

Investigation of a Fuzzy Imperfect Production Model for Deteriorating Items with Demand Depends on Warranty Period and Green Levels

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Abstract. A lot of attention has been paid to the integration of quality, environmental factors, product warranty, and greenness into industrial processes in research on production and inventory management for goods that are becoming worse. The current marketing situation has high uncertainty in nature. Combining the above-mentioned factors a fuzzy production model is developed. In order to assist decision-makers in making more informed choices regarding pricing and replenishment, this paper evaluates the effects on the overall profit of greenness, product warranty, and carbon-reduction technology. An algorithm will need to be used to determine the optimal values for five decision variables. In one experiment, the mathematical formula is numerically demonstrated, and the effects of different factors on total profit and managerial insights are examined through a sensitivity analysis.

Keywords: Fuzzy production model; Deteriorating item; Warranty and green level dependent demand

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

A comprehensive literature review is used to gather and evaluate the pertinent papers. Yadav et al. [1] and Yadav and Desai [2]. Three steps make up the main technique described in this work: (1) a comprehensive keyword search of the pertinent literature; (2) a close examination of the published manuscript and its summary; and (3) an explanation of how their research has contributed to this expansion. Scientific research manuscripts from prestigious journals like Elsevier and Springer are reviewed, and the Web of Science is used as the database. This is done in an effort to cover as much literature as possible. An inventory model for deteriorating goods was developed in a substantial study that was carried out earlier than the research suggested by Tiwari et al. [3]. This model simultaneously considered carbon emissions and subpar manufacture. They looked at an integrated approach for the two-echelon inventory model, in which the transportation, storage, and disposal of damaged goods results in the release of carbon. In a different paper that looked at sustainable inventory models for deteriorating goods, Bai et al. [4] developed

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a trade-and-cap approach for emissions from production and storage processes where emissions can be decreased by using green technology. The main objective of this effort is to determine pricing and optimal replenishment in both decentralised and centralised scenarios. Different payment options, such as cash, credit, and prepayment, have an effect on a model of depreciating inventory in 2020, according to Shi et al. [5]. They have researched the possibility of investing in green technologies in conjunction with a carbon tax plan for emissions coming from manufacturing processes. Mishra et al. [6] investigated a feasible inventory problem that prioritises financing technologies that simultaneously reduce carbon emissions, ordering costs, and deterioration. This study considers a continuous emission when demand is determined by both price and hybrid stock. A Stackelberg competitive model for steady-state degradation is put forth in this work. Examining the outcomes of green investing technology to reduce emissions in accordance with cap-and-trade and its regulations is the main goal of this essay. Mishra et al. [7] study from 2021, which looked at ways to manage carbon emissions and product deterioration in a greenhouse farm concurrently, addressed other shortages. In this essay, price-dependent demand functions that are linear and non-linear are compared. Price discount inventory model based on advance and Duary et al. [8] introduced delayed payment for deteriorating inventory model under shortages. Khan et al. [9] presented a hybrid payment model for cash and advances, with time-varying holding costs and time-dependent demand, as a non-instantaneous deteriorating inventory model. Rahman et al. [10, 11] studied a declining inventory model with a discount facility in an interval environment. As a novel approach to dealing with interval uncertainty, they have employed the parametric approach of interval.

