{j=\kappa+1}^{n} d_{G_1}(\varepsilon_j)$.
Therefore for $i = 1,2, \dots, \kappa$, DDS(ε_i)= $\left(1,1, \kappa - 1 + \sum_{j=\kappa+1}^{n} d_{G_1}(\varepsilon_j)\right)$.
Consider the vertex $\varepsilon_i, i = \kappa + 1, \kappa + 2, \dots, n$. The eccentricity of ε_i in the drastic sum is 2.
$d_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$
$d_{(i+1)_1}(\varepsilon_i)$=$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_i) = \mu_1(\varepsilon\varepsilon_i) = d_{G_1}(\varepsilon_i)$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are paths of length 2 from ε_i. Therefore,
$d_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j\neq i}}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j) = \sum_{j=1}^{\kappa} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j) + \sum_{j=\kappa+1}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j)$
$=\sum_{j=1}^{\kappa} 1 + \sum_{\substack{j=\kappa+1 \\ j\neq i}}^{n} \mu_1(\varepsilon\varepsilon_j) = \kappa + \sum_{\substack{j=\kappa+1 \\ j\neq i}}^{n} d_{G_1}(\varepsilon_j)$.
Therefore for $i = \kappa + 1, \kappa + 2, \dots, n$, DDS($\varepsilon_i$)= $(\sigma_1(\varepsilon_i), d_{G_1}(\varepsilon_i), \kappa + \sum_{\substack{j=\kappa+1 \\ j\neq i}}^{n} d_{G_1}(\varepsilon_j))$.

Theorem 3.1.4. Let G_1:(σ_1, μ_1) and G_2: (σ_2, μ_2) have same vertex sets $(\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$. Then the DDS of the vertices in the drastic sum are
DDS(ε) = $(1, n)$ and DDS(ε_i) = $(1,1, n - 1), i = 1,2, \dots, n$
Proof: Since G_1 and G_2 are fuzzy graphs on same star graph, by the definition of the drastic sum $(\sigma_{G_1 \hat{\cup} G_2})(\vartheta) = 1$ for all vertices ϑ and$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_i) = 1$ for all edges $\varepsilon\varepsilon_i$.
Consider the vertex ε. The eccentricity of the vertex ε is 1.
$d_{1_0}(\varepsilon)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$; $\quad d_{1_1}(\varepsilon_i)$=$\sum_{i=1}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_i) = \sum_{i=1}^{n} 1 = n$
Therefore, DDS(ε) = $(1, n)$
Consider the vertex $\varepsilon_i, i = 1,2, \dots, n$. The eccentricity of ε_i in the drastic sum is 2.
$d_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1$; $\quad d_{(i+1)_1}(\varepsilon_i)$=$(\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_i) = 1$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are paths of length 2 from ε_i in the drastic sum. Therefore,

$$d_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j\neq i}}^{n} (\mu_{G_1 \hat{\cup} G_2})(\varepsilon\varepsilon_j) = \sum_{\substack{j=1 \\ j\neq i}}^{n} 1 = n - 1$$

Therefore, DDS(ε_i)= $(1,1, n - 1), i = 1,2, \dots, n$.

Remark 3.1.5. If G_1 and $G_2$are two different fuzzy graphs with no vertex in common, then the DDS of a vertex ϑ in the drastic sum is,

$$DDS_{G_1 \hat{\cup} G_2}(\vartheta) = \begin{cases} DDS_{G_1}(\vartheta), \text{if } \vartheta \in V_1 \\ DDS_{G_2}(\vartheta), \text{if } \vartheta \in V_2 \end{cases}$$

3.2. DnDS of drastic sum of fuzzy graphs on stars

In this section, the DnDS of the vertices in drastic sum of two fuzzy graphs on star graphs are obtained in terms of the parameters of the given graphs.

