Intern. J. Fuzzy Mathematical Archive
Vol. 22, No. 2, 2024, 65-73
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 25 September 2024
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v22n2a03245

International Journal of Fuzzy Mathematical Archive

Certain Concepts of Intuitionistic Fuzzy Tree

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Received 6 August 2024; accepted 22 September 2024

Abstract. Fuzzy graphs play a crucial role in analyzing and understanding complex systems characterized by uncertainty and imprecise information. Among the various types of fuzzy graphs, intuitionistic fuzzy graphs stand out for their ability to represent the membership degrees of both vertices and edges using intervals and fuzzy numbers, respectively. In this paper, we explore the concepts of intuitionistic fuzzy cycles (IFCs) and intuitionistic fuzzy trees (IFTs). Additionally, we discuss several properties of IFTs and examine the relationship between intuitionistic fuzzy trees and intuitionistic fuzzy cycles.

Keywords: Fuzzy set, intuitionistic fuzzy set, intuitionistic fuzzy tree, intuitionistic fuzzy cycle, intuitionistic fuzzy graph

AMS Mathematics Subject Classification (2010): 03C72, 05C99

1. Introduction

Graph theory started its journey from the famous Konigsberg bridge problem. This problem is been the birth of graph theory. Finally, Euler solved this problem with the help of graphs. Though graph theory is a relatively old subject, its growing applications are shown in research. Graph theory is a vital field in various domains including mathematics, engineering, physics, social sciences, biology, computer science linguistics, etc. The notion of a fuzzy graph arises from the idea that networks can sometimes be unclear or uncertain. This is an important field of research. Traditional graphs are limited when it comes to capturing the uncertain nature of network measurements, like strong connections, accomplished individuals and influential figures in social networks. Fuzzy graphs, on the other hand, provide a better representation of these less clear aspects. Fuzzy graph models take on the presence being ubiquitous in environmental and fabricated structures by humans, specifically the vibrant processes in physical, biological, and social systems. Owing to the unpredictable and indiscriminate data which are intrinsic in real life, problems are often ambiguous, so it is very challenging for an expert to exemplify those problems by applying an FG. Intuitionistic fuzzy graph, belonging to the fuzzy graphs family has good capabilities when facing problems that fuzzy graphs cannot express. Intuitionistic fuzzy graphs can handle the vagueness connected with the incompatible and determinate information of any real-world problem, whereas fuzzy graphs may not succeed in bearing satisfactory results. The existence of uncertainty in certain aspects of graph theory problems has led to the development of fuzzy theory. In 1965, Zadeh [55] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, robotics, computer networks, expert systems, decision making and automata theory. Building on this idea, Rosenfeld [27] and Yeh and Bang [54] introduce concept of fuzzy graphs. Bhattacharya [9] gave some remarks on fuzzy graphs. Fuzzy trees were characterized by Sunitha and Vijayakumar [44,45]. The authors have characterized fuzzy trees using its unique maximum spanning tree. Bhutani and Rosenfeld [11] have introduced the concepts of strong arcs [12,13]. They have studied the strong arcs of a fuzzy tree.

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Parvathy and M.G.Karunambigai [26] introduced the concept of intuitionistic fuzzy graph (IFG) as a special case of Atanassov's IFGs and analyzed its components. Many results in different kinds of fuzzy graphs studied in [28-37, 47-52]. Lakdashti et al. [18] presented some results on edge irregular product vague graphs. Shao et al. [43] defined certain concepts of vague graphs with application to medical diagnosis. Talebi and Rashmanlou [53] given domination set in vague graphs with application. Borzooei and Rashmanlou [6, 7, 8] defined Cayley interval-valued fuzzy graphs and novel concepts in vague graphs. Topological indices in fuzzy graphs studied by Kosari et al. [16]. Also, they defined some types of domination in vague graphs with applications in medicine [17]. Talebi et al. [47-52] investigated new results in interval-valued fuzzy graphs and bipolar fuzzy graphs. In this paper, we define an intuitionistic fuzzy cycle (IFC) and intuitionistic fuzzy tree (IFT). Also, we study about some properties of an IFT and the relationship between an intuitionistic fuzzy tree and intuitionistic fuzzy cycle with some examples.

