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On Spectrum and Energy of Fuzzy Graphs

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Abstract. The idea of fuzzy graphs was proposed by Rosenfeld as a generalization of Euler's graph theory. The relationship between a fuzzy graph and its adjacency matrix presents the spectrum formula of a fuzzy graph. The concept of energy is closely related to the spectrum of a graph This study focuses on the weighted fuzzy graph. We provide the eigenvalue, spectrum, spectral radius, and energy of a fuzzy graph associated with the adjacency matrix. Ultimately, we utilize these tools to construct an algorithm that effectively addresses energy determination from fuzzy graphs.

Keywords: Fuzzy graph, energy of a graph, adjacency matrix.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

In our daily lives, we face many situations and difficulties that involve enormous uncertainty. To overcome these difficulties, scientists around the world are starting to design mathematical tools that can deal with these difficulties accurately. The first inventor was Zadeh (1965) who discovered the idea of fuzzy set theory. Fuzzy sets are a powerful logic designed to quantify uncertain information and play an important role in solving real-life problems such as decision-making. In some life situations, users find it difficult to deal with the uncertainty of a single framework. For example, when someone is asked about the temperature forecast for tomorrow here, that person will find it difficult to specify the answer in a single frame, but at the same time, he or she will find greater flexibility and ease when asked to specify the answer in interval form. This reason prompted Zadeh once again to redefine the concept of fuzzy sets in interval form when he defined the meaning of interval-valued fuzzy sets.

Zadeh [20] introduced the notion of a fuzzy set (FS) as a way to expand on the idea of a crisp set in order to show how items might belong to particular categories even when there is uncertainty involved. The concept of fuzzy graph theory has captivated the attention of numerous scholars, motivating them to make significant contributions. For instance, Mordeson and Chang-Shyh [9] demonstrated various operations that may be performed on fuzzy graphs. The combination of these two concepts resulted in the emergence of a novel concept known as fuzzy graph energy, as proposed by Narayanan and Mathew [10].

Furthermore, Qiang et al. [11] studied concepts of domination in vague graphs with application in medical sciences. Some properties of complex Pythagorean fuzzy graph proposed by Shoaib et al. [19]. A Survey on domination in vague graphs with application in transferring cancer patients between countries defined by Rao et al. [12]. Shi et al. [17] presented the applications. Kosari et al. [8] investigated novel concepts in vague graphs and cubic graphs.

The development of research in decision-making theory is very rapid. The development of the degree of collection function of alternative two-dimensional universal sets in algebraic mathematical form (matrix form) turns out to provide more useful results for better choices in signals and systems as well as other fields such as decision-making problems and medical diagnosis [2]. Furthermore, two algorithms have also been devised to handle this group of decision-making problems, the first relying on the score function and the second based on the corresponding matrix aggregation operator. Apart from that, decision-making techniques can also be based on similarity measures from fuzzy interval values . A new algorithm has been discovered which is applied to medical diagnosis to determine whether a patient is suffering from respiratory disease [1]. Using positive and negative pooling functions and multi-argument functions, this structure works best for testing it. This makes them better at solving real-world problems, especially problems that have good and bad sides [3]. Decision-making algorithms involving energy from interval-valued fuzzy graphs can be seen in [5,14].

Graphs model many real-world systems. Graphs show these systems' entity relationships. Electrical, computer, and ecosystem connections might be physical or relationship-based. Graphs represent networks. The concept of graph theory was initially developed by Euler in the year 1736. The concept of energy is closely related to the spectrum of a graph. The name of this concept is derived from its inspiration in the field of chemistry, specifically energy. In 1978, Gutman [6] mathematically characterized the study of electron energy in chemistry for graphs. Molecular graphs are graphical representations that can be used to depict organic compounds.

Additionally, linking matrices to various graphs has grown in popularity as a field of study in recent years. This topic can be seen in [15,16]. Romdhini et al. [14] in keeping with the preceding pattern of contributions on fuzzy graph environments, concentrate on expressing the signless Laplacian matrix for the interval-valued fuzzy graph. Moreover, Shi et al. [18] described an investigation of the main energies of picture fuzzy graphs and their applications..

The structure of this paper is as follows. In Section 2, we present many established definitions and discuss fundamental concepts that are pertinent to our study. In Section 3, we present the spectrum of the fuzzy graph together with illustrative calculation examples. The conclusions of this investigation are summarized in Section 4.