Chen and Ouyang [12] introduced a deteriorating inventory model with Fuzzy valued inventory parameters. Dutta and Kumar [13] studied a fuzzy inventory model for fully backlogged items that are deteriorating and experiencing shortages. Kumar et al. [14] proposed a time-dependent demand and partial backlog for deteriorating items in a fuzzy inventory model. Vijai Stanly and Uthayakumar [15] introduced a fuzzy inventory model with exponential demand and shortages for deteriorating items. Hou et al. [16] state that the first study in the field examining quality improvement in a sustainable production model ought to be written. The contribution of this work to previous research is minimal, though, as no suggestions for reducing emissions are provided. As a trade-and-cap mechanism for defective items in a production firm where carbon is released due to the production, transportation activities, and remanufacturing, Battini et al. [17] proposed the study that was expanded by Shu et al. [18]. They concluded that the cost and emissions associated with remanufacturing activities are reduced when carbon emission regulations are enforced. Subsequently, Kazemi et al. [19] looked into a model that showed how processes like scrapping and warehousing release carbon. This paper's main innovation is its examination of the relationship between inspection and long-term inventory management. Indrajitsingha et al. [20] proposed a fuzzy inventory model that considers stock dependence for deteriorating items' demand rate. According to Tiwari et al. [21], inventory management can reduce carbon emissions by investing in items of imperfect quality. The Rout et al. [22] paper, which developed an integrated production model accounting for objects deteriorating when defective manufacturing is accessible, was one of the articles that aided in our investigation. Analysing the impact of each method of reducing carbon emissions on the total amount of money generated by the inventory system

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has been the main objective of this study. But no one has yet talked about how to reduce uneven manufacturing, deterioration of goods, or carbon emissions. Finally, Sarkar et al. [23] suggested a coordinated three-tier supply chain that deteriorates in relation to expiration dates. The goal of this research is to reduce the incidence of subpar products by utilising technology and quality improvement strategies. In a fuzzy environment, Manna et al. [24, 25] investigated an imperfect production model. Fuzzy differential and fuzzy integral methods have been employed to depict this model. Rani et al. [26] proposed Fuzzy inventory model in a green supply chain for deteriorating new and refurbished items that can be cannibalised. Kumar et al. [27] suggested under the influence of learning, a sustainable fuzzy inventory model for deteriorating items with partial backordering and social and environmental responsibility is provided. Kumar et al. [14] research on fuzzy inventory models for deteriorating products with seasonal demand patterns that repeat.

2. Problem description and notation

In order to close the gap, a manufacturing model based on Rout et al. [22] was developed for a system that takes into consideration a single production system and single customer. The producer maintains a consistent rate of production in order to fulfil the retailer's demand. Actually, it is not insignificant that defective (lower-quality) goods are produced as a result of equipment failures or human error. Hence, a certain amount of defective products (ξ) remain, and the defective items have undergone reworking. Some of the refurbished items are still beyond repair and will be thrown away, even if they are deemed flawless. The beginning defective rates of goods (ξ) might be decreased by investing of the improvement of quality enhancement technology, according to the research written by Sarkar et al. [23]. Furthermore, the manufacturer's goods deteriorate with time. To slow down deterioration and decrease the quantity of items that deteriorate and become useless, investments are made in preservation technologies (Mishra et al., [6]). Carbon emissions are produced during the production, recycling, disposal, deterioration, and storage of the items. In line with changing the sustainability of the production system, the manufacturer makes investments in green technology to reduce emissions (Jaber et al., [28]). Furthermore, the effects of green investment technology and investments that prevent carbon emissions are discussed together. Eventually, perfect things will have a set selling price. In this connection, we may refer to some works related to crisp, fuzzy and interval environments [29-36].

Notation

For accurate model analysis, the terms and underlying presumptions used are listed below.

Notation	Units	Usual meanings
\tilde{A}	\$/order	Fuzzy ordering cost
$\tilde{\xi}$	%	Fuzzy defective rate
$\tilde{\mu}$	%	Fuzzy rate of discount
$I(t)$	Units	Inventory level over the time period $0 \leq t \leq T$
$\tilde{\delta}$	constant	Fuzzy rate of deterioration
\tilde{h}	\$/unit/time unit	Fuzzy unit holding cost per time unit

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\tilde{I}_e	\$/time unit	Fuzzy loan interest rate
\tilde{Q}	Units	Fuzzy ordering size in each cycle
$\tilde{\eta}$	constant	Fuzzy cost of green technology
\tilde{c}_p	\$/unit	Fuzzy unit cost of purchase
\tilde{M}	Time unit	Fuzzy advance payment time point
$\tilde{\pi}(g, p, T)$	\$/time unit	Fuzzy average profit
Decision variables		
T	Time unit	Business period.
P	\$/unit	selling price per unit
g	--	green level

