Intern. J. Fuzzy Mathematical Archive
Vol. 21, No. 2, 2023, 43-56
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 15 July 2023
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v21n2a42394

International Journal of Fuzzy Mathematical Archive

Solving Linear Programming Problems with Neutrosophic Coefficients

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Received 13 June 2023; accepted 12 July 2023

Abstracts. In real-world scenarios, both determinate and indeterminate information are present, making it essential to address indeterminate problems in optimization tasks. Neutrosophic numbers (NNs) are particularly effective in representing this mix of information. An NN, expressed as $(x + yI)$, includes a determinate component x and an indeterminate component bI, where $x, y \in \mathbb{R}$, and I symbolizes indeterminacy, with $\mathbb{R}$ representing the set of all real numbers.

This paper introduces the basic operations of NNs and a corresponding NF, which involves NNs and is termed simply as a NF. While few methods exist for solving neutrosophic linear programming problems (NLPPs), most focus on the three components truth, falsity, and neutrality within the coefficients. Here, we propose a novel approach for solving NLPPs where the coefficients in both the objective function and the constraints are modeled as NNs.

To demonstrate the efficacy of our method, we present a numerical example along with an application in production planning, showcasing how NLPPs can be solved and applied in practice. Additionally, we explore the potential ranges for the optimal solution when the indeterminacy I is defined as a possible interval that corresponds to real-world application requirements. This approach provides a more comprehensive framework for addressing uncertainty in optimization problems, particularly in contexts where indeterminate information cannot be ignored.

Keywords: fuzzy number, neutrosophic number, fuzzy linear programming problem, neutrosophic linear programming problem

AMS Mathematics Subject Classification (2010): 90B05, 90C05

1. Introduction

After the development of fuzzy set (FS) theory, it became a significant tool for addressing problems involving non-random uncertainty. However, researchers observed that FS theory was often inadequate in solving certain problems. This inadequacy became apparent, particularly when dealing with incomplete, inconsistent, or indeterminate information.

To address these issues, Smarandache proposed the concept of Neutrosophic Sets (NSs), characterized by three key components: the truth membership function (t), the indeterminacy membership function (i), and the falsity membership function (f) [38, 39, 40, 41, 42]. NS theory has found extensive application in various fields, proving especially effective in dealing with indeterminate and inconsistent information in real-world scenarios [48]. NSs generalize several existing concepts, including classical fuzzy sets, interval-valued FS, and intuitionistic FS. In response to the introduction of NSs, several researchers have explored its various branches, such as interval neutrosophic sets [46].

Subsequently, interest in NSs grew, leading to the development of Neutrosophic Fuzzy Numbers (NFN) and their corresponding matrices [10]. In 2010, Wang et al. introduced the concept of a single-valued neutrosophic set (SVNS), where the three componentsâ€"truth, indeterminacy, and falsityâ€"are independent and can be expressed separately [46]. The SVNS framework closely aligns with human cognition, as it accounts for the imperfections in knowledge that people acquire from external information sources.

For more detailed and systematic information on neutrosophic sets, numbers, and matrices, refer to [30, 31]. Some researchers have noted that NSs extend FS theory by introducing three distinct functions: the membership function, the indeterminacy function, and the non-membership function. Additionally, Smarandache introduced the concept of a neutrosophic number (NN), which extends the classical real number or serves as an alternative representation of an interval number. These NNs are particularly useful in handling various types of uncertainties.

2. Neutrosophic number

Smarandache first introduced the concept of a NN, which consists of two parts: the determinate part and the indeterminate part. A NN is typically represented as $N = x + yI$, where x and y are real numbers, and I represents indeterminacy, such that $I^2 = I$, $I \cdot 0 = 0$, and $\frac{I}{I}$ is undefined. A NN is called a pure NN when $x = 0$.

For instance, consider the NN $N = 50 + 30I$. If I belongs to the interval $[0,0.02]$, then N is equivalent to being within the range $[50,50.6]$ for $N \geq 50$. This indicates that the determinate part of N is 50, while the indeterminate part, represented by $30I$, accounts for the possibility that N could be slightly larger than 50, depending on the value of I within its specified interval.

