Intern. J. Fuzzy Mathematical Archive
Vol. 22, No. 2, 2024, 53-63
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 8 September 2024
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v22n2a02244

International Journal of Fuzzy Mathematical Archive

Some Results in Intuitionistic Fuzzy Graphs

***A.A.Talebi*[1] and *Hossein Rashmanlou*[2]**

[1]Department of Mathematics, University of Mazandaran, Babolsar, Iran
Email: a.talebi@umz.ac.ir
\[2]Department of Mathematics, University of Mazandaran, Babolsar, Iran
Email: H.Rashmanlou@stu.umz.ac.ir

Received 3 August 2024; accepted 7 September 2024

Abstract. The concept, tools and techniques, of an intuitionistic fuzzy graph have found many applications in different areas such as topology, all kinds of systems and networks, computer science, etc. An intuitionistic fuzzy graph is a generalized structure of a fuzzy graph that provides more flexibility, adaptability and compatibility to real human-centric systems than a simpler fuzzy graph. So, in this paper, we study some results in the intuitionistic fuzzy graph (IFG) and present some basic definitions. We investigate several kinds of arcs, for example, α_μ-strong, β_μ-strong, δ_μ-arc in an intuitionistic fuzzy graph and analyse some properties. Also, we give intuitionistic fuzzy bridge, intuitionistic fuzzy cut nodes and some interesting properties of an intuitionistic fuzzy bridge.

Keywords: Intuitionistic fuzzy set, μ-strong, γ-strong, intuitionistic fuzzy bridge, intuitionistic fuzzy tree

AMS Mathematics Subject Classification (2010): 03C72, 05C99

1. Introduction

In 1965, Zadeh [54] introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, signal processing, multiagent systems, pattern recognition, robotics, computer networks, expert systems, decision making and automata theory.

Ten years after Zadeh's landmark paper, Rosenfeld [26] and Yeh and Bang [53] introduced the concept of fuzzy graphs. Models based on fuzzy graphs are powerful tools and techniques to deal with many combinatorial problems in a wide array of different domains in science and technology exemplified by algebra, topology, optimization, computer science, social science, etc. The fuzzy graph models often outperform graph models, that is, non-fuzzy graph models, because of the inherent existence of vagueness, imprecision and ambiguity in virtually all practical problems, and fuzzy graphs provide in this respect an additional modelling capability. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya [3] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by fuzzy graph theory is now finding numerous applications in modern science and technology, especially in the fields of information theory, neural networks, expert systems, cluster analysis, medical diagnosis, control theory, etc. Fuzzy trees were characterized by Sunitha and Vijayakumar [38]. The authors have characterized fuzzy trees using their unique maximum spanning tree. Bhutani and Rosenfeld [4] have introduced the concepts of strong arcs [5], fuzzy end nodes [6], etc. They have shown the existence of a strong path between any two nodes of a fuzzy graph and have studied the strong arcs of a fuzzy tree. In [5], the concepts of fuzzy end nodes and multimin and locamin cycles were studied.

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing exponential growth in mathematics and its applications. This ranges from traditional Mathematics to Information Sciences. This leads to consider IFGs and their applications. Parvathy and Karunambigai [25] introduced the concept of the intuitionistic fuzzy graph (IFG) as a special case of Atanassov's IFGs and analyzed its components. In [20], some important operations on IFGs are defined and their properties are studied. Ghorai et al. [10, 11] studied certain graph parameters in bipolar fuzzy environment and results of m-polar fuzzy graphs with application. Khatun et al. [15,16] investigated a comprehensive study on m-polar picture fuzzy graphs and its application and picture fuzzy cubic graphs and their applications. Lakdashti et al. [17] presented some results on edge irregular product vague graphs. Shao et al. [45] defined certain concepts of vague graphs with application to medical diagnosis. Talebi and Rashmanlou [52] gave domination set in vague graphs with application. Borzooei and Rashmanlou [9] defined Cayley interval-valued fuzzy graphs. Topological indices in fuzzy graphs studied by Kosari et al. [13]. Also, they defined some types of domination in vague graphs with applications in medicine [14]. Talebi et al. [46-51] investigated new results in interval-valued fuzzy graphs and bipolar fuzzy graphs.

