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Neutrosophic Functions and Solution of Neutrosophic Ordinary Differential Equations by Predictor-Corrector Method

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Abstract. In the realm of neutrosophic mathematics, solving differential equations introduces complexities due to the uncertain nature of the parameters involved. This study explores the application of Milne's predictor and corrector formula within the neutrosophic environment to solve ordinary differential equations (ODEs) with initial conditions. The approach leverages the flexibility of neutrosophic sets to represent vague or imprecise initial conditions and parameters. Milne's method, known for its efficiency in predicting and correcting numerical solutions, is adapted to accommodate neutrosophic uncertainties, enhancing its applicability in practical scenarios where precise initial conditions may be lacking. Through theoretical analysis and computational experiments, this research demonstrates the effectiveness of combining neutrosophic calculus with Milne's formula, offering insights into its potential for solving ODEs under uncertain conditions.

Keywords: fuzzy number, neutrosophic number, neutrosophic function, neutrosophic ODE

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

We are often faced with many ambiguous situations because of the limited, vague and uncertain knowledge available in our daily lives. It becomes impossible to depict and characterize any phenomenon in precise manner. In order to deal with these circumstances, Zadeh proposed fuzzy set theory in 1965. In fuzzy set theory, each intellectual word is assigned a membership grade, and all of these intellectual forms may then be simply fitted into the fuzzy environment. Basically, fuzzy set theory allows each element of a set "A" to have a specific degree of membership, represented by $\mu_A(x)$, which denotes that each element x of set A has a membership value lying in the closed interval [0,1]. When we want to fit distinct intellectual words into a fuzzy set then we assign a numerical value to them between 0 and 1, and call them fuzzy numbers. Chang and Zadeh developed fuzzy numbers in 1972, while Dubois and Prade studied generalization of fuzzy numbers in 1978.

In practice, we normally consider the membership value, although this is insufficient. In such cases, the non-membership value must also be taken into account. But fuzzy sets are established solely for membership values, they do not take non-membership values into account. Atanassov presented the intuitionistic fuzzy set (IFS) in 1986, which is an extension of fuzzy sets that encompassed both situations. Because it contains information that belongs to the set as well as information that does not belong to the set, intuitionistic fuzzy sets are regarded as an extension of fuzzy sets.

In the real-life uncertainty, there is also the possibility of a different situation, known as indeterminacy. When the knowledge on which items belong to the set and do not belong to the set is insufficient, a neutral state condition known as indeterminacy arises. In order to comprehend this scenario in real life, Smarandache was the first to establish neutrosophic set theory which consider truth value, indeterminate value, and false value in 2006 [16]. In neutrosophic set, grade of membership of Truth values (T), Indeterminate values (I) and False values (F) has been defined within the non-standard interval -]0,1[+. However it is difficult to handle data with non-standard interval, and hence, the single-valued neutrosophic set was introduced which takes the values in the standard interval [0,1].

In this paper, we describe how to solve differential equations in a neutrosophic setting using calculus features of the neutrosophic set, which was discussed. He was first introduced neutrosophic derivative which is an extension of fuzzy derivative. Neutrosophic derivative has new type of the granular derivative (gr-derivative). Also, he gave the gr-partial derivative of neutrosophic-valued several variable functions and investigated the if and only if condition for the existence of gr-derivative of neutrosophic-valued function. In the recent time, a lot of effort is done in the neutrosophic environment to describe many real-life occurrences using differential equations. For example, Sumanthi et al. [17, 18] has discussed the solution of neutrosophic differential equation using trapezoidal neutrosophic numbers, many methods are available to solve fuzzy differential equations [8, 11, 9]. Moi [10] discussed boundary value problem for second order differential equation in neutrosophic environment, and many other researchers discussed similar problems. In this study, we addressed theory for the solution of first order differential equations using numerical approach, namely, Milne's Predictor and Corrector formula in neutrosophic environment, which was inspired by these researches.

Definition 1. *Let X be a universal set. A neutrosophic set A on X is defined as*

$$A = \langle T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where $T_A(x), I_A(x), F_A(x) : X \to [0,1]$ represents the degree of membership, degree of indeterministic, and degree of non-membership, respectively, of the element $x \in X$ such that $0^- \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3^+$.