It's important to note that while a NN might resemble a complex number in form, it differs significantly because $I^2 = I$ rather than -1, as in the case of a complex number.

The three basic operations defined on NNs $S = s_1 + s_2 I$ and $T = t_1 + t_2 I$ are as follows:

1. $S + T = (s_1 + t_1) + (s_2 + t_2)I$
2. $S - T = (s_1 - t_1) + (s_2 - t_2)I$
3. $S \times T = s_1 t_1 + (s_1 t_2 + s_2 t_1 + s_2 t_2)I$

In real neutrosophic algebra, we use K to denote the neutrosophic field over certain neutrosophic vector spaces. The smallest field generated by $K \cup I$, or $K(I)$, is referred to as the neutrosophic field, as it incorporates the indeterminacy factor I, which has the unique properties $I^n = I$, $I + I = I$, and for any scalar $t \in K$, $t \cdot I = tI$ and $0 \cdot I = 0$. Consequently, we generally denote the neutrosophic field generated by $K \cup I$ as $K(I) = \langle K \cup I \rangle$.

Therefore, across different algebraic fields, various types of neutrosophic fields can be defined, each generated by the field of a corresponding neutrosophic vector space.

3. Literature review

Linear programming problems (LPPs) in fuzzy environments are typically categorized into two groups: symmetric and non-symmetric problems, as classified by Zimmermann [52]. Various models for fuzzy linear programming (FLP) problems were introduced by Lnuiguchui et al. [24]. Kumar et al. [22] extended this by introducing FLP problems with both equality and inequality constraints.

Numerous authors have explored several properties of FLP problems and proposed various models for solving them. The foundational theory of fuzzy programming was first suggested by Tanaka et al. [45]. Wu [47] advanced the theory by introducing the concept of a non-dominated solution in multi-objective programming, which provided a more realistic approach for FLP problems. Researchers like Maleki et al. [26] have developed ranking functions to convert FLP problems into equivalent crisp LP models for solution. Zimmermann [51] introduced a method using multi-objective linear programming techniques to address FLP problems, leading to further advancements in the field. Subsequent contributions include new defuzzification techniques from Maleki et al. [26], Maleki [27], Liu [23], Jimenez et al. [18], and Nasseri [29], as well as methods utilizing Millenâ€™s transform by Peraei et al. [32]. Ebrahimnejad et al. [11] developed a novel primal-dual algorithm for FLP problems, and Saneifard and Saneifard [35] focused on defuzzifying fuzzy numbers using Mellinâ€™s transform.

In the realm of intuitionistic FLP problem (IFLPP), Kabiraj et al. [19] device a method using the concept of Zimmermanâ€™s approach used for solving FLP problems. They further developed a technique using the (α, β)-cut for IFLP problems, when the coefficients are triangular intuitionistic fuzzy numbers [20].

Bera and Mahapatra proposed the Big-M simplex method for NLPPs [6]. Das and Dash [9] developed a new ranking method for NLPPs with mixed constraints using triangular NNs. Nafei et al. [28] presented interval NLPPs with triangular interval NNs, employing a new ranking technique to transform interval NLPPs into crisp LPPs. Basumatary and Broumi [5] worked on interval-valued triangular NLPPs based on interval-valued triangular numbers, and Abdelfattah [1] proposed a parametric approach for solving NLPPs. Sagayakavitha and Sudha [34] introduced a new approach for solving NLPPs with symmetric triangular NNs. Finally, Tamilarasi and Paulraj [44] developed an improved method for solving NLPPs using Mellinâ€™s transform.

Despite the extensive research on LPPs, only one paper [49] addresses NNs. Most existing work on NLPPs focuses on the three types of neutrosophic parameters true, false, and neutral values. This paper proposes a new approach to solving NLPPs where the coefficients are represented as NNs, offering an innovative method for addressing these types of problems.

