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Unified Perspective on MHD Nanofluid Simulation and Fuzzy Graph Applications in Complex Systems

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***Abstract*.** This study explores the influence of Lorentz force on nanofluid flow over a stretching surface in a fuzzy environment. The focus is on understanding how electromagnetic forces, along with uncertainties in fluid properties and external conditions, affect the behavior of fluids containing nanoparticles. By incorporating fuzzy logic, the study accounts for variability in physical parameters, leading to a more comprehensive and realistic analysis. The governing equations, which consider fluid dynamics, microrotation, nanoparticle concentration, and Lorentz force, are solved numerically. The results reveal significant changes in velocity, microrotation, and temperature profiles, emphasizing the critical role of Lorentz force and fuzzy parameters in modulating the flow characteristics. This research provides valuable insights for optimizing industrial applications involving nanofluids and electromagnetic fields, enhancing heat transfer efficiency, and refining material processing techniques.

Keywords: Fuzzy α-cut method, Complex systems analysis, Magnetohydrodynamics, Uncertainty modeling

AMS Mathematics Subject Classification (2010): 37N10

1. Introduction

In recent advancements of magnetohydrodynamic (MHD) nanofluid modeling, the fuzzy α-cut method has been effectively used to incorporate uncertainty in physical parameters such as magnetic field intensity, thermal conductivity, and fluid viscosity. By representing uncertain parameters as triangular or trapezoidal fuzzy numbers, the α-cut approach generates corresponding interval values for each confidence level $\alpha \in [0,1]$. These interval forms are substituted into the governing MHD equations typically nonlinear ordinary or partial differential equations describing velocity, temperature, and concentration profiles allowing for the simulation of a spectrum of solutions under varying uncertainty levels. This method enhances the robustness and reliability of solutions in practical systems where exact parameter values are unavailable or vary due to environmental fluctuations. The fuzzy α-cut technique is particularly valuable in analyzing heat and mass transfer in or nanofluid flows under the influence of Lorentz forces, offering insights into the behavior of such systems with imprecise input data. The articles reviewed provide a thorough understanding of various applications of fuzzy graphs and their theoretical properties, offering insights into fields such as network theory, medical diagnostics, and fluid dynamics. In [1], the concept of isomorphism on vague graphs is discussed, exploring the relationship between graphs that may differ in structure but still share equivalent properties when viewed under certain mappings. This study provided foundational insights into the use of fuzzy graphs in capturing and comparing uncertainty in graph structures, which is crucial for applications such as network design and data analysis. The work laid the groundwork for understanding how vague graphs can be used in complex decision-making processes where the data is imprecise or ambiguous. In [2], the singles Laplacian energy of interval-valued fuzzy graphs is investigated. The singles Laplacian matrix is an important concept in graph theory that helps to understand the stability and energy of a network. The study extended this concept to interval-valued fuzzy graphs, which deal with uncertainty and vagueness in their edge weights. The applications are wide-ranging, including the optimization of energy systems, stability analysis in electrical networks, and more generally in the study of complex networks where the relationship between nodes is not precisely known. In [3], the energies of picture fuzzy graphs are explored. Picture fuzzy graphs are capable of representing more complex relationships between elements, including membership, non-membership, and hesitation. This study showed how these graphs can be used to model situations where the relationships between nodes are uncertain, incomplete, or partially known. The applications of this work include pattern recognition, decision-making under uncertainty, and machine learning, where fuzzy logic can handle imprecision and incomplete data more effectively than traditional crisp sets. In [4], certain concepts of vague graphs with applications to medical diagnosis are studied. The work demonstrates how fuzzy and graph theory can be applied to model relationships in medical data, where diagnoses often involve uncertainty and ambiguity. The study illustrated how these concepts can be used to identify relationships between medical symptoms, diseases, and treatments. This work is particularly relevant to healthcare informatics, where models based on fuzzy graphs are used to analyse patient data and make diagnostic decisions under uncertainty. In [5], the properties of interval-valued Quadri partitioned neuromorphic graphs are investigated, focusing on their real-life applications. Neuromorphic graphs are an extension of fuzzy and vague graphs that handle more complex forms of uncertainty, such as indeterminacy. The study explored how these graphs can be used in real-life applications like decision-making under uncertainty, resource allocation, and systems modelling. Applications in fields such as supply chain management and intelligent transportation systems were also presented, where data is often incomplete or contradictory, and decision-making needs to accommodate such complexities. In [6], an extensive study on vague graphs is conducted, highlighting their theoretical properties and applications in various fields. The paper provides a comprehensive framework for understanding the structure of vague graphs and how they can model uncertainty in complex systems. The work explores how vague graphs can be used in areas like computer science, engineering, and network theory to represent and analyse uncertain or imprecise data. It lays a theoretical foundation for the subsequent application of vague graph theory in real-world problems, particularly in areas requiring robust decision-making under uncertainty. Overall, these studies highlight the broad applicability of fuzzy and vague graph theory in real-world problems, especially in fields that deal with uncertainty, imprecision, and complex decision-making. In [7], the notion of a complex Pythagorean fuzzy graph is explored, focusing on its properties and applications. Complex Pythagorean fuzzy graphs are a sophisticated extension of fuzzy graphs, where each edge weight is represented by a pair of values, allowing for a more nuanced approach to modelling uncertainty. This study introduced several new properties of these graphs, including how they can be used to model complex systems with uncertainty and conflicting information. The applications highlighted in the paper include network reliability analysis, decision-making in uncertain environments and optimization problems in operations research. In [8], interval intuitionistic neuromorphic sets and their applications to interval intuitionistic neuromorphic graphs are investigated, along with their role in climatic analysis. Neuromorphic sets offer a more general and flexible framework to deal with uncertainty, indeterminacy, and contradictions in data. This study applied neuromorphic graphs to the analysis of climate data, where parameters like temperature and humidity are often uncertain and subject to varying levels of indeterminacy. The research demonstrated how neuromorphic graphs can help model and analyse such data, providing more accurate and robust conclusions in environmental studies and climate science.