2. Preliminaries

Definition 2.1. Let Xbe a fixed set, an Intuitionistic Fuzzy set (IFS) A in X is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ determine the degree of memership and the degree of non-memership of the element $x \in X$, respectively and for every $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2. An intuitionistic fuzzy graph (IFG) is of the form $G = (V, E)$ where

(i) $V = \{v_0, v_1, \dots, v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$, respectively and $0 \leq \gamma_1(v_i) + \mu_1(v_i) \leq 1$ for $v_i \in V$,$(i = 1, 2, \dots, n)$.

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ such that

(a) $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,

(b) $\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$,

(c) $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, for every$(v_i, v_j) \in E$.

Definition 2.3. Let $G = (V, E)$ be an IFG. An IFG$H = (V', E')$ is said to be an intuitionistic fuzzy subgraph (IFSG) of G, if $V' \subseteq V$ and $E' \subseteq E$ such that for $x \in V'$, if $\mu_1'(x) > 0$ or $\gamma_1'(x) > 0$, then $\mu_1'(x) = \mu_1(x)$ and $\gamma_1'(x) = \gamma_1(x)$and for $(x, y) \in E'$, if $\mu_2'(x, y) > 0$ or $\gamma_2'(x, y) > 0$, then $\mu_2'(x, y) = \mu_2(x, y)$ and $\gamma_2'(x, y) = \gamma_2(x, y)$. Also H is said to be an intuitionistic spanning fuzzy subgraph (IFSS) of G, if $V' = V$.

Definition 2.4. An IFG$G = (V, E)$ is strong, if for all $(v_i, v_j) \in E \mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$ and is complete if for all $v_i, v_j \in V \mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$.

3. Intuitionistic fuzzy trees

In this section we introduce types of intuitionistic fuzzy trees. We recognize types of arcs in an intuitionistic fuzzy tree.

Definition 3.1. A μ-connected $G = (V, E)$, is an intuitionistic fuzzy μ-tree (IFμ-tree), if it has an intuitionistic fuzzy spanning subgraph F, which is a μ-tree,such that for all arcs (u, v) not in F, $\mu_2(u, v) < \mu_F^\infty(u, v)$. Also F is called a spanning μ-treeof G.

Definition 3.2. A γ-connected $G = (V, E)$, is an *intuitionistic fuzzy γ-tree* ($IF\gamma$-tree), if it has an intuitionistic fuzzy spanning subgraph F', which is a γ-tree, such that for all arcs (u, v) not in F', $\gamma_2(u, v) > \gamma_{F'}^\infty(u, v)$. Also F' is called a *spanning γ-tree* of G.

Definition 3.3. Let $G = (V, E)$ be a strong connected IFG. Then $G$is an *intuitionistic fuzzy tree* (IFT), if it has an intuitionistic fuzzy spanning subgraph F'', which is a tree, such that for all arcs (u, v) not in F'', $\mu_2(u, v) < \mu_{F''}^\infty(u, v)$ and $\gamma_2(u, v) > \gamma_{F''}^\infty(u, v)$. Also F'' is called a*spanning tree* of G.

Proposition 3.4. *If $G = (V, E)$ is an IFT, then G is an $IF\mu$-tree and $IF\gamma$-tree.*

Example 3.5. In Figure 1, $G = (V, E)$ is an IF μ-tree and $IF\gamma$-tree, but is not an IFT, because there is not a spanning tree F''.