4. Neutrosophic functions

Smarandache defined an interval function, also referred to as a neutrosophic function (NF) or thick function, denoted by

$$g(y): \mathbb{R} \to \mathbb{R}^2,$$

where $\mathbb{R}$ represents all real numbers. This function is given by

$$g(y) = [g_1(y), g_2(y)] \text{for} y \in \mathbb{R}.$$

In addition to this, he also defined open interval functions $g(y) = (g_1(y), g_2(y))$ and semi-open/semi-closed interval functions $g(y) = [g_1(y), g_2(y))$ and $g(y) = (g_1(y), g_2(y)]$ for any $y \in \mathbb{R}$.

For instance, consider the NF

$$g(y) = [10y, 10y + 2] \text{for} y \in \mathbb{R}.$$

Here, $g(y)$ represents a thick (interval) area between the two parallel lines $g_1(y) = 10y$ and $g_2(y) = 10y + 2$. Specifically, when $y = 5$, $g(5) = [50,52]$ represents an interval value. However, Smarandache's NF does not accommodate NNs, which are useful for expressing determinate and indeterminate information in cases of incomplete, uncertain, and indeterminate problems. Therefore, we introduce the concept of NFs to formulate a neutrosophic linear programming problem (NLPP) model in such indeterminate contexts.

We define a NF as follows:

Definition 1. *A NF with n variables (unknowns) is defined by*

$$f(Y, I): Z^n \to Z,$$

where $Y = [y_1, y_2, \dots, y_n]^T$ is an n-dimensional vector with $Y \in Z^n$, and I represents indeterminacy. If $f(Y, I)$ is a linear function, it is termed a linear NF. Conversely, if $f(Y, I)$ is nonlinear, it is termed a neutrosophic nonlinear function.

This article focuses solely on linear NFs.
For example, a linear NF with a two variables y_1 and y_2 is given by

$$f(Y, I) = (3 + I)y_1 + (4 + 3I)y_2 + 6 \text{ for } X \in Z^2.$$

Here, the NF comprises two variables y_1 and y_2, and two NNs (coefficients) $3 + I$ and $4 + 3I$. In practical situations, we can specify a range for the indeterminacy $I \in [I^L, I^U]$ corresponding to actual requirements. For instance, if I is within the range $[0,1]$, the linear NF becomes

$$f(Y, I) = [3,4]y_1 + [4,7]y_2 + 6 \text{ for } I \in [0,1].$$

Thus, a function with interval coefficients can be expressed as a function with neutrosophic coefficients, and vice versa.

5. Neutrosphic linear programming problem

In classical mathematical programming, coefficients and variables in both the objective function and constraints are usually considered as precise, determinate values. However, practical problems often involve both determinate and indeterminate information, necessitating methods to address these indeterminate aspects. This section introduces a method for solving Neutrosophic Linear Programming Problems (NLPP) to handle such cases.

Definition 2. *An NLPP is categorized as a general maximum-type problem if it meets the following conditions:*

a) The objective function is linear with NNs as coefficients and it's value is to be maximized.

b) All decision variables are either positive or zero.
c) The constraints are expressed as $a_{i1}y_1 + a_{i2}y_2 + \cdots + a_{in}y_n \leq b_i$, where y_j (for $j = 1,2,\ldots,n$) are decision variables and $a_{i1}, a_{i2}, \ldots, a_{in}$ are NNs.

For example, a NLPP with two decision variables y_1 and y_2 is given below.

$$\begin{aligned} &Max\ \ f(Y,I) = (3+2I)y_1 + (5+7I)y_2 \\ &subject\ \ to,\ \ (1+0I)y_1 + (1+2I)y_2 \leq 1 \\ &(2+0I)y_1 + (3+I)y_2 \leq 1 \\ &y_i \geq 0, i = 1,2. \end{aligned} \tag{1}$$

where y_1 and y_2 are two decision variables and I is indeterminacy.