Theorem 3.2.1. Let G_1 and G_2 have vertex sets $\{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ and $\{\varepsilon, \vartheta_1, \vartheta_2, \dots, \vartheta_m\}$ respectively, where ε is the apex vertex of both $G_1$and G_2 and $\varepsilon_i \neq \vartheta_j$ for any i and j. Then the DnDS of drastic sum are as follows:
DnDS(ε)= $(1, nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon))$
For $i = 1,2, \dots, n$, DnDS(ε_i)= $\left(\sigma_1(\varepsilon_i), 1, nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon) - \sigma_1(\varepsilon_i)\right)$
For $j = 1,2, \dots, m$, DnDS(ϑ_j)= $\left(\sigma_2(\vartheta_j), 1, nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon) - \sigma_2(\vartheta_j)\right)$
Proof: Since the apex vertex ε is the only common vertex of G_1 and G_2, the drastic sum of $G_1$and G_2 is also a star with apex vertex ε and vertices $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n, \vartheta_1, \vartheta_2, \dots, \vartheta_m$. Also, there is no edges in $E_1 \cap E_2$. By the definition of the drastic sum,
$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$, $(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$, for all i and $(\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sigma_2(\vartheta_j)$, for all j.
Let the vertices of $G_1 \hat{\cup} G_2$ be arranged as $\varepsilon, \varepsilon_1, \dots, \varepsilon_n, \vartheta_1, \vartheta_2, \dots, \vartheta_m$.
Consider the vertex ε. The eccentricity of ε in $(G_1 \hat{\cup} G_2)^*$ is 1.

$$nd_{i_0}(\varepsilon) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$$

$nd_{i_1}(\varepsilon) = \sum_{w \epsilon V_1 \cup V_2} (\sigma_{G_1 \hat{\cup} G_2})(w) = \sum_{i=1}^{n} \sigma_1(\varepsilon_i) + \sum_{j=1}^{m} \sigma_2(\vartheta_j) = nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon)$.
Therefore, DnDS(ε)= $(1, nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon))$.
For any $i = 1,2, \dots, n$, the eccentricity of ε_i is 2.
$nd_{(i+1)_0}(\varepsilon_i) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$, $\quad nd_{(i+1)_1}(\varepsilon_i) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
The paths of length 2 are $\varepsilon_i \varepsilon \varepsilon_\kappa, \kappa = 1,2, \dots, n, \kappa \neq i$ and $\varepsilon_i \varepsilon \vartheta_j, j = 1,2, \dots, m$.
$nd_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{\kappa=1 \\ \kappa \neq i}}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_\kappa) + \sum_{j=1}^{m} (\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sum_{\substack{\kappa=1 \\ \kappa \neq i}}^{n} \sigma_1(\varepsilon_\kappa) + \sum_{j=1}^{m} \sigma_2(\vartheta_j)$
$= nd_{G_1}(\varepsilon) - \sigma_1(\varepsilon \varepsilon_i) + nd_{G_2}(\varepsilon)$
Therefore, DnDS(ε_i)= $\left(\sigma_1(\varepsilon_i), 1, nd_{G_1}(\varepsilon) + nd_{G_2}(\varepsilon) - \sigma_1(\varepsilon_i)\right)$.
For any $j = 1,2, \dots, m$, the eccentricity of ϑ_j is 2.
$nd_{(j+n+1)_0}(\vartheta_j) = (\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sigma_2(\vartheta_j)$; $\quad nd_{(j+n+1)_1}(\vartheta_j) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
The paths of length 2 are $\vartheta_j \varepsilon \varepsilon_i, i = 1,2, \dots, n,$ and $\vartheta_j \varepsilon \vartheta_\kappa, \kappa = 1,2, \dots, m, \kappa \neq j$.
$nd_{(j+n+1)_2}(\vartheta_j) = \sum_{i=1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) + \sum_{\substack{\kappa=1 \\ \kappa \neq j}}^{m} (\sigma_{G_1 \hat{\cup} G_2})(\vartheta_\kappa) = \sum_{i=1}^{n} \sigma_1(\varepsilon_i) + \sum_{\substack{\kappa=1 \\ \kappa \neq j}}^{m} \sigma_2(\vartheta_j)$
$= nd_{G_1}(\varepsilon) - \sigma_2(\vartheta_j) + nd_{G_2}(\varepsilon)$
Therefore, DnDS(ϑ_j)= $(\sigma_2(\vartheta_j), 1, nd_{G_1}(\varepsilon) - \sigma_2(\vartheta_j) + nd_{G_2}(\varepsilon))$