Definition 2. *Let X be a universal set. A single valued neutrosophic set A on X is defined as*

$$A = \{\langle T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where $T_A(x), I_A(x), F_A(x) : X \to [0,1]$ represents the degree of membership, degree of indeterministic, and degree of non-membership, respectively, of the element $x \in X$ such that $0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3$.

1.1. Neutrosophic number

Samrandche first proposed a concept of neutrosophic number which consists of the determinant part and the indeterminate part. It is usually denoted by $N = a + bI$, where a

and b are real numbers and I is the indeterminacy such that $I^2 = I$, $I \cdot 0 = 0$ and $\frac{I}{I}$ is undefined. We call $N = a + bI$ as a pure neutrosophic number if $a = 0$.

For example, let us consider a neutrosophic number as $z = 3 + 2I$. Then, it indicates that its determinate value is 3 and its indeterminate value is $2I$. In actual applications, persons usually specify some possible interval range of indeterminacy I to satisfy some actual requirements. Assume that the indeterminacy I is considered as such a possible interval $[0,0.01]$. Then, it is equivalent to $z \in [3,3.02]$.

Some basic operations on neutrosophic numbers

For two neutrosophic number $z_1 = a_1 + b_1 I$ and $z_2 = a_2 + b_2 I$ their some basic operational laws are presented below:

Addition

$$z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)I$$

Substraction

$$z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)I$$

Multiplication by a scalar

For a scalar c, $c(z_1) = ca_1 + cb_1 I$

Multiplication of two neutrosophic numbers

$$z_1 z_2 = (a_1 a_2) + (a_1 b_2 + a_2 b_1 + b_1 b_2)I$$

Division

$$\frac{z_1}{z_2} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)}.$$

Neutrosophic numbers satisfied the commutative, associative, distributive, identity and inverse properties.

2. Neutrosophic function on $R(I)$

A neutrosophic function is a neutrosophic relation in which the vertical line test does not necessarily work. However, in this case, the neutrosophic function coincides with the neutrosophic relation. Generally, a neutrosophic function is a function that has some indeterminacy (with respect to one or more of its formula, domain, or range).

Smarandache defined an interval function (neutrosophic function/thick function) $f: R \to R^2$ where R is all real numbers, as follows:

$$h(x) = [h_1(x), h_2(x)] \quad for \quad x \in R.$$

Example 1.

Let's consider $h: R \to R^2$ a different type of neutrosophic function defined as $\forall x \in R$ $h(x) \in [2,3]$ so we can write $h(x) = I$ Therefore, we just know that this function is bounded by the horizontal lines $y = 2$ and $y = 3$

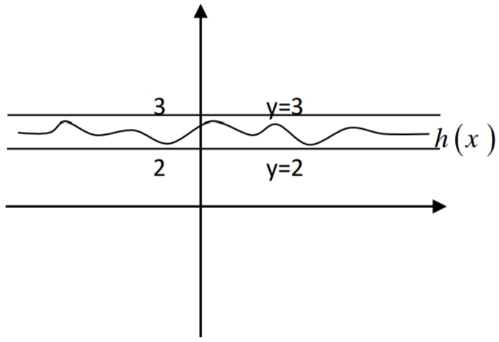

Figure 1:

We can modify $h(x)$ and get a constant neutrosophic function (or thick function): $l: R \to p(R)$ defined as $\forall x \in R$, $l(x) = [2,3]$ where $p(R)$ is the set of all subset of R .

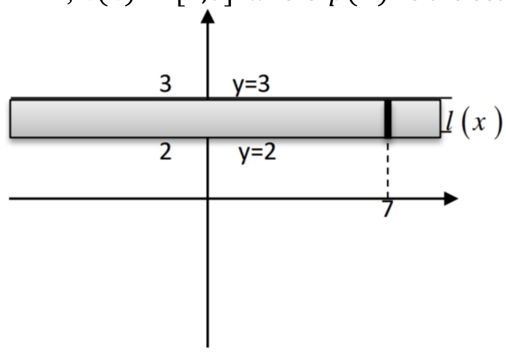

Figure 2:

Example 2

A non-constant neutrosophic thick function $K: R \to p(R)$ defined as $\forall x \in R, K(x) = [2x, 2x+1]$. Then, the neutrosophic function $h(x)$ represents a thick (interval area) between two parallel lines $K_1(x) = 2x$ and $k_2(x) = 2x + 1$.