Like crisp LPP, to convert the constraints into equalities, slack variables are introduced. By adding these slack variables, such as y_3 and y_4, the original constraints can be transformed into equalities or slack equations. Consequently, the constraints are redefined in this modified form to facilitate further analysis and solution of the Neutrosophic Linear Programming Problem (NLPP). This approach ensures that all constraints are handled consistently within the optimization framework.

$$\begin{aligned} &y_1 + (1+2I)y_2 + y_3 + 0y_4 = 1 \\ &2y_1 + (3+I)y_2 + 0y_3 + y_4 = 1 \\ &y_i \geq 0, i = 1,2,3,4 \end{aligned}$$

The adjusted objective function is given by

$$Max\ f(Y,I) = (3+2I)y_1 + (5+7I) + (0+0I)y_3 + (0+0I)y_4$$

The standard form of the given NLPP is

$$\begin{aligned} &Max\ f(Y,I) = (3+2I)y_1 + (5+7I)y_2 + 0 \times y_3 + 0 \times y_4 \\ &subject\ to, y_1 + (1+2I)y_2 + y_3 + 0 \times y_4 = 1 \\ &2y_1 + (3+I)y_2 + 0 \times y_3 + y_4 = 1 \\ &y_i \geq 0, i = 1,2,3,4 \end{aligned} \tag{2}$$

In solving Neutrosophic Linear Programming Problems (NLPPs), slack variables are always non-negative. For an NLPP with two variables, these slack variables remain non-negative at the corner points of the feasible region. If a slack variable turns negative, it suggests an error in the formulation or solution process. It's important to note that slack variables are tools to facilitate the resolution of NLPPs. Generally, an NLPP with n variables $(y_1, y_2, \ldots, y_n)$ is structured as follows:

$$\begin{aligned} &Max\ \ z = \textstyle\sum_{j=1}^{n} c_j y_j \\ &subject\ \ to\ \textstyle\sum_{j=1}^{n} a_{ij} y_j \leq\ \ or\ \ =\ \ or\ \ \geq b_i,\ i = 1,2,\ldots,m \\ &y_j (\geq 0)\ \ are\ \ the\ \ decision\ \ varibales\,. \end{aligned} \tag{3}$$

When c_j, b_i, or a_{ij} are interval numbers of the form $[p_i, q_i] = \{y_i : p_i \leq y_i \leq q_i\}$ 3, one can solve the problem using existing methods. These methods produce a set of solutions that are of the interval type. Alternatively, a set of interval numbers can be converted into

NNs, which allows the application of techniques designed for handling neutrosophic data. This conversion facilitates solving problems involving indeterminate and uncertain information.

Any interval number can be written as a NNs. For example, $[p_i, q_i] = [p_i, p_i] + (q_i - p_i)I$ where $I = [0,1]$. That is, $[p_i, q_i] = p_i + h_i I$, $h_i = q_i - p_i$, then length of the interval. Similarly, a set of interval numbers $[3,5], [8,15], [2,16]$ can be written as $[3,5] = [3,3] + 2[0,1] = 3 + 2I$, where $I = [0,1]$ and $[3,3] = 3$, a degenerate number. Similarly, $[8,15] = 8 + 7I$, $[2,16] = 2 + 14I$, etc.

In uncertain domain, an LPP can be represented in the following form:

a) The cost vector is uncertain and the other coefficients and variables remain certain.
b) The coefficients a_{ij} are uncertain and other remains certain.
c) The right hand vector is uncertain and other remains certain.
d) All coefficients are uncertain, but variables are certain.
e) All coefficients and variables are uncertain.

Here, we address the scenario where the coefficients are NNs of the form $a + bI$. If I is considered as 0, there is no uncertainty, and the problem is termed "underachievement." Conversely, when I is set to 1, representing full uncertainty, the problem is referred to as "overachievement." In this context, the underachievement and overachievement of the NF $f(Y, I)$ are denoted by $f(Y, I = 0)$ and $f(Y, I = 1)$, respectively.