Theorem 3.2.2. Let G_1:(σ_1, μ_1) and G_2: (σ_2, μ_2) have κ common end vertices with vertex sets $(\varepsilon, w_1, w_2, \dots, w_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n)$ and $(\vartheta, w_1, w_2, \dots, w_\kappa, \vartheta_{\kappa+1}, \dots, \vartheta_m)$, where ε and ϑ are the apex vertices of $G_1$and G_2 respectively. Then the DnDS of drastic sum are as follows:
DnDS(ε)= $(\sigma_1(\varepsilon), \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i), \sigma_2(\vartheta), \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j))$
DnDS(ϑ)= $(\sigma_2(\vartheta), \kappa + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j), \sigma_1(\varepsilon), \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$
For $i = 1,2, \dots, \kappa$,
DDS(w_i)= $(1, \sigma_1(\varepsilon) + \sigma_2(\vartheta), \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i) + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j) + \kappa - 1)$

For $i = \kappa+1, \kappa+2, \dots, n$,
DnDS(ε_i)= $(\sigma_1(\varepsilon_i), \sigma_1(\varepsilon), \kappa + \sum_{\substack{l=\kappa+1 \\ l\neq i}}^{n} \sigma_1(\varepsilon_l), \sigma_2(\vartheta), \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j))$
For $j = \kappa+1, \kappa+2, \dots, m$,
DnDS(ϑ_j)= $(\sigma_2(\vartheta_j), \sigma_2(\vartheta), \kappa + \sum_{\substack{l=\kappa+1 \\ l\neq j}}^{m} \sigma_2(\vartheta_l), \sigma_1(\varepsilon), \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$.

Proof: Since $w_1, w_2, \dots, w_\kappa$ are the only κ common vertices of G_1 and G_2. Therefore, by the definition of the drastic sum,
$(\sigma_{G_1 \hat{\cup} G_2})(w_i) = 1, i = 1,2,\dots,\kappa$; $(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = \sigma_1(\varepsilon)$,
$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i), i = \kappa+1, \kappa+2, \dots, n$,
$(\sigma_{G_1 \hat{\cup} G_2})(\vartheta) = \sigma_2(\vartheta)$; $(\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sigma_2(\vartheta_j), j = \kappa+1, \kappa+2, \dots, m$,
Let the vertices be arranged in the order $\varepsilon, \vartheta, w_1, \dots, w_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n, \vartheta_{\kappa+1}, \dots, \vartheta_m$.
Consider the vertex ε. The eccentricity of ε in $(G_1 \hat{\cup} G_2)^*$ is 3.

$$nd_{i_0}(\varepsilon) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = \sigma_1(\varepsilon)$$

$$nd_{i_1}(\varepsilon) = \sum_{x \epsilon N_1(\varepsilon)} (\sigma_{G_1 \hat{\cup} G_2})(x) = \sum_{i=1}^{\kappa} (\sigma_{G_1 \hat{\cup} G_2})_1(w_i) + \sum_{i=\kappa+1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i)$$

$= \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i)$
$\varepsilon w_j \vartheta$ is the path of length 2 for $j = 1,2,\dots,\kappa$. Therefore
$nd_{i_2}(\varepsilon) = \sigma_2(\vartheta)$.
$\varepsilon w_i \vartheta \vartheta_j$ is the path of length 3 for $i = 1,2,\dots,\kappa$ and $j = \kappa+1, \kappa+2, \dots, m$.
Therefore,

$$nd_{i_3}(\varepsilon) = \sum_{j=\kappa+1}^{m} (\sigma_{G_1 \hat{\cup} G_2})(\vartheta_j) = \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

Therefore, DnDS(ε)= $(\sigma_1(\varepsilon), \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i), \sigma_2(\vartheta), \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j))$
Similarly, DnDS(ϑ)= $(\sigma_2(\vartheta), \kappa + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j), \sigma_1(\varepsilon), \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$
Consider the vertex $w_i, i = 1,2,\dots,\kappa$. The eccentricity of w_i in the underlying graph $(G_1 \hat{\cup} G_2)^*$ of the drastic sum is 2.
For $i = 1,2,\dots,\kappa$,
$nd_{(i+2)_0}(w_i) = (\sigma_{G_1 \hat{\cup} G_2})(w_i) = 1$; $nd_{(i+2)_1}(w_i) = \sigma_1(\varepsilon) + \sigma_2(\vartheta)$
The paths of length 2 are $w_i \varepsilon \varepsilon_l, l = \kappa+1, \kappa+2, \dots, n$ and $w_i \varepsilon \vartheta_j, j = \kappa+1, \kappa+2, \dots, m$.
Therefore, for $i = 1,2,\dots,\kappa$