Many results in different kinds of fuzzy graphs were studied in [27-37, 41-45]. Fuzzy graphs and algorithms based on them can be very useful for the solution of many problems of practical interest. Since uncertain and imprecise information is an essential characteristic feature of virtually all real-life problems, mostly uncertain, the modelling of such problems using fuzzy graphs is difficult, even for an expert. An intuitionistic fuzzy graph, an extension of the basic concept of a fuzzy graph, can be employed to deal with deeper aspects of uncertainty and imprecision for which the use of fuzzy graphs would not fully succeed. In this paper, we define types of arcs in intuitionistic fuzzy graphs and intuitionistic fuzzy trees. Also, we study intuitionistic fuzzy bridges and intuitionistic fuzzy cut nodes and we investigate some important properties IFB. we define the intuitionistic fuzzy cycle and intuitionistic fuzzy tree and examine the relationship between an IFT and IFC, we also discuss about important properties of an IFT and IFC.

In this research work, we study some properties of the intuitionistic fuzzy graph(IFG) and some basic definitions. We investigate several kinds of arcs, for example: α_μ-strong, β_μ-strong, δ_μ-arc in an intuitionistic fuzzy graph and present some properties of theirs. Also, we present an intuitionistic fuzzy bridge (IFB), intuitionistic fuzzy cut nodes (IFCN), and some interesting properties of an intuitionistic fuzzy bridge.

2. Preliminaries

Definition 2.1. *Let X be a fixed set, An intuitionistic fuzzy set (IFS) A in X is an object of the form* $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, *where* $\mu_A: X \to [0,1]$ *and* $\nu_A: X \to [0,1]$ *determine the degree of membership and the degree of non-membership of the element* $x \in X$*, respectively and for every* $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2. An intuitionistic fuzzy graph (IFG)is of the form $G = (V, E)$ where

(i) $V = \{v_0, v_1, \ldots, v_n\}$ such that $\mu_1: V \to [0,1]$ and $\gamma_1: V \to [0,1]$ denote the degree of membership and non-membership of the element $v_i \in V$,respectively and $0 \leq \gamma_1(v_i) + \mu_1(v_i) \leq 1$ for $v_i \in V$,$(i = 1, 2, \ldots, n)$.

(ii) $E \subseteq V \times V$ where $\mu_2: V \times V \to [0,1]$ and $\gamma_2: V \times V \to [0,1]$ such that

(a) $\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$,

(b) $\gamma_2(v_i, v_j) \leq \max[\gamma_1(v_i), \gamma_1(v_j)]$,

(c) $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$.

Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree ofnon-membership of the edge $e_{ij} = (v_i, v_j)$ on V. If we consider arcs of G only with the degree of memberships, then is called μ-arcs and if we consider arcs of G only with the degree of non-memberships, then is called γ-arcs.

Remark 2.3. Let $G = (V, E)$ be an IFG. If we consider all of the vertices and arcs only with the degree of memberships, then G induce a fuzzy graph than the degree of membership. All of the definitions and theorems for a fuzzy graph satisfy it.

Definition 2.4. Let $G = (V, E)$ be an IFG. An $IFGH = (V', E')$ is said to be an intuitionistic fuzzy subgraph ($IFSG$) of G, if $V' \subseteq V$ and $E' \subseteq E$ such that for $x \in V'$, if $\mu_1'(x) > 0$ or $\gamma_1'(x) > 0$, then $\mu_1'(x) = \mu_1(x)$ and $\gamma_1'(x) = \gamma_1(x)$ and for $(x, y) \in E'$, if

$\mu_2'(x,y) > 0$ or $\gamma_2'(x,y) > 0$, then $\mu_2'(x,y) = \mu_2(x,y)$ and $\gamma_2'(x,y) = \gamma_2(x,y)$. Also, H is said to be an intuitionistic spanning fuzzy subgraph ($IFSS$) of G, if $V' = V$.