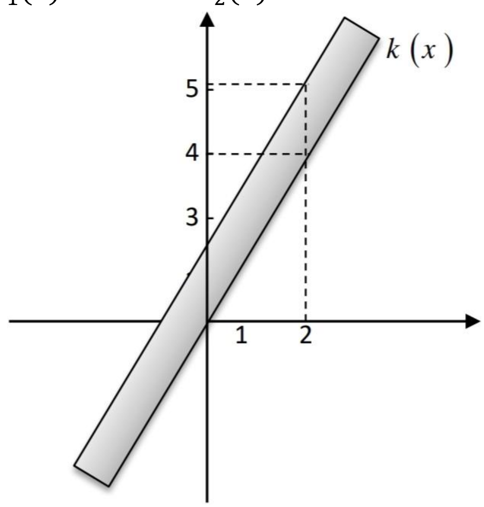

Figure 3:

Let $f: R(I) \to R(I)$, $f = f(X)$ where $X = x_1 + x_2 I \in R(I)$, the f is called neutrosophic real function with one neutrosophic variable.

A neutrosophic real function $f(X)$ can be written as follows:

$$f(X) = f(x_1 + x_2 I) = f(x_1) + [f(x_1 + x_2) - f(x_1)]I.$$

Neutrosophic exponential and logarithmic functions

Let $R(I)$ be the fields of reals. Then

1. Exponential function

$$\begin{aligned} f(X) &= e^X = e^{x_1 + x_2 I} \\ &= e^{x_1} + [e^{x_1 + x_2} - e^{x_1}]I \end{aligned}$$

2. Logarithmic function

$$\begin{aligned} f(x) &= \log(X) = \log(x_1 + x_2 I) \\ &= \log(x_1) + [\log(x_1 + x_2) - \log(x_2)]I \end{aligned}$$

where $a + bI > 0$.

Neutrosophic trigonometric functions

Let $R(I)$ be the fields of reals, then we have

1. Sine function

$$\begin{aligned} f(X) &= \sin X = \sin(x_1 + x_2 I) \\ &= \sin x_1 + [\sin(x_1 + x_2) - \sin x_1]I \end{aligned}$$

2. Cosine function

$$\begin{aligned} f(x) &= \cos X = \cos(x_1 + x_2 I) \\ &= \cos x_1 + [\cos(x_1 + x_2) - \cos x_1]I \end{aligned}$$

3. Tangent function

$$\begin{aligned} f(x) &= \tan X = \tan(x_1 + x_2 I) \\ &= \tan x_1 + [\tan(x_1 + x_2) - \tan x_1]I \end{aligned}$$

3. Derivative of a neutrosophic function on $R(I)$

Definition 3. *Let $f(X) = f(x_1 + x_2 I) = f(x_1) + [f(x_1 + x_2) - f(x_1)]I$ be a neutrosophic function on $R(I)$. Then the derivative of the neutrosophic function $f(X)$ is defined by*

$$f'(X) = f'(x_1 + x_2 I) = f'(x_1) + [f'(x_1 + x_2) - f'(x_1)]I.$$

Examples

1. If $f(X) = X^N = (x_1 + x_2 I)^{(n_1 + n_2 I)} = x_1^{(n_1 + n_2 I)} + [(x_1 + x_2)^{(n_1 + n_2 I)} - x_1^{(n_1 + n_2 I)}]I$ where $N = n_1 + n_2 I$ be any neutrosophic constant. Then

$$\begin{aligned} f'(X) &= (x_1^{(n_1+n_2I)})' + [((x_1+x_2)^{(n_1+n_2I)})' - (x_1^{(n_1+n_2I)})']I \\ &= (n_1+n_2I)\{x_1^{(n_1+n_2I-1)} + [(x_1+x_2)^{(n_1+n_2-1)} - x_1^{(n_1+n_2I-1)}]I\} \\ &= (n_1+n_2I)X^{(n_1-n_2I-1)} \\ &= NX^{(N-1)} \end{aligned}$$