$$nd_{(i+1)_2}(w_i) = \sum_{l=\kappa+1}^{n} \sigma_1(\varepsilon_l) + \sum_{\substack{j=1 \\ j\neq i}}^{\kappa} (\sigma_{G_1 \hat{\cup} G_2})(w_i) + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

$$= \sum_{l=\kappa+1}^{n} \sigma_1(\varepsilon_l) + \sum_{\substack{j=1 \\ j\neq i}}^{\kappa} (\sigma_{G_1 \hat{\cup} G_2})(w_i) + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

$$= \sum_{l=\kappa+1}^{n} \sigma_1(\varepsilon_l) + \sum_{\substack{j=1 \\ j\neq i}}^{\kappa} 1 + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

$$nd_{(i+1)_2}(w_i) = \sum_{l=\kappa+1}^{n} \sigma_1(\varepsilon_l) + \kappa - 1 + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

Therefore, $i = 1,2, \dots, \kappa$,

DnDS(w_i)= $(1, \sigma_1(\varepsilon) + \sigma_2(\vartheta), \sum_{l=\kappa+1}^{n} \sigma_1(\varepsilon_l) + \kappa - 1 + \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j))$

Consider the vertex ε_i, $i = \kappa + 1, \kappa + 2, \dots, n$.

The eccentricity of ε_i in the drastic sum is 4.

$nd_{(i+\kappa+2)_0}(\varepsilon_i) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$; $\quad nd_{(i+\kappa+2)_1}(\varepsilon_i) = \sigma_1(\varepsilon)$

The paths of length 2 are $\varepsilon_i \varepsilon w_j, j = 1,2, \dots, \kappa$ and $\varepsilon_i \varepsilon \varepsilon_j, j = \kappa + 1, \kappa + 2, \dots, n, j \neq i$. Therefore,

$$nd_{(i+\kappa+1)_2}(\varepsilon_i) = \sum_{i=1}^{\kappa} (\sigma_{G_1 \hat{\cup} G_2})(w_i) + \sum_{\substack{l=\kappa+1 \\ l \neq i}}^{n} \sigma_1(\varepsilon_l) = \sum_{i=1}^{\kappa} 1 + \sum_{\substack{l=\kappa+1 \\ l \neq i}}^{n} \sigma_1(\varepsilon_l)$$

$$= \kappa + \sum_{\substack{l=\kappa+1 \\ l \neq i}}^{n} \sigma_1(\varepsilon_l).$$

The paths of length 3 from ε_i in the drastic sum are $\varepsilon_i \varepsilon w_j \vartheta, j = 1,2, \dots, \kappa$. Therefore,

$$nd_{(i+\kappa+1)_3}(\varepsilon_i) = \sigma_2(\vartheta)$$

The paths of length 4 from ε_i in the drastic sum are $\varepsilon_i \varepsilon w_l \vartheta \vartheta_j, l = 1,2, \dots, \kappa$, $j = \kappa + 1, \kappa + 2, \dots, m$. Therefore,

$$nd_{(i+\kappa+1)_4}(\varepsilon_i) = \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j)$$

Hence, for $i = \kappa + 1, \kappa + 2, \dots, n$,

DnDS(ε_i)=$(\sigma_1(\varepsilon_i), \sigma_1(\varepsilon), \kappa + \sum_{\substack{l=\kappa+1 \\ l \neq i}}^{n} \sigma_1(\varepsilon_l), \sigma_2(\vartheta), \sum_{j=\kappa+1}^{m} \sigma_2(\vartheta_j))$

Similarly, for $j = \kappa + 1, \kappa + 2, \dots, m$,

DnDS(ϑ_j)=$\left(\sigma_2(\vartheta_j), \sigma_2(\vartheta), \kappa + \sum_{\substack{l=\kappa+1 \\ l \neq i}}^{m} \sigma_2(\vartheta_j), \sigma_1(\varepsilon), \sum_{j=\kappa+1}^{n} \sigma_1(\varepsilon_i)\right)$.