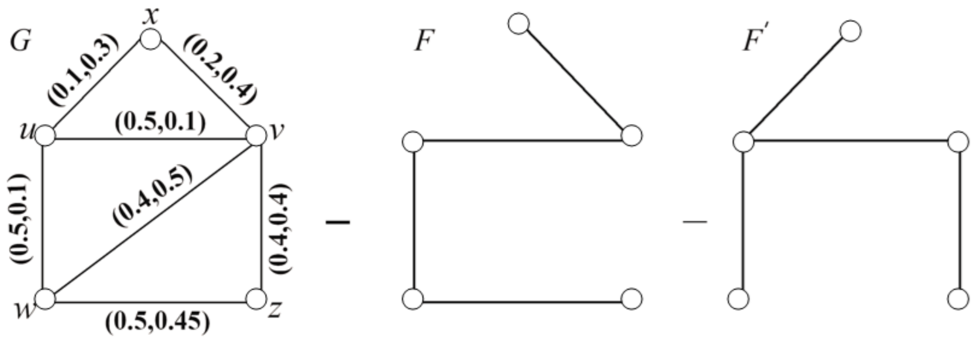

Figure 1: Intuitionistic fuzzy μ-tree and γ-tree G

Theorem 3.6. *An arc (x, y) in an IFμ-tree $G = (V, E)$ is α_μ-strong iff (x, y) is an arc of the spanning μ-tree $F$of G.*
Proof: It is clear.

Theorem 3.7. *An arc (x, y)in an IF γ-tree $G = (V, E)$ is α_γ-strong iff (x, y) is an arc of the spanning γ-tree $F'$of G.*
Proof: Assume that $(x, y)$is a α_γ-strongarc in G, then by Definition 3.2, $\gamma_2(x, y) < \gamma^{\infty}_{G-(x,y)}(x, y)$. If (x, y) not in F', then $\gamma_2(x, y) > \gamma^{\infty}_{F'}(x, y)$. Also the spanning γ-tree F' is an *IFSS* of $G - (x, y)$. Hence $\gamma^{\infty}_{F'}(x, y) \geq \gamma^{\infty}_{G-(x,y)}(x, y)$. It follows $\gamma_2(x, y) > \gamma^{\infty}_{G-(x,y)}(x, y)$, which contradicts the assumption. Hence (x, y) is in F'. Conversely, let the arc(x, y)be in F'. If (x, y) is not a α_γ-strong arc in G, then $\gamma_2(x, y) \geq \gamma^{\infty}_{G-(x,y)}(x, y)$. We consider C a γ-cycle consist of (x, y), hence there exists the arc (u, v) in C, which is not in F'. Therefore, $\gamma_2(u, v) > \gamma^{\infty}_{F'}(u, v)$. We have γ-path $P = C - (u, v)$ from u to v in F', hence $\gamma^{\infty}_{P}(u, v) = \gamma^{\infty}_{F'}(u, v)$, because F' is a γ-tree. Also we have $\gamma^{\infty}_{P}(u, v) \geq \gamma_2(x, y)$, thus $\gamma^{\infty}_{F'}(u, v) \geq \gamma_2(x, y)$, which implies that $\gamma_2(u, v) > \gamma_2(x, y)$. Therefore, (x, y) is not the weakest γ-arc of every cycle in G. Hence (x, y) is an IF γ-bridge.Thus(x, y) is α_γ-*strong*, which complete the proof.

Proposition 3.8. *In an $IFTG = (V, E)$, there exist unique spanning tree F''such that $F'' = F' = F$.*
Proof: If $G = (V, E)$ is an IFT, then there exists a spanning treeF'' such that for all arcs (u, v) not in F'',

$$\mu_2(u, v) < \mu^{\infty}_{F''}(u, v) \tag{1}$$

and

$$\gamma_2(u, v) > \gamma^{\infty}_{F''}(u, v) \tag{2}$$

By (1), there exists an unique spanning μ-tree F such that $F^{"} = F$ and by (2), there exists an unique spanning γ-tree F' such that $F^{"} = F'$. Hence, $F^{"}$is unique spanning tree and $F^{"} = F' = F$.

Proposition 3.9. *Let $G = (V, E)$ be an IFG, then we have:*
(i) *If G is an IF μ-tree and the arc (x, y) not in F, then $\mu_F^\infty(x, y) = \mu_2'^\infty(x, y)$.*
(ii) *If G is an IF γ-tree and the arc (x, y) not in F', then $\gamma_{F'}^\infty(x, y) = \gamma_2'^\infty(x, y)$.*
(iii) *If G is an IFT and the arc (x, y) not in $F^{"}$, then $\mu_{F^{"}}^\infty(x, y) = \mu_2'^\infty(x, y)$ and $\gamma_{F^{"}}^\infty(x, y) = \gamma_2'^\infty(x, y)$.*
Proof: (i) Let P be a μ-path from x to y in F. All arcs of P are α_μ-strong. Hence P is a α_μ-strong path. Thus P is a μ-strongest$(x - y)$ path. It follows that $\mu_F^\infty(x, y) = \mu_2'^\infty(x, y)$.
(ii) Let P be a γ-path from x to y in F'. All arcs of P are α_γ-strong. Hence, P is a α_γ-strong path. Thus, P is a γ-strongest $(x - y)$ path. It follows that $\gamma_{F'}^\infty(x, y) = \gamma_2'^\infty(x, y)$.
(iii) It follows obviously from (i) and (ii).