2. If $f(X) = e^X = e^{x_1+x_2I} = e^{x_1} + [e^{x_1+x_2} - e^{x_1}]I$ then

$$\begin{aligned} f'(X) &= (e^{x_1})' + [(e^{x_1+x_2})' - (e^(x_1)')]I \\ &= e^{x_1} + [e^{x_1+x_2} - e^{x_1}]I \\ &= e^X \end{aligned}$$

3. If $f(X) = \ln(X) = ln(x_1+x_2I) = \ln(x_1) + [\ln(x_1+x_2) - \ln(x_1)]I$ then

$$\begin{aligned} f'(X) &= (\ln(x_1))' + [(\ln(x_1+x_2))' - (\ln(x_1))'] \\ &= \frac{1}{x_1} + [\frac{1}{x_1+x_2} - \frac{1}{x_1}]I \\ &= \frac{1}{x_1} + [\frac{-x_2}{x_1(x_1+x_2)}]I \\ &= \frac{1+0I}{x_1+x_2I} \\ &= \frac{1}{X} \end{aligned}$$

4. If $f(X) = \sin X = \sin(x_1+x_2I) = \sin x_1 + [\sin(x_1+x_2) - \sin x_1]I$ then

$$\begin{aligned} f'(X) &= (\sin x_1)' + [(\sin(x_1+x_2))' - (\sin x_1)']I \\ &= \cos x_1 + [\cos(x_1+x_2) - \cos x_1]I \\ &= \cos(x_1+x_2I) \\ &= \cos X \end{aligned}$$

5. If $f(X) = \cos X = \cos(x_1+x_2I) = \cos x_1 + [\cos(x_1+x_2) - \cos x_1]I$ then

$$\begin{aligned} f'(X) &= (\cos x_1)' + [(\cos(x_1+x_2))' - (\cos x_1)']I \\ &= -\sin x_1 + [-\sin(x_1+x_2) + \sin x_1]I \\ &= -\{\sin x_1 + [\sin(x_1+x_2) - \sin x_1]I\} \\ &= -\sin(x_1+x_2I) \\ &= -\sin X \end{aligned}$$

4. Integral of a neutrosophic function on $R(I)$

Definition 4. *Let* $f(X) = f(x_1+x_2I) = f(x_1) + [f(x_1+x_2) - f(x_1)]I$ *be a neutrosophic function on* $R(I)$*, then we define integration of the neutrosophic function* $f(X)$ *as follows:*

$$\int f(X)\,dX = \int f(x_1)\,dx_1 + [\int f(x_1+x_2)\,d(x_1+x_2) - \int f(x_1)\,dx_1]I + a + bI$$

where $a+bI$ is a neutrosophic constant.

Examples:

1. If $f(X) = e^X = e^{x_1} + [e^{x_1+x_2} - e_1^x]$ then

$$\begin{aligned}\int e^X\,dX &= \int e^{x_1}\,dx_1 + [\int e^{x_1+x_2}\,d(x_1+x_2) - \int e^{x_1}\,dx_1]I \\ &= e^{x_1} + [e^{x_1+x_2} - e^{x_1}]I + a + bI \\ &= e^X + a + bI\end{aligned}$$

where $a + bI$ be any neutrosophic constant.

2. If $f(X) = X^3 lnX = x_1^3 lnx_1 + [(x_1+x_2)^3 ln(x_1+x_2) - x_1^3 lnx_1]$ then

$$\int X^3 \ln X\,dX$$
$$= \int x_1^3 \ln x_1\,dx_1 + [\int (x_1+x_2)^3 \ln(x_1+x_2)\,d(x_1+x_2) - \int x_1^3 \ln x_1\,dx_1]I$$
$$= \frac{x_1^4}{4}\ln x_1 - \frac{x_1^4}{16} + [\frac{(x^1+x_2)^4}{4}\ln(x_1+x_2) - \frac{(x_1+x_2)^4}{16} - \frac{x_1^4}{4}\ln x_1 + \frac{x_1^4}{16}]I$$
$$+a + bI$$