Theorem 3.2.3. Let G_1:(σ_1, μ_1) and G_2:(σ_2, μ_2) have $n + 1$ and $\kappa + 1$ vertices respectively $(n > \kappa)$ such that G_2 is a fuzzy subgraph of G_1, then the DnDS of drastic sum are as follows:

If ε is the apex vertex of G_1, DnDS(ε)= $(1, \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$

For $\varepsilon_i \in V_1 \cap V_2$, DnDS($\varepsilon_i$)= $(1,1, \kappa - 1 + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$

For $\varepsilon_i \in V_1 - V_2$, DnDS($\varepsilon_i$)= $(\sigma_1(\varepsilon_i), 1, \kappa + \sum_{\substack{j=\kappa+1 \\ i \neq j}}^{n} \sigma_1(\varepsilon_j))$

Proof: Let $\{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ be the vertex set of G_1. Since both G_1 and G_2 are fuzzy graphs on star graph and G_2 is a fuzzy subgraph of G_1, ε is also the apex vertex of G_2.

Without loss of generality, let $V_2 = \{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa\}$ be the vertex set of G_2 and let $V_1 = \{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n\}$ be the vertex set of G_1.

Then, $\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa \in V_1 \cap V_2$ and $\varepsilon_{\kappa+1}, \varepsilon_{\kappa+2}, \dots, \varepsilon_n \in V_1 - V_2$.

By the definition of the drastic sum,

$$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1, (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1, i = 1,2, \dots, \kappa$$

$$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i), i = \kappa + 1, \kappa + 2, \dots, n$$

Here, the drastic sum $G_1 \hat{\cup} G_2$ is a fuzzy graph on a star graph with vertices $\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_\kappa, \varepsilon_{\kappa+1}, \dots, \varepsilon_n$ having ε as the apex vertex.
Consider the vertex ε. The eccentricity of ε in the drastic sum is 1.

$$nd_{i_0}(\varepsilon) = (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$$

$$nd_{i_1}(\varepsilon) = \sum_{i=1}^{\kappa} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) + \sum_{i=\kappa+1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i)$$

Therefore, DnDS(ε)= $(1, \kappa + \sum_{i=\kappa+1}^{n} \sigma_1(\varepsilon_i))$.
Consider the vertex $\varepsilon_i, i = 1,2, \dots, \kappa$. The eccentricity of ε_i in the drastic sum is 2.
$nd_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1$; $nd_{(i+1)_1}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are paths of length 2. Therefore,

$$nd_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_j) = \sum_{j=1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_j) - (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i)$$
$$= nd_{G_1 \hat{\cup} G_2}(\varepsilon) - 1$$

Therefore, for $i = 1,2, \dots, \kappa$, DnDS(ε_i)= $(1,1, nd_{G_1 \hat{\cup} G_2}(\varepsilon) - 1)$.
Consider the vertex $\varepsilon_i, i = \kappa + 1, \kappa + 2, \dots, n$. The eccentricity of ε_i in the drastic sum is 2.
$nd_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sigma_1(\varepsilon_i)$; $nd_{(i+1)_1}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are the paths of length 2 from ε_i. Therefore,

$$nd_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_j) = \sum_{j=1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_j) - (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i)$$
$$= nd_{G_1 \hat{\cup} G_2}(\varepsilon) - \sigma_1(\varepsilon_i)$$

Therefore for $i = \kappa + 1, \kappa + 2, \dots, n$,
DnDS(ε_i)= $(\sigma_1(\varepsilon_i), 1, nd_{G_1 \hat{\cup} G_2}(\varepsilon) - \sigma_1(\varepsilon_i))$.