5.1. Example 1

Let us consider a NLPP

$$\begin{aligned} &Max\ f(Y, I) = (3 + 2I)y_1 + (5 + 7I)y_2 + 0y_3 + 0y_4 \qquad (4)\\ &subject\ to, y_1 + (1 + 2I)y_2 + y_3 + 0y_4 = 1\\ &2y_1 + 3y_2 + 0y_3 + y_4 = 1\\ &y_i \geq 0, i = 1,2,3,4 \end{aligned}$$

The under achivement NLPP is given by

$$\begin{aligned} &Max\ f(Y, I = 0) = 3y_1 + 5y_2 + 0y_3 + 0y_4\\ &subject\ to,\ y_1 + y_2 + y_3 \qquad = 1\\ &2y_1 + 3y_2 \qquad + y_4 = 1\\ &y_i \geq 0, i = 1,2,3,4 \end{aligned}$$

The optimal solution is $\underline{f_{max}} = \frac{5}{3}$, for $y_1 = 0, y_2 = \frac{1}{3}$.

The over achievement NLPP is given by

$$\begin{aligned} &Max\ f(Y, I = 1) = 5y_1 + 12y_2 + 0y_3 + 0y_4\\ &subject\ to,\ y_1 + 3y_2 + y_3 \qquad = 1\\ &2y_1 + 3y_2 \qquad + y_4 = 1\\ &y_i \geq 0, i = 1,2,3,4 \end{aligned}$$

The optimal solution is $\overline{f_{max}} = 4$, for $y_1 = 0, y_2 = \frac{1}{3}$.

Now, we solve the NLPP (4) by using direct simplex method. The successive simplex tables are shown below:

		c	(3+2I)	(5+7I)	0	0	
c_B	y_B	b	a_1	a_2	a_3	a_4	Min ratio
	y_3	1	1	1+2I	1	0	$\frac{1}{(1+2I)}$
	y_4	1	2	(3)	0	1	$\frac{1}{3}\rightarrow$
	$z_j - c_j$		$-3-2I$	$-5-7I\uparrow$	0	0	
	y_3	$\frac{2}{3}(1-I)$	$\frac{1-4I}{3}$	0	1	$-\frac{1+2I}{3}$	
+7I	y_2	$\frac{1}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	
	$z_j - c_j$	$\frac{5+7I}{3}$	$\frac{1+8I}{3}$	0	0	$\frac{5+7I}{3}$	

Since all $z_j - c_j \geq 0$, so the problem becomes in its optimal stage.
Hence, the optimal solution is $f_{max} = \frac{5+7I}{3}$, for $y_1 = 0, y_2 = \frac{1}{3}$.

Now, if the indeterminacy value is 0, we get the optimal value which is same as the under achievement $(\underline{f_{max}} = \frac{5}{3})$ and if the indeterminacy value is 1, i.e. $I = 1$ we see the optimal value is equal to over achievement value $(\overline{f_{max}} = 4)$.

Notice that, both the under and over achievement values are same as the solution obtained by applying simplex method directly to an NLPP. So, the direct arithmetic can be used to solve the NLPP. But, the computation with nuetrosophic number is little bit complicated and need some theories for optimality.

Further, since $I = [0,1]$, let $\theta \in [0,1]$ be a parameter, so the solution of the NLPP (4) is $\frac{5}{3} + \frac{7}{3}\theta, \theta \in [0,1]$. It means the solution lies between $\frac{5}{3}$ and 4.

5.2. Example 2

Let us consider a NLPP whose cost coefficients and basis vectors are real NNs,

$$Max\ f(Y, I) = (9 + I)y_1 + 10y_2 + (6 + I)y_3 \quad (5)$$
$$subject\ to, y_1 + y_2 + 2y_3 \leq 2 + 2I$$
$$2y_1 + 3y_2 + 4y_3 \leq 3 + 3I$$
$$6y_1 + 6y_2 + 2y_3 \leq 8 + 5I$$
$$y_i \geq 0, i = 1,2,3, I \in [0,1]$$

Introducing three slack variables y_4, y_5 and y_6 to the L.H.S. of the first, second, and third constraints respectively, we get the converted constrains as

$$y_1 + y_2 + 2y_3 + y_4$$
$$2y_1 + 3y_2 + 4y_3 + y_5 = 3 + 3I$$
$$6y_1 + 6y_2 + 2y_3 + y_6 = 8 + 5I$$