Assumption

- i) Under a single-item model, which is characterised by the instantaneous replenishment of inventory, a single producer supplies goods to a single retailer.
- ii) Lead times are disregarded along with shortages. According to Rout et al. (2020), everything degenerates at the same rate (δ).
- iii) The combination of a product's price, warranty duration, and green level determines its demand is fuzzy in nature. In terms of mathematics, it is represented as $\tilde{D}(g, p, w) = \tilde{a} + \tilde{c}g^\alpha - \tilde{b}p + \tilde{\beta}w^\gamma$ where $0 < \alpha, \gamma \leq 1$.
- iv) Fuzzy imperfect production is taken with the rate (ξ) and the fuzzy rate of production (P) is considered as constant.
- v) Manufacturers offer some fuzzy discount ($\nu\%$) on the purchase price of the product.

3. Mathematical formulation

Let us consider a manufacturing firm that produces a single item with the rate P . During the production process they will also produce some imperfect item with the rate ξ . All the items are stored in a stocking point in order to satisfy the retailer's demand. This production process is continued up to the time period $t = t_1$. In this instance, the customer purchases the goods by paying an advance of the total purchase cost before receiving a large quantity. Prior to receiving goods from a seller who gives an early payment discount, a customer first purchases items, or Q units, by paying the full purchase price. Due to the combined effects of the demand $D(g, p, w)$ and the deterioration, the inventory level decreased, and at time $t = T$ the stock level was zero. A pictorial representation of the inventory system is shown in Figure 1.

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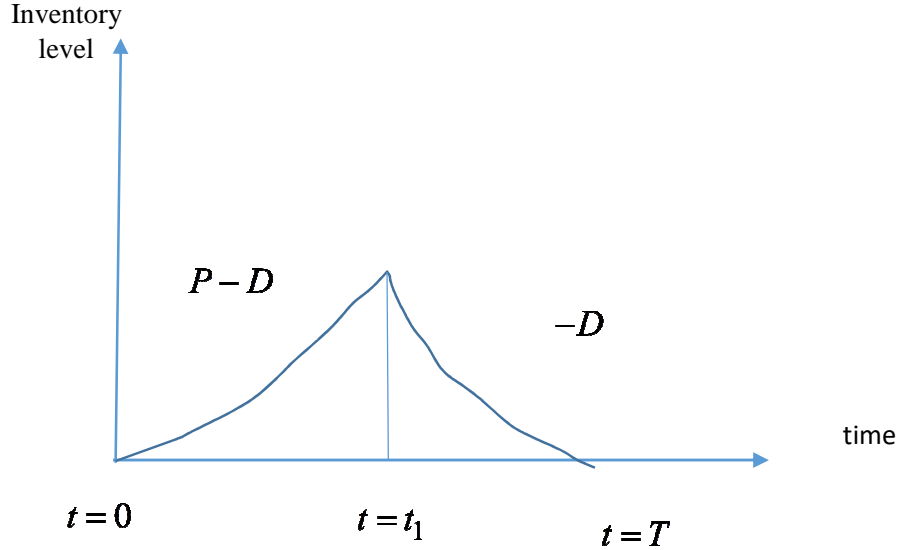


Figure 1: Pictorial representation of the production system.

As a result, the suggested model followed the given differential equation.

$$\frac{d\tilde{q}(t)}{dt} + \delta\tilde{q}(t) = (1-\xi)\tilde{P} - \tilde{D}(g, p, w), \quad 0 \leq t \leq t_1 \quad (1)$$

$$\frac{d\tilde{q}(t)}{dt} + \delta\tilde{q}(t) = -\tilde{D}(g, p, w), \quad t_1 < t \leq T \quad (2)$$

with the boundary conditions $\tilde{q}(0) = \tilde{Q}$, $q(T) = 0$ and $\tilde{q}(t)$ is continuous at $t = t_1$.