2. Preliminaries

In this section, we will discuss several basic concepts of fuzzy sets. Zadeh introduced the concept of fuzzy sets in 1965 to describe the belonging of objects to certain sensations under uncertainty. The Zadeh fuzzy set is characterized by a section called single truth membership so that its values fall into the closed interval [0, 1].

Definition 2.1. [20] Fuzzy set A on universal set V is an ordered pair set

$$A = \{\langle v, \mu_A(v)\rangle : v \in V\}$$

where $\mu_A(v)$ is a membership function of v in A which links every element of V with a real number in the interval [0,1].

Zimmermann [21] has provided a comprehensive explanation of fuzzy set theory and its applications, explaining the basic concepts underlying fuzzy theory and its relationship to fuzzy set theory.

Definition 2.2. [21] For fuzzy sets A dan B and for all $v \in V$, the following hold:

i. $A = B$ if and only if $\mu_A(v) = \mu_B(v)$.
ii. $A \subseteq B$ if and only if $\mu_A(v) \leq \mu_B(v)$.
iii. Complement of A is defined as $\mu_A^C(v) = 1 - \mu_A(v)$.
iv. If $Q = A \cup B$, then $\mu_Q(v) = max\{\mu_A(v), \mu_B(v)\} = \mu_A(v) \vee \mu_B(v)$.
v. If $C = A \cap B$, then $\mu_C(v) = min\{\mu_A(v), \mu_B(v)\} = \mu_A(v) \wedge \mu_B(v)$.
vi. $1 - max\{\mu_A(v), \mu_B(v)\} = min\{1 - \mu_A(v), 1 - \mu_B(v)\}$.

Furthermore, we include some basic concepts of fuzzy graphs in this section. The idea of fuzzy graphs was proposed by Rosenfeld as a generalization of Euler's graph theory. The idea of fuzzy graph theory aroused the interest of many researchers and encouraged them to make many contributions.

Definition 2.3. [10] Let $\Gamma = (P, Q)$ be a weighted fuzzy graph with $|P| = n$. The weight is a function $w: P \times P \to [0,1]$ with $w_{ij} = \mu(v_i, v_j) \in [0,1]$ be the weight between the vertex v_i and v_j in P. Then the adjacency matrix of a fuzzy graph Γ is a square matrix of order n, denoted by $A = [a_{ij}]$ whose (i,j) −entries are

$$a_{ij} = \begin{cases} w_{ij}, & \text{if } v_i \neq v_j \text{ and they are adjacent} \\ 0, & \text{otherwise.} \end{cases}$$

Since we are aware that A is a square matrix, we can determine its eigenvalues. We now arrive at the graph concept's energy thanks to this formulation. We now go on to A 's eigenvalue definition.
For an $n \times n$ identity matrix I_n, the characteristic polynomial of A is

$$P_A(\lambda) = |\lambda I_n - A|.$$

Let $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of Γ as the roots of $P_A(\lambda) = 0$.

The definition of the spectrum of fuzzy graph is given below.

Definition 2.4. [10] Let A be an $n \times n$ adjacency matrix of fuzzy graph Γ. The spectrum of Γ associated with A is defined as the list of eigenvalues of A, $\lambda_1, \lambda_2, \cdots, \lambda_n$, with respective multiplicities $k_1, k_2, \cdots, k_n$ and denoted by

$$\sigma = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ k_1 & k_2 & \cdots & k_n \end{pmatrix}.$$

Based on Definition 2.4, we present the following definitions for determining the spectral radius and energy of fuzzy graph.

Definition 2.5. [7] Let $\Gamma = (P, Q)$ be a weighted fuzzy graph, where $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of A. Then the spectral radius of Γ is denoted by ρ and defined as

$$\rho(\Gamma) = max\{|\lambda| : \lambda \in \sigma\}.$$

Definition 2.6. [6] Let $\Gamma = (P, Q)$ be a weighted fuzzy graph, where $\lambda_1, \lambda_2, \dots, \lambda_n$ are the eigenvalues of A. Then the energy of Γ is denoted by E and defined as

$$E = \sum_{i=1}^{n} |\lambda_i|.$$

3. Main results

This section will present several results on the fuzzy graph energy corresponding to the adjacency matrix as defined in Definition 2.3. We begin with two results that provide the energy information of the adjacency matrix.