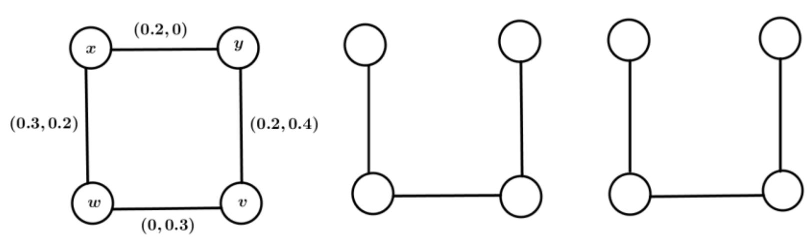
Definition 2.5. An $IFGG = (V, E)$ is *strong*, if for all $(v_{\mathrm{i}}, v_{\mathrm{j}}) \in E \mu_{2\mathrm{ij}} = \min(\mu_{1\mathrm{i}}, \mu_{1\mathrm{j}})$ and $\gamma_{2\mathrm{ij}} = \max(\gamma_{1\mathrm{i}}, \gamma_{1\mathrm{j}})$ and is *complete* if for all $v_{\mathrm{i}}, v_{\mathrm{j}} \in V \mu_{2\mathrm{ij}} = \min(\mu_{1\mathrm{i}}, \mu_{1\mathrm{j}})$ and $\gamma_{2\mathrm{ij}} = \max(\gamma_{1\mathrm{i}}, \gamma_{1\mathrm{j}})$.

Remark 2.6. When $\mu_{2\mathrm{ij}} = \gamma_{2\mathrm{ij}} = 0$ for some i and j, then there is no edge between v_{i} and v_j. Otherwise, there exists an edge between v_{i} and v_j.

Definition 2.7. An $IFGG = (V, E)$ is a *μ-tree (also a γ-tree)*, if G_μ^* and G_γ^* are tree, respectively and$G$ is a *tree*, if is a μ-tree, (also a γ-tree) and $G_\mu^* = G_\gamma^*$. Also G is a *μ-cycle* (*also γ-cycle)*, if G_μ^* and G_γ^* are cycle, respectively and G is a *cycle*, if is a μ-cycle, (also a γ-cycle) and $G_\mu^* = G_\gamma^*$.

Remark 2.8. Let $G = (V, E)$ be an IFG. If $G$is a μ-cycle, then the weakest μ-arc of G is the arc with minimum degree of membership. If G is a γ-cycle, then the weakest γ-arc of G is the arc with maximum degree of non-membership.

Example 2.9. In Figure 1, let$G = (V, E)$be an IFG, such that $V = \{x, u, v, w\}$,
$E = \{(x,u), (u,v), (v,w), (x,w)\}$. Then G is a μ-tree and γ-tree, but is not a tree.



$G: G_\mu^* G_\gamma^*$

Figure 1: μ-tree and γ-tree G

Definition 2.10. Let$P: x = v_0,\ v_1, \dots, v_n = y$ be a sequence of distinct vertices in an intuitionistic fuzzy graph, then P is a *μ-path* from x to y, if $\mu_2(v_{i-1}, v_i) > 0$, and is a *γ-path*, if $\gamma_2(v_{i-1}, v_i) > 0$, for $i = 1, 2, \dots, n$. Also, P is a *path*, if is bothμ-path andγ-path, then P is called a $(x - y)$path and the *length* of P for $n > 0$ is n. If $x = y$ and $n > 3$, P is called μ-cycle,γ-cycle and cycle, respectively.

Definition 2.11. An intuitionistic fuzzy graph $G = (V, E)$ is *μ-connected*, if there exists a μ-path between every pair of vertices in G and is *γ-connected*, if there exists a γ-path between every pair of vertices in G. Also, G is called *strong connected*, if there exists a path between every pair of vertices in G.

Remark 2.12. In Figure 1, G is μ-connected and γ-connected, but is not connected. There exist a μ-path and γ-path from x to v, but there no exist a path.

Definition 2.13. If $v_i, v_j \in V \subseteq G$, the *$\mu$-strength of connectedness* between v_i and v_j is $\mu_2^{\infty}(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) \mid k = 1, 2, \ldots, n\}$ and the *γ-strength of connectedness* between v_i and v_j is $\gamma_2^{\infty}(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) \mid k = 1, 2, \ldots, n\}$. $\mu_2^k(u, v)$ is defined as $\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \ldots \wedge \mu_2(v_{k-1}, v) \mid u, v_1, v_2, \ldots, v_{k-1}, v \in V\}$, if u, v are connected by μ-paths of length k. Also if u, v are connected by γ-paths of length k, then $\gamma_2^k(u, v)$ is defined as

$$\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \ldots \vee \gamma_2(v_{k-1}, v) \mid u, v_1, v_2, \ldots, v_{k-1}, v \in V\}.$$

The μ-strength and γ-strength of connectedness between v_i and v_j in $G = (V, E)$ is denoted by $\mu_G^{\infty}(v_i, v_j)$ and $\gamma_G^{\infty}(v_i, v_j)$, respectively. Also, $\mu_2'^{\infty}(v_i, v_j)$ and $\gamma_2'^{\infty}(v_i, v_j)$ denote $\mu_{G-(v_i,v_j)}^{\infty}(v_i, v_j)$ and $\gamma_{G-(v_i,v_j)}^{\infty}(v_i, v_j)$, where $G - (v_i, v_j)$ obtain from G by deleting the arc (v_i, v_j).