Let us assume that $\tilde{k} = (1-\xi)\tilde{P} - \tilde{D}(g, p, w)$

Solving the equation (1)-(2) with the help of boundary condition $q(0) = Q$ and $q(T) = 0$, we have

$$\tilde{q}(t) = \frac{\tilde{k}}{\delta} + \left(\tilde{Q} - \frac{\tilde{k}}{\delta} \right) e^{-\delta t} \quad (3)$$

$$q(t) = \frac{\tilde{D}(g, p, w)}{\delta} \left\{ -1 + e^{\delta(T-t)} \right\} \quad (4)$$

Now, using the condition of continuity the maximum stock can be written as:

$$\tilde{Q} = \frac{\tilde{k}}{\delta} + \left[-\frac{\tilde{k}}{\delta} + \frac{\tilde{D}(g, p, w)}{\delta} \left\{ -1 + e^{\delta T} \right\} \right] e^{\delta t_1} \quad (5)$$

The associated costs of the inventory system are given below:

- i) Ordering cost(OC)= \tilde{A}
- ii) Production cost (PC)= $\tilde{p}_c \tilde{P} t_1$

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$$\begin{aligned} \text{iii) Holding cost (HC)} &= \tilde{h} \left[\int_0^{t_1} q(t) dt + \int_{t_1}^T q(t) dt \right] \\ &= \tilde{h} \left[\frac{\tilde{k}t_1}{\tilde{\delta}} + \frac{1}{\tilde{\delta}} \left(Q - \frac{\tilde{k}}{\tilde{\delta}} \right) (1 - e^{-\tilde{\delta}t_1}) \right] + \frac{\tilde{h}\tilde{D}(p, g, w)}{\tilde{\delta}^2} \{ e^{\tilde{\delta}(T-t_1)} - \tilde{\delta}(T-t_1) - 1 \} \end{aligned}$$

$$\text{iv) Warranty cost (WC)} = \tilde{\kappa}\tilde{\eta}\tilde{D}(g, p, w)\tilde{w}T$$

$$\text{v) Green level cost (GLC)} = \tilde{\mu}\tilde{g}T$$

$$\text{vi) Revenue earned from sales (SR)} = p(1 - \tilde{d})\tilde{D}(g, p, w)T$$

Hence, the profit of the system is as follows:

$$T\tilde{P} = SR - A - PC - HC - WC - GLC \quad (4)$$

Therefore, the average profit is given by:

$$\tilde{\pi}(g, p, T) = \frac{T\tilde{P}}{T} \quad (5)$$

i.e.,

$$\tilde{\pi}(g, p, T) = \frac{1}{T} \left[\begin{aligned} &p(1 - \tilde{d})\tilde{D}(g, p, w)T - \tilde{A} - \tilde{p}_c\tilde{P}t_1 - \tilde{h} \left[\frac{\tilde{k}t_1}{\tilde{\delta}} + \frac{1}{\tilde{\delta}} \left(Q - \frac{\tilde{k}}{\tilde{\delta}} \right) (1 - e^{-\tilde{\delta}t_1}) \right] \\ &- \frac{\tilde{h}\tilde{D}(g, p, w)}{\tilde{\delta}^2} \{ e^{\tilde{\delta}(T-t_1)} - \tilde{\delta}(T-t_1) - 1 \} - \tilde{\kappa}\tilde{\eta}\tilde{D}(g, p, w)\tilde{w}T - \tilde{\mu}\tilde{g}T \end{aligned} \right] \quad (6)$$

Now, we have to optimize the objective function (6) in terms of the decision variables p, g and T . In the next section, we are going to discuss about the theoretical derivations of the objective function (6).