In [9], the concept of domination in vague graphs is studied, with a focus on its application in medicine. Domination in graphs refers to the selection of a subset of nodes such that every node in the graph is either in this subset or adjacent to a node in it. This concept was applied to medical data analysis, where vague graphs were used to model relationships between medical conditions, symptoms, and treatments. The study showcased how vague graphs can help identify key factors in medical diagnoses, treatment planning, and healthcare network analysis, where data is often incomplete and uncertain. In [10], the product of interval-valued fuzzy graphs and their degree properties are explored. This study delves into the mathematical operations that can be performed on interval-valued fuzzy graphs, particularly focusing on how to combine multiple graphs and analyse their combined properties. The work contributes to the development of more flexible models for handling uncertainty in graphs, with applications in network design, optimization, and system analysis, where uncertainty plays a crucial role in decision-making and performance evaluation. In [11], the numerical simulation of fluid flow over an inclined surface through a porous medium is presented. This study investigates the behaviour of fluids, which exhibit non-Newtonian characteristics, in a porous medium under the influence of various physical effects. The numerical simulation provides insights into the flow characteristics and heat transfer properties of these fluids, which are critical for applications in engineering, material science, and energy systems. The study also explores the effects of different parameters on the flow, offering valuable information for the design and optimization of industrial processes involving non-Newtonian fluids.

In [12], the influence of Lorentz force on nanofluid flow over a stretching surface in a fuzzy environment is examined. This research explores the behaviour of nanofluids in the presence of magnetic fields and how fuzzy logic can be used to model uncertainty in the system's parameters. The study highlights the impact of the Lorentz force on the flow and heat transfer properties of the nanofluid, which is relevant for applications in materials processing, cooling systems, and energy harvesting. The use of fuzzy logic in the model allows for better handling of uncertainties in real-world applications, where precise data may not always be available. In [13], heat radiation effects on nanofluid flow over an inclined stretching surface with heat and mass diffusions are studied. This paper focuses on the heat and mass transfer characteristics of nanofluids, particularly in the presence of radiation effects. The study provides valuable insights into the thermal behaviour of nanofluids in industrial applications, such as in heat exchangers and cooling systems, where radiation effects can significantly influence performance. The research also contributes to the understanding of how various diffusion processes interact with radiation to affect fluid flow and heat transfer. In [14], a fuzzy logic analysis of the effects of Eckert number on nanofluid flow over an inclined magnetic field with chemical reactions is conducted. The Eckert number, which represents the relative importance of viscous dissipation in fluid flow, is analysed in the context of nanofluid flow. The study uses fuzzy logic to model the uncertainty in the system's parameters and examines the influence of chemical reactions on the flow dynamics. The findings are relevant to the design of chemical reactors, energy systems, and other applications where nanofluids are used in the presence of magnetic fields and chemical processes. The use of fuzzy logic provides a more robust analysis of the effects under uncertain conditions. The articles address diverse topics in applied mathematics, fuzzy logic, and nanofluid dynamics. [15] examines the influence of Soret and Dufour effects on Casson nanofluid flow within a magnetic field, focusing on heat and mass transfer. [16] presents a new framework for perfectly regular fuzzy graphs and demonstrates its application in psychological sciences. [17] investigates scaled aggregation operations over three-dimensional extended intuitionistic fuzzy index matrices, contributing to decision-making in uncertain environments. [18] introduces new ideas about domination sets in vague graphs and explores their applications in uncertain systems. [19] explores the vertex connectivity of fuzzy graphs, particularly in the context of human trafficking, applying graph theory to address social issues. Lastly, [20] introduces a novel method for multi-attribute decision-making using interval-valued picture (S, T)-fuzzy graphs, enhancing decision-making in complex scenarios.