Remark 2.14. If P is a μ-path in $G = (V, E)$ from x to y, then the *μ-strength of P* is denoted by $\mu_P^{\infty}(x, y)$ and if P is a γ-path in G, then the *γ-strength of P* is denoted by $\gamma_P^{\infty}(x, y)$. A path between a pair of vertices x and y is the *μ-strongest $(x - y)$ path* and *γ-strongest $(x - y)$ path*, if the μ-strength and γ-strength of it is equal to $\mu_2^{\infty}(x, y)$ and $\gamma_2^{\infty}(x, y)$, respectively.

3. Types of arcs and paths

Definition 3.1. An arc (x, y) in an IFGG $= (V, E)$ is called μ-strong and γ-strong, if $\mu_2(x, y) \geq \mu_2'^{\infty}(x, y)$ and $\gamma_2(x, y) \leq \gamma_2'^{\infty}(x, y)$, respectively. Also, (x, y) is called strong, if it μ-strong or γ-strong.

Definition 3.2. An arc (x, y) in $G = (V, E)$ is called α_μ-strong, β_μ-strong, δ_μ-arc, α_γ-strong, β_γ-strong and δ_γ-arc, if $\mu_2(x, y) > \mu_2'^{\infty}(x, y)$, $\mu_2(x, y) = \mu_2'^{\infty}(x, y)$, $\mu_2(x, y) < \mu_2'^{\infty}(x, y)$, $\gamma_2(x, y) < \gamma_2'^{\infty}(x, y)$, $\gamma_2(x, y) = \gamma_2'^{\infty}(x, y)$ and $\gamma_2(x, y) > \gamma_2'^{\infty}(x, y)$, respectively.

Example 3.3. In Figure 2, let $G = (V, E)$ be an IFG, such that $V = \{x, u, v, w, z\}$, $E = \{(x, u), (x, v), (u, v), (u, w), (v, z), (w, z)\}$. Then the arc (x, u) is δ_μ-arc and α_γ-strong, the arc(x, v) is α_μ-strong and δ_γ-arc, the arc(u, v) is α_μ-strong and α_γ-strong and the arcs $(u, w), (v, z)$ and (w, z) are β_μ-strong and β_γ-strong.

Definition 3.4. Let $P: x = v_0, v_1, \ldots, v_n = y$ be a μ-path from x to y in an $G = (V, E)$. P is *μ-strong* (α_μ*-strong*), if for $i = 1, 2, \ldots, n$, the arcs (v_{i-1}, v_i) are μ-strong (α_μ-strong). Let P be a γ-path, then P is *γ-strong* (α_γ*-strong*), if for $i = 1, 2, \ldots, n$, the arcs (v_{i-1}, v_i) are γ-strong (α_γ-strong). Let P be a path, then P is *strong* (α-*strong*), if P is μ-strong or γ-strong (α_μ-strong or α_γ-strong).

Remark 3.5. In Figure 2, the path $P: x, v, u$ is a α_μ-strong path and the path $P': x, u, v$ is a α_γ-strong path. Hence P and P' are α-strong paths.

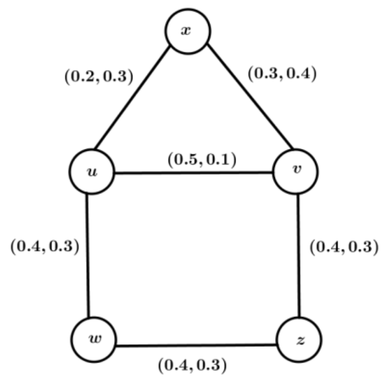


Figure 2: Intuitionistic fuzzy graph G

Proposition 3.6. *If $G = (V, E)$ be a γ-connected IFG, then there exists a γ-strong path between every vertices of G.*

Proof: An IFG G is γ-connected, hence there exists γ-path between every vertices. If (x, y) is not γ-strong arc, then we have $\gamma_2(x, y) > \gamma_2'^{\infty}(x, y)$. Hence there exists a γ-path P from x to y, which γ-strength of it is less than $\gamma_2(x, y)$. Now if some arcs of P are not γ-strong, we repeat this argument. Finally we will have a γ-path from x to y, which is γ-strong. This complete the proof.