4. Numerical illustration

To validate the proposed model, a single numerical instance is examined. The following are the values of the system parameters:

Example:

Below are the values for the different inventory parameters connected to the recommended model. To solve the problem, the GMIV technique is used to difuzzify the inventory parameter values.

$$\begin{aligned} \tilde{\alpha} &= (115, 120, 125), \tilde{b} = (2.5, 2.7, 2.9), \tilde{c} = (1.9, 2.1, 2.3), \\ \tilde{\alpha} &= (0.3, 0.5, 0.7), \tilde{\beta} = (0.3, 0.5, 0.7), \\ \tilde{\gamma} &= (0.05, 0.07, 0.09), \tilde{d} = (0.09, 0.1, 0.3), \tilde{\xi} = (0.03, 0.05, 0.07), \\ \tilde{P} &= (195, 200, 205), \tilde{\delta} = (0.05, 0.07, 0.09), \\ \tilde{\kappa} &= (13, 15, 17), \tilde{\eta} = (0, 1, 0.2, 0.3), \tilde{\mu} = (1, 2, 3), \\ \tilde{A} &= \$(495, 500, 505) / order, p_c = \$(15, 20, 25) / unit, \\ t_1 &= (0.1, 0.2, 0.3) months, w = (1.5, 2, 2.5) months, \\ h &= \$(1.5, 2, 2.5)/unit / month. \end{aligned}$$

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Table 1: Finding the best answers to solve the objective function

Decision variables/dependent variables/objective function value	Optimal values
p	\$ 35.28467
T	3.567124 Months
g	87.69437%
π	\$ 569.5859
Q	382.4573

5. Sensitivity analysis

Sensitivity tests are carried out in relation to numerical Example-1 by altering the parameter values from -20 percent to +20 percent in order to show the impact of the various production system parameters on total cycle duration (T), starting stock level", maximum green level", and profit per unit ". The ideal results of these studies are shown graphically in Figure 2-8.