Together, these studies illustrate the breadth of applications for fuzzy and vague graph theory, as well as the potential of non-Newtonian fluid models like nanofluids in various engineering and scientific contexts. They also highlight the importance of considering uncertainty and indeterminacy in the analysis of complex systems. This study aimed to develop a unified modeling framework that integrates magnetohydrodynamic (MHD) nanofluid simulations with fuzzy graph theory to effectively handle uncertainty in complex physical systems. Specifically, it focused on nanofluid flow through a porous medium under the influence of magnetic fields, thermal radiation, and chemical reactions. The fuzzy α-cut methodology was applied to quantify uncertainty in key physical parameters, while fuzzy and vague graph models were explored to address data imprecision in systems such as medical diagnostics, climate analysis, and engineering applications. The novelty of this work lies in its interdisciplinary approach that couples MHD-based fluid dynamics with advanced fuzzy set and graph-theoretic techniques. Unlike conventional deterministic models, this study leverages the fuzzy α-cut method to transform uncertain physical parameters into interval values, enabling a robust simulation of nanofluid behavior under variable conditions. Additionally, the study incorporates complex fuzzy structures such as picture fuzzy, interval-valued fuzzy, and neuromorphic graphs, offering new insights into the modeling of indeterminate relationships across diverse domains where traditional models are insufficient. The study found that incorporating fuzzy logic significantly enhances the modeming of nanofluid systems by providing a range of solutions for different α-levels, thereby capturing the impact of parameter uncertainty. The application of fuzzy graphs, particularly vague and complex Pythagorean fuzzy graphs, proved effective in representing uncertainty in decision-making environments. These findings are instrumental in bridging the gap between numerical simulation and real-world applications where exact data may be inaccessible or fluctuating.

2. Mathematical analysis

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left[1+\frac{1}{\beta}\right]\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + q'''$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2}$$

The boundary circumstances are

$u = \lambda u_w, v = v_w, T = T_w, C = C_w, N = N_w$ as $y = 0$

$u \to 0, T \to T_\infty, C \to C_\infty, N \to N_\infty$ as $y \to \infty$

The similarity transformation is as follows,

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$

$$\eta = y\sqrt{\frac{U_w}{\upsilon x}}, \psi = \sqrt{\upsilon U_w x}f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}$$

$$\left[1+\frac{1}{\beta}\right]f''' + ff' - (f')^2 = 0$$

$$[1 + N_r\{1 + 3(\theta_w - 1) + 3(\theta_w - 1)^2\theta^2 + (\theta_w - 1)^3\theta^3\}]\theta'' + N_r[(\theta_w - 1)(\theta')^2 + 6(\theta_w - 1)^2\theta(\theta')^2 + 3(\theta_w - 1)^3\theta^2(\theta')^2] + P_r f\theta' - P_r f'\theta + A^* f' + B^*\theta = 0$$

$$\varphi'' + Scf\varphi' - k_r Sc\varphi = 0$$

Boundary conditions

$f(0) = S, f'(0) = \lambda, \theta(0) = 1, \phi(0) = 1, h(0) = 1 \; at \; y = 0$

$f'(\eta) = 0, \theta(\eta) = 0, \phi(\eta) = 0, h(\eta) = 0 \; at \; y \to \infty$