$$= \{\frac{x_1^4}{4}\ln x_1 + [\frac{(x_1+x_2)^4}{4}\ln(x_1+x_2) - \frac{x_1^4}{4}\ln x_1]I\} + a + bI$$
$$= \{\frac{x_1^4}{16} + [\frac{(x^1+x_2)^4}{16} - \frac{x_1^4}{16}]I\}a + bI$$
$$= \frac{(x_1+x_2I)^4}{4}\ln(x_1+x_2I) - \frac{(x_1+x_2I)^4}{16}a + bI$$
$$= \frac{X^4}{4}\ln X - \frac{X^4}{16} + a + bI$$

where $a + bI$ be any neutrosophic constant.

3. If $f(X) = \sin X \cos^2 X = \sin x_1 \cos^2 x_1 + [\sin (x_1+x_2)\cos^2(x_1+x_2) - \sin x_1 \cos^2 x_1]$ then

$$\int \sin X \cos^2 X\,dX$$
$$= \int \sin x_1 \cos^2 x_1\,dx_1 + [\int \sin(x_1+x_2)\cos^2(x_1+x_2)\,d(x_1+x_2)$$
$$- \int \sin x_1 \cos^2 x_1\,dx_1]$$
$$= -\frac{1}{3}\cos^3 x_1 + [-\frac{1}{3}\cos^3(x_1+x_2) + \frac{1}{3}\cos^3 x_1]$$
$$= -\frac{1}{3}\cos^3(X)$$

4.1. The definite integral

Definition 5. *Let* $f(X) = f(x_1 + x_2I) = f(x_1) + [f(x_1+x_2) - f(x_1)]I$ *be a neutrosophic function on* $R(I)$, *then we define the definite integration of the neutrosophic function* $f(X)$ *as follows:*

$$\int_{a+bI}^{c+dI} f(X)\,dX = \int_a^c f(x_1)\,dx_1 + [\int_{a+b}^{c+d} f(x_1+x_2)\,d(x_1+x_2) - \int_a^c f(x_1)\,dx_1]I$$

Examples:

If $f(X) = e^X = e^x + [e^(x+y) - e^x]I$ then

$$\begin{aligned}\int_{0+0I}^{1+I} e^X\,dX &= \int_0^1 e^x\,dx + [\int_0^2 e^{(x+y)}\,d(x+y) - \int_0^1 e^x\,dx]I \\ &= [e^x]_0^1 + \{[e^{(x+y)}]_0^2 - [e^x]_0^1\} \\ &= (e-1) + [(e^2-1) - (e-1)]I \\ &= (e-1) - (e^2-e)I\end{aligned}$$

5. Solution of ordinary differential equation in neutrosophic environment

Differentiation plays an important role in the field of science and engineering. Many problems arise with uncertain or imprecise parameters.To model this uncertainty, we develop the differential equation with imprecise parameters. Neutrosophic differential equation has been introduced to model this uncertainty. for finding a solution to a differential equation, which has previously been done in a classical and fuzzy environment, has prompted us to consider similar forms of expansion in a neutrosophic environment. The analytic methods to solve neutrosophic differential equations are used for a limited classes of differential equations. Most of the differential equations governed by physical problems do not possess closed form solutions. For these types of problems the numerical methods are used. The commonly used multistep method is Milne's predictor-corrector method which is discussed below.

5.1. Milne's predictor-corrector method

Let the differential equation be

$$Y' = F(X,Y) \tag{1}$$

with the initial condition $Y(X_0) = Y_0$ where $Y = y_l + y_k I$ and $X = x_l + x_k I$.

Now (1) is integrated between X_{i-3} and Y_{i+1} and find

$$Y_{i+1} = Y_{i-3} + \int_{X_{i-3}}^{X_{i+1}} F(X,Y)\, dX \tag{2}$$

Now, the function $F(X,Y)$ is replaced by Newtonâ€™s forward difference formula in the form

$$F(X,Y) = F_{i-3} + U\Delta F_{i-3} + \frac{U(U-1)}{2!}\Delta^2 F_{i-3} + \frac{U(U-1)(U-2)}{3!}\Delta^3 F_{i-3} \tag{3}$$