Proposition 3.7. If a path P from x to y in an IFG $G = (V, E)$ is α_γ-strong, then P is a γ-strongest $(x - y)$ path.

Proof: Let $P: x = v_0, v_1, \ldots, v_n = y$ be a α_γ-strong path and suppose that P is not a γ-strongest $(x - y)$ path in G. Let $P': x = v_0', v_1', \ldots, v_n' = y$ be a γ-strongest $(x - y)$ path in G. Hence $\gamma_2(v_{i-1}', v_i') < \gamma_P^{\infty}(x, y)$, for $i = 1, 2, \ldots, n$. Also P and P' form a γ-cycle, called C. The weakest γ-arc of C is in P. Let (u, v) be a weakest γ-arc in P. We consider P'' be the $(u - v)$ path in C not include (u, v). It follows that $\gamma_2(u, v) \geq \gamma_{P''}^{\infty}(u, v) \geq$

$\gamma_2'^{\infty}(u,v)$ which implies that (u,v) is not a α_γ-strong arc, this contradicts the assumption. Therefore, P is γ-strongest $(x-y)$ path in G.

Definition 3.8. An arc (x,y) in an $IFGG=(V,E)$ is said to be an *intuitionistic fuzzy* μ-*bridge* ($IF\mu$-*bridge*), if deleting (x,y) reduces the μ-strength of connectedness between some pair of vertices. Equivalently, there exist $u,v \in V$ such that (x,y) is an arc of every μ-strongest $(u-v)$ path. An arc (x,y) is said to be an *intuitionistic fuzzy* γ-*bridge* ($IF\ \gamma$-*bridge*), if deleting (x,y) increases the γ-strength of connectedness between some pair of vertices. Equivalently, there exist $u,v \in V$ such that (x,y) is an arc of every γ-strongest $(u-v)$ path. An arc (x,y) is said to be an *intuitionistic fuzzy bridge* (IFB), if it is an $IF\mu$-bridge or $IF\gamma$-bridge.

Definition3.9. A vertex $x \in V$ is an $IFGG=(V,E)$ is an *intuitionistic fuzzy* μ-*cut vertex* ($IF\mu$-*cut vertex*), if deleting it reduce the μ-strength of connectedness between some pair of vertices. Equivalently, there exist $u,v \in V$ such that x is a vertex of every μ-strongest $(u-v)$ path. A vertex $x \in V$ is an *intuitionistic fuzzy* γ-*cut vertex* ($IF\gamma$-*cut vertex*), if deleting it increase the γ-strength of connectedness between somepair of vertices. Equivalently, there exist $u,v \in V$ such that $x$is a vertex of every γ-strongest $(u-v)$ path. A vertex $x \in V$ is an *intuitionistic fuzzy cut vertex* ($IFCV$), if it is an $IF\mu$-cut vertex or $IF\ \gamma$-cut vertex.

Example 3.10. In Figure 3, let $G=(V,E)$ be an IFG, such that $V=\{x,u,v,w,z\}$, $E=\{(x,u),(x,v),(u,v),(u,w),(v,z),(w,z)\}$. Then the arcs $(x,u)$and $(x,v)$are β_μ-strong and α_γ-strong, the arcs (u,v) and (u,w) are β_μ-strong andδ_γ-arc, the arcs(v,z) and (w,z) are α_μ-strong and α_γ-strong. Hence all arcs are strong. Also the arcs(v,z) and (w,z) are $IF\mu$-bridge, and the arcs $(x,u),(x,v),(v,z)$ and (w,z) are $IF\ \gamma$-bridge. Hence all arcs except (u,v) are IFB. Also, z is an $IF\ \mu$-cut vertex and x,z are $IF\ \gamma$-cutvertex. Hence x,z are $IFCV$.