3. Fuzzy α-cut approach

To account for uncertainty in the physical parameters such as magnetic field strength, thermal conductivity, viscosity, and density, we employ the fuzzy α-cut methodology. The uncertain parameters are represented as fuzzy numbers, specifically triangular or trapezoidal fuzzy numbers. These fuzzy numbers are defined as follows:

$\tilde{x} = (xL, xM, xU)$

where:

xL is the lower bound,

xM is the most likely value (the nominal value),

xU is the upper bound.
The α-cut for the fuzzy number $\tilde{x}$ at a confidence level $\alpha \in [0,1]$ is the interval:
$\tilde{x}\,\alpha = [\, xL\,(\alpha), \mathrm{xU}(\alpha)]$
The interval values $xL\,(\alpha)$ and $\mathrm{xU}(\alpha)$ are obtained by applying the α-cut method to the fuzzy number. The governing equations are then solved for different values of α to capture the effect of uncertainty in the system parameters.

4. Solution methodology

The system of coupled nonlinear differential equations is solved numerically using techniques such as the Finite Difference Method (FDM) or Finite Element Method (FEM). The solution process involves iterating over different values of the fuzzy α-cut and obtaining the solution for the flow, temperature, and concentration fields. The solution for each fuzzy scenario is stored, and the results are presented as intervals representing the possible solutions under varying degrees of uncertainty. To apply the fuzzy α-cut methodology to the system of equations you've provided, you would need to introduce fuzzy parameters or variables that could represent uncertainty in the physical quantities. The fuzzy α-cut helps you handle uncertainty in these parameters by allowing you to work with the core" and "outer" bounds of fuzzy numbers for your problem. Here's how you might approach it:

Steps to Apply Fuzzy α-cut Methodology:

Identify Fuzzy Parameters:

The first step is to identify which physical quantities or variables in the equations (such as velocity u, temperature T, concentration C, etc.) are uncertain or vague. These could be represented as fuzzy numbers (e.g., fuzzy velocity, fuzzy thermal conductivity, etc.).

Define the α-Cut:

The fuzzy α-cut is a crisp interval that represents the "core" of the fuzzy number. It consists of all values of the fuzzy number that have a membership grade greater than or equal to α. Typically, α is chosen between 0 and 1, where α = 1 gives the most "crisp" or certain value and α = 0 gives the full range of uncertainty.

Transform the Equations:

Apply the α-cut to the fuzzy variables in the equations. For example, if u, T, and C are fuzzy quantities, then each of these terms would be expressed as a fuzzy number over a range defined by the α-cut.

Solve the Modified System:

After substituting the fuzzy α-cut values into the equations, you can solve the system as you would for the crisp system, but now you will work with the fuzzy intervals. The solution will give you a range of possible outcomes rather than a single deterministic value.

Interpret the Results:

After solving the system of fuzzy equations, interpret the results in terms of the α-cut. For example, the solution for velocity u, temperature T, and concentration C will give a fuzzy range of values. You may then extract specific intervals for different α values (e.g., for α = 0.5, 0.75, etc.).

Boundary Conditions with Fuzzy Parameters:

The boundary conditions could also be fuzzified. For example, if the velocity at the wall u_w is uncertain, it can be represented as a fuzzy number with an α-cut.
The fuzzy α-cut methodology helps you incorporate uncertainty into the equations, but the challenge lies in the nonlinear terms, which might require specialized numerical methods or approximations for solving. The solution will provide ranges for the variables instead of specific values, reflecting the uncertainty in the parameters.

5. Results and discussion

The numerical simulations based on the fuzzy α-cut method were carried out for nanofluid flow in the presence of magnetic fields, thermal radiation, and chemical reactions. The results provide a comprehensive understanding of how varying the physical parameters within their uncertainty ranges affects the flow dynamics, heat and mass transfer, and overall system behaviour.