where $U = \frac{X-X_{i-3}}{H} = u_l + u_k I$ (Say) and $H = h_l + h_k I$

The value of $F(X,Y)$ is substituted from (3) to (2) and find

$$Y_{i+1} = Y_{i-3} + H\int_{0+0I}^{4+0I} [F_{i-3} + \quad U\Delta F_{i-3} + \frac{U^2-U}{2}\Delta^2 F_{i-3} + \frac{U+3+3U^2+2U}{6}\Delta^3 F_{i-3}]\, dU$$

$$= Y_{i-3} + H[4F_{i-3} + 8\Delta F_{i-3} + \frac{20}{3}\Delta^2 F_{i-3} + \frac{8}{3}\Delta^3 F_{i-3}]$$

$$= Y_{i-3} + H[4f_(i-3) + 8(F_{i-2} - F_{i-3}) + \frac{20}{3}(F_{i-1} - 2F_{i-2} + F_{i-3}) + \frac{8}{3}(F_i - 3F_{i-1} + 3F_{i-2} - F_{i-3})$$

$$= Y_{i-3} + \frac{4H}{3}[2F_{i-2} - F_{i-1} + 2F_i]$$

Thus the Milne's predictor formula is

$$Y_{i+1}^p = Y_{i-3} + \frac{4h}{3}[2F_{i-2} - F_{i-1} + 2F_i] \tag{4}$$

The corrector formula is developed in a similar way. The value of Y_{i+1}^p will now be used. Again, the given differential equation is integrated between X_{i-1} and X_{i+1} and the function $F(X,Y)$ is replaced by the Newton's formula (3) Then

$$Y_{i+1} = Y_{i-1} + \int_{X_{i-1}}^{X_{i+1}} [F_{i-1} + U\Delta F_{i-1} + \frac{U(U-1)}{2}\Delta^2 F_{i-1}]\, dx$$
$$Y_{i-1} + H\int_{0+0I}^{2+0I} [F_{i-1} + U\Delta F_{i-1} + \frac{U^2-U}{2}\Delta^2 F_{i-1}]\, dU$$
$$= Y_{i-1} + H[2F_{i-1} + 2\Delta F_{i-1} + \frac{1}{3}\Delta^2 F_{i-1}]$$
$$= Y_{i-1} + H[2F_{i-1} + 2(F_i - F_{i-1}) + \frac{1}{3}(F_{i+1} - 2F_i + F_{i-1})]$$
$$= Y_{i-1} + \frac{H}{3}[F_{i-1}4F_i + F_{i+1}]$$

This formula is known as corrector formula and it is denoted by Y_{i+1}^c. That is,

$$Y_{i+1}^c = Y_{i-1} + \frac{H}{3}[F(X_{i-1}, Yi-1) + 4F(X_i, Y_i) + F(X_{i+1}, Y_{i+1}^p)] \quad (5)$$

When Y_{i+1}^p is computed using the formula (4), formula (5) can be used iteratively to obtain the value of Y_{i+1} to the desired accuracy.

5.2. Illustration

Find the value of $Y(2+2I)$ for the initial value problem

$$\frac{dY}{dX} = (0.3+0.1I)Y^2 sinX \text{ with } Y(0+0I) = 1+I$$

using Milnes's predictor-corrector method, taking $h = 0.5 + 0.5I$.

Let $F(X,Y) = (0.3+0.1I)Y^2 sinX,\ X_0 = 0+0I,\ Y_0 = 1+I,\ H = 0.5+0.5I$

Fourth-order Runge-Kutta method is used to compute the starting values Y_1, y_2 and Y_3.

So now,

$$K_1^{(0)} = HF(X_0, Y_0)$$
$$= (0.5+0.5I)(0.3+0.1I)(1+I)^2 sin(0+0I)$$
$$= 0$$

$$K_2^{(0)} = HF(X_0 + \frac{H}{2}, Y_0 + \frac{K_1^{(0)}}{2})$$
$$= (0.5+0.5I)(0.3+0.1I)(1+I)^2 sin(0.25+0.25I)$$
$$= 0.0006545 + 0.013308I$$

$$K_3^{(0)} = HF(X_0 + \frac{H}{2}, Y_0 + \frac{K_2^{(0)}}{2})$$
$$= (0.5+0.5I)(0.3+0.1I)(1.000327+1.013635I)^2 sin(0.25+0.25I)$$
$$= 0.000655 + 0.013405I$$

$$K_4^{(0)} = HF(X_0 + H, Y_0 + K_3^{(0)})$$
$$= (0.5+0.5I)(0.3+0.1I)(1.000655+1.013405I)^2 sin(0.5+0.5I)$$
$$= 0.0013107 + 0.027007I$$