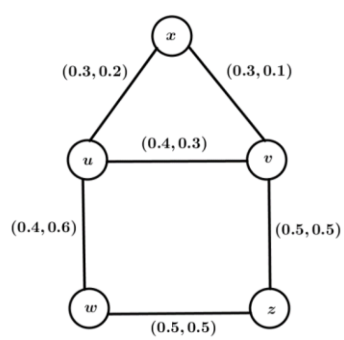


Figure 3: Intuitionistic fuzzy graph G

Theorem 3.11. *Let* (x, y) *be an arc in an* $IFGG = (V, E)$*, then*
(i) (x, y) *is an* $IF\mu$*-bridge iff* $\mu_2(x, y) > \mu_2'^{\infty}(x, y)$.
(ii) (x, y) *is an* $IF\gamma$*-bridge iff* $\gamma_2(x, y) < \gamma_2'^{\infty}(x, y)$.
(iii) (x, y) *is an* IFB *iff* $\mu_2(x, y) > \mu_2'^{\infty}(x, y)$ or $\gamma_2(x, y) < \gamma_2'^{\infty}(x, y)$.

Proof: (*i*) It is obvious.

(*ii*) Assume that (x, y) is an $IF\gamma$-bridge, hence there exist $u, v \in V$ such that (x, y) is an arc of γ-strongest $(u - v)$ path, which is called P. Now let P' be a γ-path from u to v not include (x, y) and the γ-strength of it be minimum between all the γ-paths from u to v not include (x, y). Then P and P' form a cycle called C and $C - (x, y)$ is a γ-path called P''. We claim that P'' is the γ-strongest path between x and y. Let ρ' be a γ-strongest path between x and y, then deleting (x, y) not increase the γ-strength of u and v. This contradicts the assumption. Hence, $\gamma_{P''}^{\infty}(x, y) = \gamma_2'^{\infty}(x, y)$. Also, the weakest γ-arc of C is on P', therefore $\gamma_2(x, y) < \gamma_{P''}^{\infty}(x, y)$ implies that $\gamma_2(x, y) < \gamma_2'^{\infty}(x, y)$. Conversley, if $\gamma_2(x, y) < \gamma_2'^{\infty}(x, y)$, then deleting (x, y) increase γ-strength of connectedness between x and y, hence (x, y) is an IF γ-bridge.
(*iii*) It follows from (*i*) and (*ii*).

4. Conclusions

Graphs theory has a wide range of applications in diverse fields. Fuzzy graph theory has numerous applications in modern sciences and technology, especially in the fields of operations research, neural networks, artificial intelligence and decision making. In this paper, the IFT, IFC, IFB, IFCN and types of arc in an IFG has been investigated. The concepts of intuitionistic fuzzy graphs can be applied in various areas of engineering, computer science: database theory, expert systems, neural networks, artificial intelligence, signal processing, pattern recognition, robotics, computer networks, and medical diagnosis.

Acknowledgement. We would like to provide our cordial thanks to the honourable referees for their valuable comments which helped us to enrich the quality of the paper.

Conflicts of interest. The authors declare no conflict of interest.

Authors' Contributions. All authors contributed equality.

REFERENCES

1. K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica-Verlag, New York, (1999).
2. S. Ashraf, S.Naz, H.Rashmanlou and M. Aslam. Malik, Regularity of graphs in single valued neutrosophic environment, *Journal of Intelligent & Fuzzy Systems*, 33 (1) (2017) 529-542.
3. P. Bhattacharya, Some remarks on fuzzy graphs, *Pattern Recognition Letter*, 6 (1987) 297-302.