Definition 3.10. Let $G = (V, E)$ be a μ-cycle, then G is *an intuitionistic fuzzy μ-cycle* (*IF μ-cycle*), if contains more than one weakest μ-arcs. Let G be a γ-cycle, then G is *an intuitionistic fuzzy γ-cycle* (*IF γ-cycle*), if contains more than one weakest γ-arcs. Let G be a cycle, then $G$is an *intuitionistic fuzzy cycle* (IFC), if is an IF μ-cycle or IF γ-cycle.

Theorem 3.11. *Let $G = (V, E)$ be an IFG. Then*
(i) *If G is an $IF\mu$-cycle, then G has no δ_μ-arcs.*
(ii) *If G is an $IF\gamma$-cycle, then G has no δ_γ-arcs.*
(iii) *If G is an IFC, then G has no δ_μ-arcs or δ_γ-arcs.*
Proof: (i) If (u, v) is a δ_μ-arc in G, then it becomes the unique weakest μ-arc in G, which contradicts the Definition 3.10.
(ii) If (u, v) is a δ_γ-arc in G, then it becomes the unique weakest γ-arc in G, which contradicts the Definition 6.1.
(iii) We get from (i) and (ii) clearly.

Theorem 3.12. *Let $G = (V, E)$ be an IFG. Then*
If G is a μ-cycle, then G is an $IF\mu$-cycle iff has at least two β_μ-strong arcs.
(ii) *If $G$is a γ-cycle, then G is an $IF\gamma$-cycle iff has at least two β_γ-strong arcs.*
(iii) *If G is a cycle, then G is an IFC iff has at least two β_μ*-strong *arcs or β_γ-strong arcs.*
Proof: (i) If G is an $IF\mu$-cycle, then there exists at least two weakest μ-arcs such that are β_μ-arcs. Hence, G has at least two β_μ-strong arcs. Conversely, gets clearly.
(ii) If G is an IF γ-cycle, then there exists at least two weakest γ-arcs such that are β_γ-arcs. Hence, G has at least twoβ_γ-strong arcs. Conversely, gets clearly.

(iii) We gets from (i) and (ii) clearly.

Theorem 3.13. *Let an IFG $G = (V, E)$ be a cycle. If G is an IFC, then G is not an IFT.*

Proof: If G is an IFC, then G is an IF μ-cycle or IF γ-cycle. Let G be an IF μ-cycle, then G is not an $IF\mu$-tree, hence G is not an IFT. Let G be an IF γ-cycle, then G is not an $IF\gamma$-tree, hence G is not an IFT. This complete the proof.

4. Conclusions

Many problems of practical interest can be modeled and solved by using fuzzy graph algorithms. A fuzzy graph is a very useful and effective tool for studying various calculations, fields of intelligence and computer science such as networking, imaging and other fields such as biological sciences. In different appropriate, they present a appropriate construction means. Intuitionistic fuzzy graphs are important in other sciences, including psychology, life sciences, medicine, social studies, and can help researchers with optimization and save time and money. Also, intuitionistic fuzzy graphs belonging to FGs family has good abilities since facing with problems that can not be explained by fuzzy graphs. In graph theory, trees and cycles are conveniently used in many combinatorial applications. In various situations they present a suitable construction means. So, in this paper, the IFT and IFC in an IFG has been investigated. Likewise, we discussed about some properties of an IFT and the relationship between an intuitionistic fuzzy tree and intuitionistic fuzzy cycle.

Acknowledgement. We would like to provide our cordial thanks to the honorable referees for their valuable comments which help us to enrich the quality of the paper.

Conflicts of interest. This is the author sole paper and there is no conflict of interest.

Authors' Contributions. It is author's full contribution.

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