So,

$$Y_1 = Y(X_1)$$
$$= Y(0.5+0.5I)$$
$$= Y_0 + \frac{1}{6}[K_1^{(0)} + 2K_2^{(0)} + 2K_3^{(0)} + K_4^{(0)}]$$
$$= 1.000655 + 1.0134055I$$

Now

$$\begin{aligned} K_1^{(1)} &= HF(X_1, Y_1) \\ &= (0.5+0.5I)(0.3+0.1I)(1.0006555+1.0134055I)^2 sin(0.5+0.5I) \\ &= 0.0013107+0.027007I \end{aligned}$$

$$\begin{aligned} K_2^{(1)} &= HF(X_1+\tfrac{H}{2}, Y_1+\tfrac{K_1^{(1)}}{2}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.00131035+1.026909I)^2 sin(0.75+0.75I) \\ &= 0.001968+0.041102I \end{aligned}$$

$$\begin{aligned} K_3^{(1)} &= HF(X_1+\frac{H}{2}, Y_1+\frac{K_2^{(1)}}{2}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.001639+1.033957I)^2 sin(0.75+0.75I) \\ &= 0.001969+0.041415I \end{aligned}$$

$$\begin{aligned} K_4^{(1)} &= HF(X_1+H, Y_1+K_3^{(1)}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.002624+1.054821I)^2 sin(1+I) \\ &= 0.002632+0.056466I \end{aligned}$$

So,

$$\begin{aligned} Y_2 &= Y(X_2) \\ &= Y(1+I) \\ &= Y_1+\tfrac{1}{6}[K_1^{(1)}+2K_2^{(1)}+2K_3^{(1)}+K_4^{(1)}] \\ &= 1.002624+1.054821I \end{aligned}$$

$$\begin{aligned} K_1^{(2)} &= HF(X_2, Y_2) \\ &= (0.5+0.5I)(0.3+0.1I)(1.002624+1.054821)^2 sin(1+I) \\ &= 0.002632+0.056466I \end{aligned}$$

$$\begin{aligned} K_2^{(2)} &= HF(X_2+\tfrac{H}{2}, Y_2+\tfrac{K_1^{(2)}}{2}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.00394+1.083054I)^2 sin(1.25+1.25I) \\ &= 0.003298+0.072698I \end{aligned}$$

$$\begin{aligned} K_3^{(2)} &= HF(X_2+\tfrac{H}{2}, Y_2+\tfrac{K_2^{(2)}}{2}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.004273+1.09117I)^2 sin(1.25+1.25I) \\ &= 0.0033+0.073312I \end{aligned}$$

$$\begin{aligned} K_4^{(2)} &= HF(X_2+H, Y_1+K_3^{(2)}) \\ &= (0.5+0.5I)(0.3+0.1I)(1.005924+1.128133I)^2 sin(1.5+1.5I) \\ &= 0.026488+0.2118605I \end{aligned}$$

So,

$$\begin{aligned} Y_3 &= Y(X_3) \\ &= Y(1.5 + 1.5I) \\ &= Y_2 + \frac{1}{6}[K_1^{(2)} + 2K_2^{(2)} + 2K_3^{(2)} + K_4^{(2)}] \\ &= 1.009668 + 1.148212I \end{aligned}$$

The predictor value is

$$\begin{aligned} Y_4^p &= Y_0 + \tfrac{4H}{3}[2F(X_1, Y_1) - F(X_2, Y_2) + 2F(X_3, Y_3)] \\ &= 1.010661 + 1.246013I \end{aligned}$$

The corrector value is

$$\begin{aligned} Y_4^c &= Y_2 + \tfrac{H}{2}[2F(X_2, Y_2) - 4F(X_3, Y_3) + F(X_4, Y_4)] \\ &= 1.0106695 + 1.246018I \end{aligned}$$

Thus the required solution is

$$Y_4 = Y(2 + 2I) = 1.0106695 + 1.246018I$$

6. Conclusion

This paper has solved the first-order ordinary differential equation in the neutrosophic environment with initial conditions. We have developed a theory in a neutrosophic environment supplemented with an example showing the solution for a first-order linear homogeneous differential equation using a numerical approach. Here we use Milne's predictor and corrector method.