4. K.R. Bhutani, On automorphism of Fuzzy graphs, *Pattern Recognition Letter*, 9 (1989)159-162.
5. K.R. Bhutani, A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, *Information Sciences*, 152 (2003) 323-326
6. K.R. Bhutani, A. Rosenfeld, Strong arcs in fuzzy graphs, *Information Sciences*, 152 (2003) 319-322.
7. K.R. Bhutani, A. Battou, On M-strong fuzzy graphs, *Information Sciences*, 155 (2003) 103-109.
8. J.A. Bondy and U.S.R. Murthy, *Graph Theory with Applications*, American Elsevier Publishing Co., New York (1976).
9. R.A. Borzooei and H. Rashmanlou, Cayley interval-valued fuzzy graphs, *UPB Scientific Bulletin, Series A: Applied Mathematics and Physics,* 78(3) (2016) 83-94.
10. G. Ghorai and M. Pal, Some isomorphic properties of m-polar fuzzy graphs with applications, *SpringerPlus*, 5 (2016) 1-21.
11. G. Ghorai, S. Sahoo, M. Pal, Certain graph parameters in bipolar fuzzy environment, *International Journal of Advanced Intelligence Paradigms,* 25 (4) (2023) 234-247.
12. G. Ghorai, M. Pal, Results of m-polar fuzzy graphs with application, *Journal of Uncertain Systems*, 12 (1) (2018) 47-55.
13. M.G. Karunambigai and R. Parvathi, Intuitionistic Fuzzy Graphs, Proceedings of 9th fuzzy Days International Conference on Computational Intelligence, Advances in soft computing: Computational Intelligence, Theory and Applications, Springer-Verlag, 20 (2006), 139-150.
14. S. Kosari, X. Qiang, J. Kacprzyk, Q. Ain, and H. Rashmanlou, A study on topological indices in fuzzy graphs with application in decision making problems, *Journal of Multiple-Valued Logic & Soft Computing,* 42 (2024).
15. S. Kosari, Z. Shao, Y. Rao, X. Liu, R. Cai and H. Rashmanlou, Some types of domination in vague graphs with application in medicine, *Journal of Multiple-Valued Logic & Soft Computing,* 41 (2023).
16. J. Khatun, S.k. Amanathulla and M. Pal, A comprehensive study on m-polar picture fuzzy graphs and its application, *Journal of Uncertain Systems,* doi.org/10.1142/S1752890924500168, (2024).
17. J. Khatun, S.k. Amanathulla and M. Pal, Picture fuzzy cubic graphs and their applications, *Journal of Intelligent & Fuzzy Systems*, Preprint (2024).
18. A. Lakdashti, H. Rashmanlou, P.K.K. Kumar, G. Ghorai and *M. Pal, Some results on edge irregular product vague graphs, International Journal of Advanced Intelligence Paradigms*, 27 (1) (2024) 18-28.
19. S. Mathew and M.S. Sunitha, Types of arcs in a fuzzy graph, *Information Sciences*, 179 (11) (2009) 1760-1768.
20. S. Mathew and M.S. Sunitha, Node connectivity and arc connectivity of a fuzzy graph, *Information Sciences,* 180 (4) (2010) 519-531.
21. J.N. Mordeson and C.S. Peng, Operations on fuzzy graphs, *Information Sciences*, 79 (1994) 159-170.
22. J.N. Mordeson, Fuzzy line graphs, *Pattern Recognition Letter*, 14 (1993) 381-384.

23. J.N. Mordeson and P.S. Nair, Cycles and co-cycles of fuzzy graphs, *Information Sciences*, 90 (1996) 39-49.
24. J.N. Mordeson, and P.S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag, NewYork, (2000).
25. A. Nagoor Gani and S. Shajitha Begum, Degree, order and size in intuitionistic fuzzy graphs, *International Journal of Algorithms, Computing and Mathematics*, (3)3 (2010).
26. R. Parvathi, M.G. Karunambigai and K. Atanassov, Operations on intuitionistic fuzzy graphs, *Proceedings of IEEE International Conference on Fuzzy Systems* (FUZZ-IEEE), August 2009, 1396-1401.
27. A. Rosenfeld, Fuzzy graphs, in: L.A. Zadeh, K.S. Fu, M. Shimura (Eds.), Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York, 1975, 77-95.
28. H. Rashmanlou, S. Samanta, M. Pal and R.A. Borzooei, Bipolar fuzzy graphs with categorical properties, *International Journal of Computational Intelligent Systems*, 8(5) (2015) 808-818.
29. H. Rashmanlou, S. Samanta, M. Pal and R.A. Borzooei, Product of bipolar fuzzy graphs and their degree, *International Journal of General Systems*, doi.org/10.1080/03081079.2015.1072521.
30. H. Rashmanlou and Y.B. Jun, Complete interval-valued fuzzy graphs, *Annals of Fuzzy Mathematics and Informatics*, 6(3) (2013) 677-687.
31. H. Rashmanlou and M. Pal, Antipodal interval-valued fuzzy graphs, *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, 3 (2013) 107-130.
32. H. Rashmanlou and M. Pal, Balanced interval-valued fuzzy graph, *Journal of Physical Sciences*, 17 (2013) 43-57.
33. H. Rashmanlou, S. Samanta, M. Pal and R.A.Borzooei, Intuitionistic fuzzy graphs with categorical properties, *Fuzzy Information and Engineering*, 7 (3) (2015) 317-334.
34. H. Rashmanlou, M. Pal, R. A. Borzooei, F. Mofidnakhaei and B. Sarkar, Product of interval-valued fuzzy graphs and degree, *Journal of Intelligent and Fuzzy Systems*, 35 (6) (2018) 6443-6451.
35. H. Rashmanlou, S. Samanta, M. Pal and R. A. Borzooei, A study on vague graphs, *SpringerPlus*, 5 (1) (2016) 1234.
36. H. Rashmanlou, S. Samanta, M. Pal and R. A. Borzooei, A study on bipolar fuzzy graphs, *Journal of Intelligent and Fuzzy Systems*, 28 (2) (2015) 571-580.
37. H. Rashmanlou and R. A. Borzooei, New concepts of interval-valued intuitionistic (S, T)-fuzzy graphs, *Journal of Intelligent and Fuzzy Systems*, 30 (4) (2016) 1893-1901.
38. Y. Rao, S. Kosari, J. Anitha, I. Rajasingh, and H. Rashmanlou, Forcing parameters in fully connected cubic networks, *Mathematics*, 10(8) (2022) 1263.
39. M.S. Sunitha and A. Vijayakumar, A characterization of fuzzy trees, *Information Sciences*, 113 (1999) 293-300.
40. M.S. Sunitha, A. Vijayakumar, Blocks in fuzzy graphs, *The Journal of Fuzzy Mathematics*, 13 (1) (2005) 13-23.