Theorem 3.2.4. Let G_1:(σ_1, μ_1) and G_2: (σ_2, μ_2) have same vertex set $\{\varepsilon, \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$. Then the DnDS of the vertices in the drastic sum are
DnDS(ε) $= (1, n)$ and DDS(ε_i) $= (1,1, n - 1), i = 1,2, \dots, n$
Proof: Since G_1 and G_2 are fuzzy graphs on same star graph, by the definition of the drastic sum $(\sigma_{G_1 \hat{\cup} G_2})(\vartheta) = 1$ for all vertices ϑ.
Consider the vertex ε. The eccentricity of the vertex ε is 1.
$nd_{1_0}(\varepsilon)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$; $nd_{1_1}(\varepsilon)$=$\sum_{i=1}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = \sum_{i=1}^{n} 1 = n$
Therefore, DnDS(ε) $= (1, n)$
Consider the vertex $\varepsilon_i, i = 1,2, \dots, m$. The eccentricity of ε_i in the drastic sum is 2.
$nd_{(i+1)_0}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_i) = 1$; $nd_{(i+1)_1}(\varepsilon_i)$=$(\sigma_{G_1 \hat{\cup} G_2})(\varepsilon) = 1$.
$\varepsilon_i \varepsilon \varepsilon_j, j \neq i$, are paths of length 2 from ε_i in the drastic sum. Therefore,

$$nd_{(i+1)_2}(\varepsilon_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} (\sigma_{G_1 \hat{\cup} G_2})(\varepsilon_j) = \sum_{\substack{j=1 \\ j \neq i}}^{m} 1 = n - 1$$

Therefore, DnDS(ε_i)= $(1,1, n - 1), i = 1,2, \dots, n$.

Remark 3.2.5. If G_1 and G_2 are two different fuzzy graphs with no vertex in common, then the DnDS of a vertex ϑ in the drastic sum is,

$$\mathrm{DnDS}_{G_1 \hat{\cup} G_2}(\vartheta) = \begin{cases} DnDS_{G_1}(\vartheta), if\ \vartheta \in V_1 \\ DnDS_{G_2}(\vartheta), if\ \vartheta \in V_2 \end{cases}$$

4. Conclusion

The DDS, DnDS and the drastic sum of fuzzy graphs have various real-world applications, especially DDS, which is used in molecular chemistry to differentiate and study the properties of isomers. Therefore, DDS and DnDS of drastic sum of fuzzy graphs create a better scope in the field of science. The DDS and DnDS of a drastic sum of fuzzy graphs on stars have been explored in all possible cases in this paper.

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REFERENCES

1. Frank Harary, *Graph Theory, Narosa / Addison Wesley*, Indian Student Edition, 1998.
2. S.R. Gayathri and K. Radha, On properties of distance neighborhood degree sequence of some fuzzy graph, *Indian Journal of Natural Sciences*, 15 (2024), 84601-84606.
3. J.N. Mordeson and C.S. Peng, Operations on fuzzy graphs, *Information Sciences,* 79 (1994) 159-170.
4. J.N. Mordeson and P.S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag Heidelberg, 2000.
5. M. Mizumoto, Fuzzy sets and their operations II, *Information and Control,* 50 (1981) 160-174
6. A. Nagoorgani and K. Radha. On regular fuzzy graphs, *Journal of Physical Sciences,* 12 (2008) 33-40.
7. A. Nagoorgani and K. Radha, Some properties of truncations of fuzzy graphs, *Advances in Fuzzy Sets and Systems*, 4 (2009) 215-227.
8. K. Radha and S.R. Gayathri, Drastic sum of two fuzzy graphs, *Our Heritage*, 68 (2020) 405-414.
9. K. Radha and S.R. Gayathri, On drastic sum and drastic product of two fuzzy graphs, *Advances and Applications in Mathematical Sciences*, 20 (2021) 686-696.
10. K. Radha and S.R. Gayathri, Distance degree sequence and distance neighborhood degree sequence of vertices in fuzzy graphs and some of their properties, *Indian Journal of Science and Technology,* 17 (2024) 4349-4357.
11. A. Rosenfeld, Fuzzy graphs. Zadeh, L.A., Fu, K.S., Tanaka, K., Shimura, M. (eds.), *Fuzzy Sets and Their Application to Cognitive and Decision Processes*, Academic Press, New York, 77-95 (1975).
12. Medha Itagi Huilgol and V. Sriram, New results on distance degree sequence of graphs, *Malaya Journal of Matematik*, 7(2019), 345-352.
13. Masaharu Mizumoto, Fuzzy sets and their operations II, Information and Control, 50(1981), 160-174.
14. Didier Dubois and Henri Prade, *Fuzzy sets and systems: Theory and Applications*, Mathematics in Science and Engineering, Academic Press, New York, 1980.