Theorem 3.1. Let Γ is a fuzzy star graph on n vertices. Then $E(\Gamma) = 2\sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2}$, where $w_i = \mu(v_1, v_i)$ for all $i = 2,3, \cdots, n$.

Proof: A fuzzy star graph Γ on n vertices consist of one vertex of degree $n-1$ and $n-1$ vertices of degree one. Then the adjacency matrix of a fuzzy star graph is

$$A = \begin{bmatrix} 0 & w_2 & w_3 & \cdots & w_n \\ w_2 & 0 & 0 & \cdots & 0 \\ w_3 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ w_n & 0 & 0 & \cdots & 0 \end{bmatrix},$$

where $w_i = \mu(v_1, v_i)$ for all $i = 2,3, \cdots, n$.

We get the characteristic polynomial of A as follows:

$$P_A(\lambda) = |\lambda I_n - A|$$

$$= \begin{vmatrix} \lambda & -w_2 & -w_3 & \cdots & -w_n \\ -w_2 & \lambda & 0 & \cdots & 0 \\ -w_3 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -w_n & 0 & 0 & \cdots & \lambda \end{vmatrix}.$$

By Minor-cofactor, we get the determinant:

$$P_A(\lambda) = \lambda^{n-2}\left(\lambda^2 - (w_2^2 + w_{n-3}^2 + \cdots + w_n^2)\right).$$

The roots of $P_A(\lambda) = 0$ are $\lambda_1 = 0$ of multiplicity $n-2$ and $\lambda_{2,3} = \pm\sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2}$ of multiplicity 1, respectively.
Therefore, the spectrum of Γ is

$$\sigma = \begin{pmatrix} \sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2} & 0 & -\sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2} \\ 1 & n-2 & 1 \end{pmatrix}.$$

The spectral radius of Γ is

$$\rho = \sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2}.$$

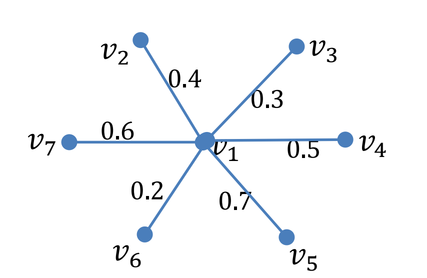
The energy of Γ is

$$E = (n-2)|0| + \left|\pm\sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2}\right| = 2\sqrt{w_2^2 + w_{n-3}^2 + \cdots + w_n^2}.$$

□

The following example is an illustration of Theorem 3.1.

Example 3.1. Let Γ be the fuzzy star graph on 7 vertices as in Figure 1



$$A = \begin{bmatrix} 0 & 0.4 & 0.3 & 0.5 & 0.7 & 0.2 & 0.6 \\ 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1: Fuzzy star graph on 7 vertices

The characteristic polynomial of A is $\lambda^5(\lambda^2 - 1.39)$. It implies that the eigenvalues of A are $\lambda = 0$ with multiplicity (5) and $\lambda = \pm\sqrt{1.39}$ with multiplicity (1), respectively. Therefore,

$$E = (5)|0| + \left|\pm\sqrt{1.39}\right| = 2\sqrt{1.39},$$

conforming Theorem 3.1.

Theorem 3.2. Let Γ is a fuzzy 1-regular graph on n vertices. Then $E(\Gamma) = 2\left(w_1 + w_2 + \cdots + w_{n/2}\right)$, where $w_i = \mu(v_{2i-1}, v_{2i})$ for all $i = 1,2,\cdots,n/2$.

Proof: A fuzzy 1-regular graph Γ on n vertices with all vertices of degree one. Hence, the adjacency matrix of a fuzzy 1-regular graph is as follows:

$$A=\begin{bmatrix} 0 & w_1 & 0 & 0 & \cdots & 0 & 0 \\ w_1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & w_2 & \cdots & 0 & 0 \\ 0 & 0 & w_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & w_{n/2} \\ 0 & 0 & 0 & 0 & \cdots & w_{n/2} & 0 \end{bmatrix},$$

for $w_i=\mu(v_{2i-1},v_{2i})$ for all $i=1,2,\cdots,n/2$. Thus, we get the characteristic polynomial of A as follows:

$$P_A(\lambda)=\begin{vmatrix} \lambda & -w_1 & 0 & 0 & \cdots & 0 & 0 \\ -w_1 & \lambda & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \lambda & -w_2 & \cdots & 0 & 0 \\ 0 & 0 & -w_2 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda & -w_{n/2} \\ 0 & 0 & 0 & 0 & \cdots & -w_{n/2} & \lambda \end{vmatrix}.$$