41. S. Samanta, B. Sarkar, D. Shin and M. Pal, Completeness and regularity of generalized fuzzy graphs, *SpringerPlus,* 5 (2016) 1-14.
42. M. Shoaib, S. Kosari, H. Rashmanlou, M. A. Malik, Y. Rao, Y. Talebi and F. Mofidnakhaei, Notion of the complex Pythagorean fuzzy graph with properties and application, *Journal of Multiple-Valued Logic & Soft Computing,* 34 (2020).
43. X. Shi, S. Kosari, A. A. Talebi, S. H. Sadati and H. Rashmanlou, Investigation of the main energies of picture fuzzy graph and its applications, *International Journal of Computational Intelligence Systems*, 15(1) (2022) 31.
44. X. Shi, S. Kosari, H. Rashmanlou, S. Broumi and S. S. Hussain, Properties of interval-valued quadri partitioned neutrosophic graphs with real-life application, *Journal of Intelligent & Fuzzy Systems*, (2023) 1-15.
45. X. Shi, S. Kosari, S. H. Sadati, A.A. Talebi and A. Khan, Special concepts of edge regularity in cubic fuzzy graph structure environment with an application, *Frontiers in Physics*, 11 (2023) 1222150.
46. Z. Shao, S. Kosari, M. Shoaib and H. Rashmanlou, Certain concepts of vague graphs with application to medical diagnosis, *Frontiers in Physics*, 8 (2020) 357.
47. A.A. Talebi, J. Kacprzyk, H. Rashmanlou and S.H. Sadati, A new concept of an intuitionistic fuzzy graph with applications, *Journal of Multiple-Valued Logic & Soft Computing*, 35 (2020).
48. A.A.Talebi, H.Rashmanlou and S.H.Sadati, Interval-valued intuitionistic fuzzy competition graph, *J. of Mult.-Valued Logic & Soft Computing*, 34 (2020) 335-364.
49. A.A.Talebi, H.Rashmanlou and S.H.Sadati, New concepts on m-polar interval-valued intuitionistic fuzzy graph, *TWMS J. App. and Eng. Math*., 10 (3) (2020) 806-818.
50. A. A. Talebi and H. Rashmanlou, Isomorphic on interval-valued fuzzy graphs, *Annals of Fuzzy Mathematics and Informatics*, 6 (1) (2013) 47-58.
51. A. A. Talebi and W. A. Dudek, Operations on level graphs of bipolar fuzzy graphs, *Bulletin Academiel De Stiinte A Republic Moldova Mathematica,* 2 (81) (2016) 107-124.
52. A.A.Talebi and H. Rashmanlou, Complement and isomorphism on bipolar fuzzy graphs, *Fuzzy Information and Engineering*, 6 (4) (2014) 505-522.
53. Y. Talebi, H. Rashmanlou, New concepts of domination sets in vague graphs with applications, *International Journal of Computing Science and Mathematics*, 10 (4) (2019) 375-389.
54. R.T. Yeh and S. Y. Bang. Fuzzy relations, fuzzy graphs and their applications to clustering analysis, In Fuzzy sets and their Applications to Cognitive and Decision Processes, Zadeh, L. A., Fu K.S. Shimara, M. Eds; Academic Press, New York.(1975), 125-49.
55. L.A. Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.