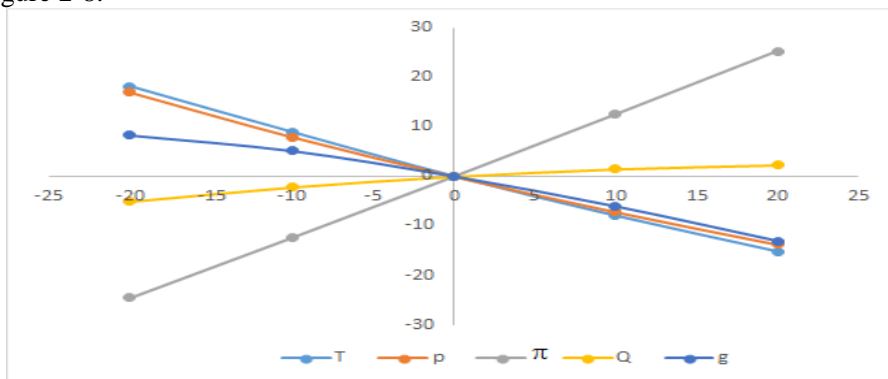


Figure 2: Post optimality analysis of \tilde{a}

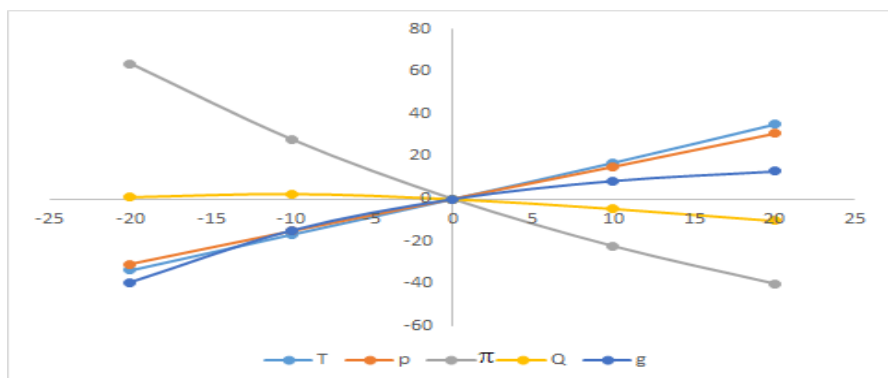


Figure 3: Post optimality analysis of \tilde{b}

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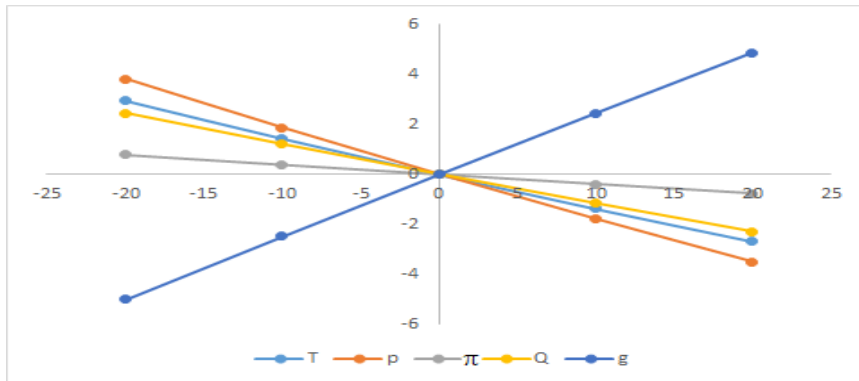


Figure 4: Post optimality analysis of $\tilde{\delta}$

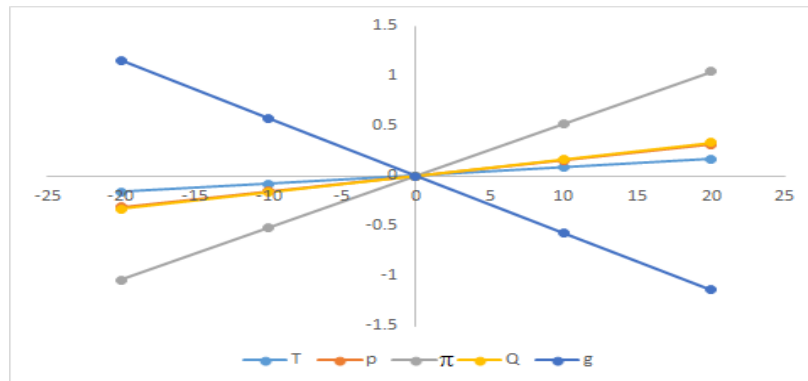


Figure 5: Post optimality analysis of $\tilde{\mu}$

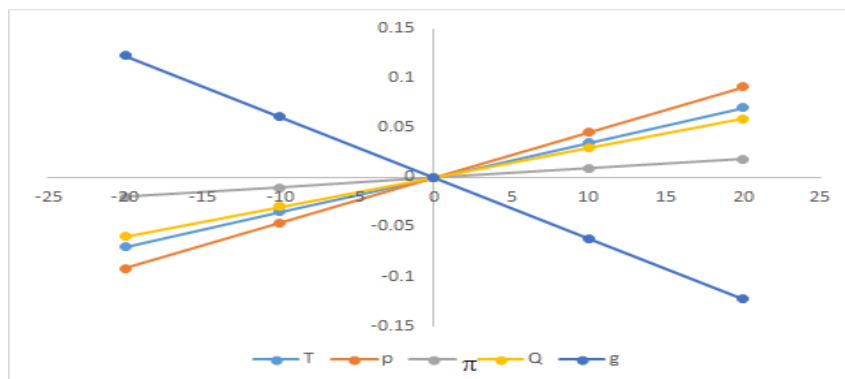


Figure 6: Post optimality analysis of \tilde{h}

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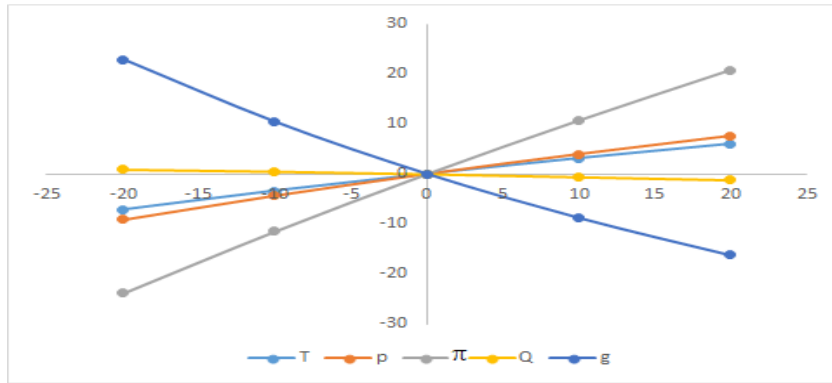