By Minor-cofactor, we get the determinant:

$$P_A(\lambda)=(\lambda^2-w_1^2)(\lambda^2-w_2^2)\cdots(\lambda^2-w_{n/2}^2).$$

The roots of $P_A(\lambda)=0$ are $\lambda_{1,2}=\pm w_1$, $\lambda_{3,4}=\pm w_2$, $\cdots$, $\lambda_{n-1,n}=\pm w_{n/2}$ of multiplicity 1, respectively. Therefore, the spectrum of Γ is

$$\sigma=\begin{pmatrix} w_1 & w_2 & \cdots & w_{n/2} & -w_1 & -w_2 & \cdots & -w_{n/2} \\ 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \end{pmatrix}.$$

The spectral radius of Γ is

$$\rho=max\{w_i|i=1,2,\cdots,n/2\}.$$

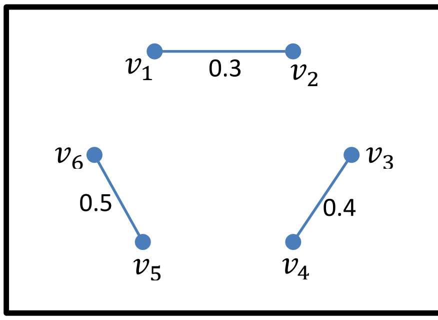
The energy of Γ is

$$\begin{aligned} E &= |w_1|+|-w_1|+|w_2|+|-w_2|+\cdots+|w_{n/2}|+|-w_{n/2}| \\ &= 2(w_1+w_2+\cdots+w_{n/2}). \end{aligned}$$

□

In order to illustrate Theorem 3.2, we present the following example.

Example 3.2. Let Γ be the fuzzy 1-regular graph on 6 vertices as in Figure 2.



$$A=\begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

Figure 2: Fuzzy 1-regular graph on 6 vertices

The characteristic polynomial of A is $(\lambda^2-0.09)(\lambda^2-0.16)(\lambda^2-0.25)$. It implies that the eigenvalues of A are $\lambda_1=0.3$, $\lambda_2=0.4$, $\lambda_3=0.5$, $\lambda_4=-0.3$, $\lambda_5=-0.4$, $\lambda_6=-0.5$ with multiplicity (1), respectively. Therefore,

$$E=|0.3|+|0.4|+|0.5|+|-0.3|+|-0.4|+|-0.5|=2.4,$$

conforming Theorem 3.2.

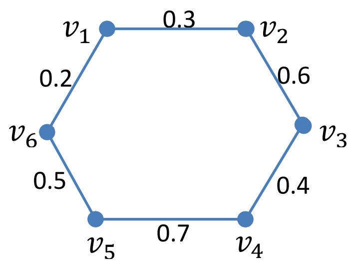
Now, let Γ be a fuzzy circle graph on n vertices which means every vertex has degree two. For $w_i=\mu(v_{2i-1},v_{2i})$, where $i=1,2,\cdots,n/2$, then the adjacency matrix of a fuzzy circle graph is as follows:

$$A=\begin{bmatrix} 0 & w_1 & 0 & 0 & \cdots & 0 & w_{n-1} \\ w_1 & 0 & w_2 & 0 & \cdots & 0 & 0 \\ 0 & w_2 & 0 & w_3 & \cdots & 0 & 0 \\ 0 & 0 & w_3 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & w_{n-2} \\ w_{n-1} & 0 & 0 & 0 & \cdots & w_{n-2} & 0 \end{bmatrix}.$$

We get the characteristic polynomial of A as follows:

$$P_A(\lambda)=\begin{vmatrix} \lambda & -w_1 & 0 & 0 & \cdots & 0 & -w_{n-1} \\ -w_1 & \lambda & -w_2 & 0 & \cdots & 0 & 0 \\ 0 & -w_2 & \lambda & -w_3 & \cdots & 0 & 0 \\ 0 & 0 & -w_3 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda & -w_{n-2} \\ -w_{n-1} & 0 & 0 & 0 & \cdots & -w_{n-2} & \lambda \end{vmatrix}.$$

Example 3.3. Let Γ be the fuzzy circle graph on 6 vertices as in Figure 3.