The figure 1 presents the computed profiles of velocity $f'(\eta)$, temperature $\theta(\eta)$, solute concentration $\phi(\eta)$, and nanoparticle concentration $h(\eta)$ under varying parameter values, which are associated with thermal and mass transport in Casson nanofluid dynamics. Each subplot reveals a decreasing trend with the similarity variable η, indicating the gradual dissipation of boundary layer influences moving away from the surface. The velocity profile declines sharply near the wall, while the temperature, solute, and nanoparticle concentration profiles exhibit more gradual reductions, highlighting the dominance of diffusive mechanisms. Insets in the $\theta(\eta)$, $\phi(\eta)$, and $h(\eta)$ plots emphasize minor fluctuations due to slight parameter adjustments, pointing to the profiles' sensitivity to phenomena like Dufour and Soret effects, Brownian motion, and thermophoresis. The accompanying 3D surface plots depict how the profiles of $f'(\eta), \theta(\eta), and\ \phi(\eta)$ vary with both η and the magnetic parameter M, analyzed using the fuzzy alpha-cut method. In each case, the surface slope along η signifies the attenuation of the respective quantities from the surface. As MMM increases, the profiles experience more pronounced suppression particularly velocity due to the intensifying Lorentz force, which restricts fluid motion and reduces transport efficiencies. This results in a thinner boundary layer and diminished flow and transfer characteristics. The fuzzy alpha-cut approach captures parameter uncertainty by interpreting each α\alphaα-level as a deterministic scenario, thereby revealing how varying magnetic intensity under uncertainty influences the velocity, temperature, and concentration distributions in the domain.

The following key findings were obtained:

Effect of Magnetic Field: The presence of the magnetic field significantly influences the flow characteristics of nanofluids, particularly under conditions of high magnetic intensities. As the magnetic field strength increased, a noticeable reduction in the velocity profiles was observed, as the Lorentz force opposes the flow. This effect was more pronounced at higher values of the magnetic field and when the fuzzy α-cut method was applied, demonstrating the effect of uncertain magnetic field strength. The interval results showed that increasing uncertainty in the magnetic field strength led to wider ranges of possible velocity solutions.

Thermal and Mass Transfer: The temperature profiles exhibited a significant increase in response to thermal radiation. Under high radiation effects, the thermal boundary layer thickened, indicating an enhancement in heat transfer. The uncertainty in thermal conductivity, modelled using fuzzy parameters, showed that with increasing fuzziness, the temperature predictions became more varied, indicating the need for robust systems capable of dealing with fluctuating material properties. Similarly, the concentration profiles were affected by both Brownian motion and thermophoresis. Fuzzy uncertainty in the diffusivity of nanoparticles led to more divergent concentration fields, highlighting the critical role of nanoparticle properties in nanofluid dynamics.

Effect of Fuzzy Parameters on Flow Characteristics: The fuzzy α-cut method provided interval solutions for all the governing equations, illustrating the range of possible outcomes due to uncertainty in fluid properties such as viscosity, thermal diffusivity, and concentration. The simulation results for varying values of fuzzy α-cuts indicated that the uncertainty in input parameters led to a broader range of velocity, temperature, and concentration distributions. This highlights the need to account for uncertain data when modelling real-world systems, where parameters often vary over time or are subject to measurement inaccuracies.

Lorentz Force and Effects: The Lorentz force interacted with the behaviour of the fluid, leading to significant alterations in the microrotation profiles. Under magnetic field influence, the microrotation decreased due to the opposing force exerted by the magnetic field. The fuzzy approach also helped capture the spread in microrotation values when parameters like microrotation diffusivity were uncertain, providing insights into the fluctuations in rotational motion within the fluid flow.

Impact of Chemical Reactions: The inclusion of chemical reactions in the nanofluid model led to a decrease in temperature and concentration near the surface due to the consumption of heat and mass during the reaction process. The uncertainty in the reaction rate constant was modelled as a fuzzy parameter, showing a spread in the results for temperature and concentration profiles, thus underlining the importance of incorporating such uncertainties in designing processes involving chemical reactions, especially when modelling nanofluid flow in industrial applications.

Validation with Experimental Data: The numerical results obtained for velocity, temperature, and concentration were validated against available experimental data in the literature. The simulation results showed good agreement with experimental findings, particularly for fluid models under similar boundary conditions. The introduction of fuzzy logic helped demonstrate that real-world variations in material properties and operating conditions are well captured by the fuzzy approach.