Figure 7: Post optimality analysis of \tilde{c}_p

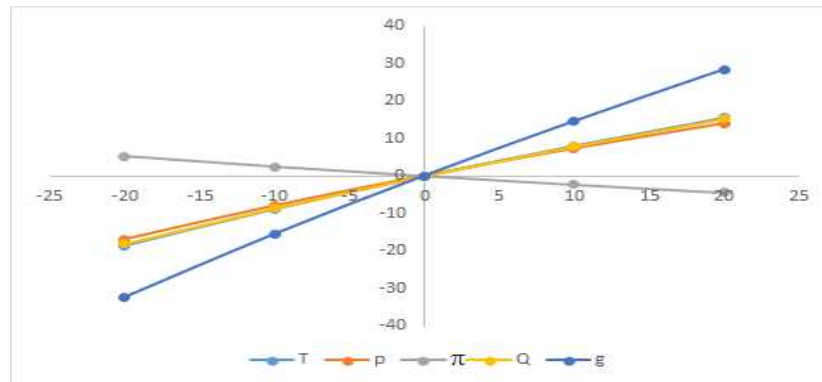


Figure 8: Post optimality analysis of \tilde{A}

According to Figure 2-8, the profit per unit (π) is quite sensible to changes in the price of selling (p), demand location parameter ' a ', and parameter scaling ' b ', whereas (π) has the opposite impact on (b). In contrast to the average profit (π), which is insensitive to changes in either direction for ordering costs ' K_o ', ' μ ' and ' c_h ' the profit per unit (π) is few sensitive to changes in either direction for costs ' δ ' and ' c '.

Figure 2-8 shows that the business length (T) significantly affects whether the demand parameter ' b ' changes positively or negatively. The business period (T) is often equally sensible to changes in the ordering cost ' A ' and the location parameter of demand ' a ', which might be positive or negative. The business period (T) is also sensitive in less to variations of the price of selling (p), constant rate of deterioration (δ), and holding cost (c_h), but the opposite is true for (δ) and (c_h).

The beginning stock level (Q) is often equally sensible to changes in ordering cost ' A ', whether they are positive or negative. The starting inventory level (Q) is less vulnerable to negative or positive changes in ' δ ', ' c_h ' and ' a ' respectively, whereas ' δ ' and are

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adversely affected. With negative or positive modifications of ' α ', ' b ' and ' μ ' respectively, the original stock level (Q) is insensitive.

6. Conclusion

This work constructs an imperfect production inventory model for degrading commodities by taking into account the nonlinear green level of products, the nonlinear warranty of an item, and the demand rate as a function of price. The model performs better when demand dependent on greenness and prepayment with a predetermined percentage discount advantage are included. The cost of obtaining environmentally friendly technologies adds even more weight to the proposed idea. With LINGO software, the goal function's concavity is solved. The results of the sensitivity analysis indicate that the location parameter, selling price, and demand scaling parameter are significantly impacted by the average profit, in both positive and negative directions. Consequently, a manager or decision maker should consider these factors more carefully when making decisions. Numerous industries, such as the food, vegetable, and pharmaceutical ones, can benefit from the idea behind this strategy.

One can extend this model by incorporating stock dependent demand, nonlinear holding cost, advance payment, and trade credit facility. Also, this model may be extended in the future to more imprecise environments, such as interval and Type-2 interval, by taking into account the uncertainty in the inventory parameters (Rahman et al., 2020).

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Conflict of interest. This is a single-author paper, so there is no conflict of interest.

Authors' Contributions. This is a single-author paper and it is fully the author's contribution.

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