$$A = \begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 & 0.2 \\ 0.3 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0.5 \\ 0.2 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

Figure 3. Fuzzy circle graph on 6 vertices

The characteristic polynomial of A is $(\lambda^2 - 0.9717)(\lambda^2 - 0.3588)(\lambda^2 - 0.05947)$. It implies that the eigenvalues of A are $\lambda_1 = 0.98575$, $\lambda_2 = 0.59902$, $\lambda_3 = 0.243868$, $\lambda_4 = -0.98575$, $\lambda_5 = -0.59902$, $\lambda_6 = -0.243868$ with multiplicity (1), respectively. Therefore,

$$\begin{aligned} E &= |0.98575| + |0.59902| + |0.243868| + |-0.98575| + |-0.59902| \\ &\quad + |-0.243868| \\ &= 3.657276. \end{aligned}$$

Let Γ be a fuzzy line graph on n vertices which means two vertices have degree one, and $n-2$ vertices of degree two. Then the adjacency matrix of a fuzzy line graph is as follows:

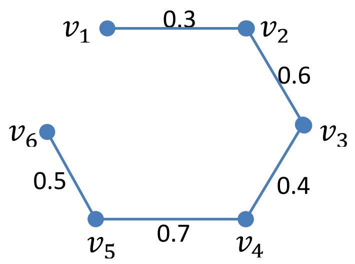
$$A = \begin{bmatrix} 0 & w_1 & 0 & 0 & \cdots & 0 & 0 \\ w_1 & 0 & w_2 & 0 & \cdots & 0 & 0 \\ 0 & w_2 & 0 & w_3 & \cdots & 0 & 0 \\ 0 & 0 & w_3 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & w_{n-2} \\ 0 & 0 & 0 & 0 & \cdots & w_{n-2} & 0 \end{bmatrix},$$

where $w_i = \mu(v_{2i-1}, v_{2i})$ for all $i = 1,2,\cdots,n/2$.

We get the characteristic polynomial of A as follows:

$$P_A(\lambda) = \begin{vmatrix} \lambda & -w_1 & 0 & 0 & \cdots & 0 & 0 \\ -w_1 & \lambda & -w_2 & 0 & \cdots & 0 & 0 \\ 0 & -w_2 & \lambda & -w_3 & \cdots & 0 & 0 \\ 0 & 0 & -w_3 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \lambda & -w_{n-2} \\ 0 & 0 & 0 & 0 & \cdots & -w_{n-2} & \lambda \end{vmatrix}.$$

Example 3.4. Let Γ be the fuzzy line graph on 6 vertices as in Figure 4.



$$A=\begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 & 0 \\ 0.3 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0.7 & 0 \\ 0 & 0 & 0 & 0.7 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

Figure 4: Fuzzy line graph on 6 vertices

The characteristic polynomial of A is $(\lambda^2-0.9434)(\lambda^2-0.39698)(\lambda^2-0.0096)$. It implies that the eigenvalues of A are $\lambda_1=0.97129$, $\lambda_2=0.630066$, $\lambda_3=0.0980429$, $\lambda_4=-0.97129$, $\lambda_5=-0.630066$, $\lambda_6=-0.0980429$ with multiplicity (1), respectively. Therefore,

$$E=|0.97129|+|0.630066|+|0.0980429|+|-0.97129|+|-0.630066| + |-0.0980429| = 3.3987978.$$

We now present the algorithm of determining the energy and spectral radius based on the tools proposed in this work, so it can be used to solve problems.

Algorithm:

1. Determine the vertex $v_1, v_2, \cdots, v_n$, the edge between v_i and v_j, and the weight of every edge ω_{ij}, for $i,j=1,2,\cdots,n$
2. Construct the fuzzy graph $\Gamma=(P,Q)$ with $P=\{v_1,v_2,\cdots,v_n\}$.
3. Analyze the connectivity of Γ.
4. Construct the adjacency matrix A of Γ.
5. Compute the eigenvalues of Γ associated to A.
6. Calculate the energy of Γ, E.
7. Select the largest value of Γ, ρ.

4. Conclusion

The fuzzy graph can enhance both flexibility and precision, making it more suitable for modelling certain situations compared to a regular graph. Graph energy has been widely utilized in various disciplines in recent times. Furthermore, we analyze the spectrum, energy, and spectral radius of the fuzzy graph by utilizing the appropriate adjacency matrix.

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