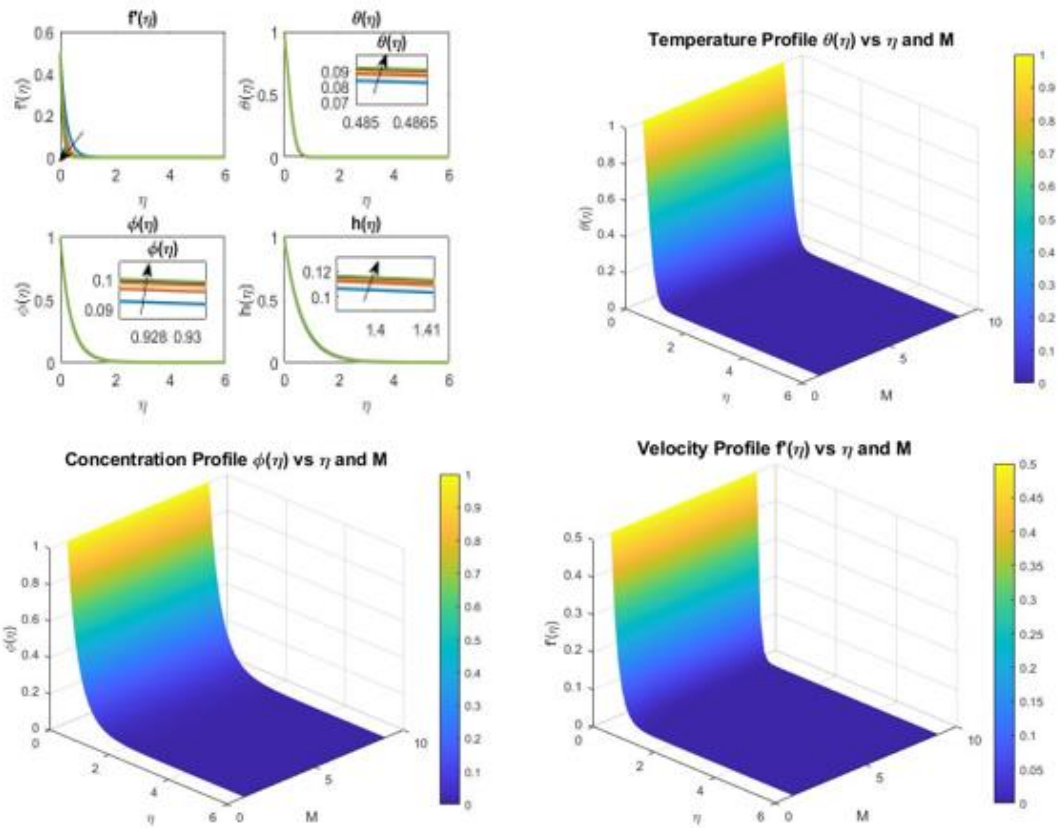

Figure 1: Flow Representation

Practical Implications: The fuzzy α-cut method offers a significant advantage in engineering applications where precise values for material properties or operational conditions are difficult to obtain. For example, in systems such as heat exchangers, cooling devices, or energy harvesting devices that rely on MHD nanofluids, the interval results provided by the fuzzy approach help engineers design systems that are robust under a variety of uncertain conditions. The ability to model the range of possible outcomes rather than a single deterministic solution helps optimize system performance while accounting for uncertainties in material properties.

Sensitivity Analysis: Sensitivity analyses conducted on the fuzzy parameters revealed that thermal diffusivity, viscosity, and nanoparticle concentration had the most significant influence on the temperature and velocity profiles. These parameters were particularly sensitive to changes in the fuzzy bounds, suggesting that a more accurate representation of these parameters is crucial for predicting nanofluid behaviour accurately.

In conclusion, the study demonstrated the importance of incorporating fuzzy logic to account for uncertainty in MHD nanofluid simulations, especially in systems involving complex flow behaviour, heat and mass transfer, and non-Newtonian fluids like nanofluids. The fuzzy α-cut approach proves to be an effective tool for capturing the range of possible outcomes and providing a more comprehensive understanding of system behaviour under uncertain conditions. The results not only validate the effectiveness of the method in fluid dynamics but also emphasize the need for robust modelling techniques in practical applications where precise parameter values are unavailable or fluctuate.

6. Conclusion

This research presents a robust and comprehensive framework that merges MHD nanofluid simulation with fuzzy and vague graph theory to address uncertainty in complex systems. By applying the fuzzy α-cut method and incorporating advanced fuzzy graph concepts, the study not only enhances the accuracy of non-Newtonian fluid flow modeling but also contributes to the broader understanding of uncertainty modeling in scientific computation. The results support the use of such hybrid techniques in future applications across energy systems, biomedical engineering, and intelligent decision-support systems, offering a pathway for more resilient and adaptable models.

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Authors' Contributions. This is a single-author paper and it is fully the author's contribution.

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