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REFERENCES

1. B. Bede and L. Stefanini, Generalized differentiability of fuzzy valued functions, *Fuzzy Sets Syst.*, 230 (2013) 119-141.
2. B. Bede and S.G. Gal, Generalizations of the differentiability of fuzzy number-valued functions with applications to fuzzy differential equations, *Fuzzy Sets Syst.*, 151 (2005) 581â€“599.
3. S.Broumi, Generalized neutrosophic soft set, *International Journal of Computer Science, Engineering and Information Technology*, 2 (2013).
4. C.Jana and M. Pal, A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making, *Symmetry*, 11(1) (2019) 110. DOI: 10.3390/sym11010110
5. C. Jana, G. Muhiuddin and M. Pal, Multiple-attribute decision making problems based on SVTNH methods, *Journal of Ambient Intelligence and Humanized Computing*, 11 (2020) 3717-3733.
6. C. Jana and M. Pal, Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators, *Soft Computing*, 25(7) (2021) 5055-5072. DOI: 10.1007/s00500-020-05509-z
7. C. Jana and M. Pal, Multiple attribute decision-making based on uncertain linguistic

operators in neutrosophic environment, *Neutrosophic Operational Research: Methods and Applications*, (2021) 315-341. DOI: 10.1007/978-3-030-57197-9_16

8. A. Khastana and R. Rodriguez-Lopezb, On the solutions to first order linear fuzzy differential equations, *Fuzzy Sets Syst.*, 295 (2016)114-135.
9. S.P. Mondal and T.K. Roy, First order homogeneous ordinary differential equation with initial value as triangular intuitionistic fuzzy number, *J Uncertain Math Sci.*, 2014 (2014) 1-17.
10. S. Moi, S. Biswas, and S. Pal(Sarkar), Second-order neutrosophic boundary-value problem, *Complex Intell. Syst.*, 7 (2021) 1079-1098.
11. S.P. Mondal and T.K. Roy, First Order linear homogeneous fuzzy ordinary differential equation based on lagrange multiplier method, *J. Soft Comput Appl.*, 2013 (2013) 1-17.
12. M.Muthulakshmi, T.G.Sindhu, C.Tamilarasi and K.Sathyaswathi, Some properties of neutrosophic real numbers, *International Journal of Creative Research Thoughts*, 10(6) (2022) 296-302.
13. M. Pal, *Recent Developments of Fuzzy Matrix Theory and Applications*, Springer, Switzerland, 2024. DOI: 10.1007/978-3-031-56936-4
14. M. Pal, *Numerical Analysis for Scientists and Engineers: Theory and C Programs*, Narosa, New Delhi, 2007.
15. M. Pal, *Neutrosophic Matrix and Neutrosophic Fuzzy Matrix*, In: Recent Developments of Fuzzy Matrix Theory and Applications. Springer, Cham. https://doi.org/10.1007/978-3-031-56936-4_10
16. A. Salamah, M.F. Alaswad and R. Dallah, A study of derivative and integration of a neutrosophic function, *Journal of Neutrosophic and Fuzzy Systems*, 5(2) (2023) 33-37.
17. F. Smarandache, Neutrosophic Set - A Generalization of the Intuitionistic Fuzzy Set, Conference 2006, IEEE International Conference on Granular Computing, GrC, Atlanta, Georgia, USA, (2006)
18. I.R.Sumathi and V.M. Priya, A new perspective on neutrosophic differential equation, *International Journal of Engineering & Technology*, 7 (4.10) (2018) 422-425.
19. R. Sumathi and C. Antony Crispin Sweety, New approach on differential equation via trapezoidal neutrosophic number, *Complex & Intelligent Systems*, 5 (2019) 417-424.