The standard form of NLPP is given by

$$Max\ f(Y,I) = (9+I)y_1 + 10y_2 + (6+I)y_3 \quad (6)$$
$$subject\ to\ , y_1 + y_2 + 2y_3 + y_4 = 2 + 2I$$
$$2y_1 + 3y_2 + 4y_3 + y_5 = 3 + 3I$$
$$6y_1 + 6y_2 + 2y_3 + y_6 = 8 + 5I$$

The successive simplex tables of the NLPP are given below:

		c	(9+I)	10	(6+I)	0	0	0	
c_B	y_B	b	a_1	a_2	a_3	a_4	a_5	a_6	Min ratio
	y_4	2+2I	1	1	2	1	0	0	$(2+2I)$
	y_5	3+3I	2	(3)	4	0	1	0	$1+I \rightarrow$
	y_6	8+5I	6	6	2	0	0	1	$\frac{(8+5I)}{6}$
	$z_j - c_j$		$-9-I$	$-10\uparrow$	$-6-I$	0	0	0	
	y_4	$(1+I)$	$\frac{1}{3}$	0	$\frac{2}{3}$	1	$-\frac{1}{3}$	0	$(3+3I)$
	y_2	$(1+I)$	$\frac{2}{3}$	1	$\frac{4}{3}$	0	$\frac{1}{3}$	0	$\frac{(3+3I)}{2}$
	y_6	$(2-I)$	(2)	0	-6	0	-2	1	$\frac{(2-I)}{2} \rightarrow$
	$z_j - c_j$		$\frac{-7-3I}{3}$	0	$\frac{22-3I}{3}$	0	$\frac{10}{3}$	0	

		c	(9+I)	10	(6+I)	0	0	0	
c_B	y_B	b	a_1	a_2	a_3	a_4	a_5	a_6	Min ratio
	y_4	$\frac{(4+7I)}{6}$	0	0	$\frac{5}{3}$	1	0	$-\frac{1}{6}$	$\frac{(4+7I)}{10}$
	y_2	$\frac{1+4I}{3}$	0	1	$(\frac{10}{3})$	0	1	$-\frac{1}{3}$	$\frac{1+4I}{10} \rightarrow$
$9+I$	y_1	$\frac{2-I}{2}$	1	0	-3	0	-1	$\frac{1}{2}$	
	$z_j - c_j$		0	0	$\frac{1-12I}{3}\uparrow$	0	$1-I$	$\frac{7+3I}{6}$	
	y_4	$\frac{(1+I)}{2}$	0	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$	0	
$6+I$	y_3	$\frac{1+4I}{10}$	0	$\frac{3}{10}$	1	0	$\frac{3}{10}$	$-\frac{1}{10}$	
$9+I$	y_1	$\frac{13+7I}{10}$	1	$\frac{9}{10}$	0	0	$-\frac{1}{10}$	$\frac{1}{5}$	
	$z_j - c_j$	$\frac{123+112I}{10}$	0	$\frac{-1+12I}{10}$	0	0	$\frac{9+2I}{10}$	$\frac{12+I}{10}$	

Since all $z_j - c_j \geq 0$, so the problem becomes of its optimal stage. Note that maximum value of I is 1. So, $(-1+12I)/10$ is positive.

Hence the optimal solution is $f_{max} = \frac{123+112I}{10}$, for $y_1 = \frac{13+7I}{10}, y_2 = 0, y_3 = \frac{1+4I}{10}$.

Here, also the solutions for under and over achievement problems are respectively $\underline{f_{max}} = \frac{123}{10}$ and $\overline{f_m ax} = \frac{235}{10}$.

Now, we check the solution of under achievement and over achievement directly. For the under achivement we choose $I = 0$ so the NLPP is

$$Max\ f(Y, I=0) = 9y_1 + 10y_2 + 6y_3 \quad (7)$$
$$subject\ to, y_1 + y_2 + 2y_3 + y_4 \quad = 2$$
$$2y_1 + 3y_2 + 4y_3 \quad + y_5 \quad = 3$$
$$6y_1 + 6y_2 + 2y_3 \quad + y_6 = 8$$
$$y_i \geq 0, i = 1,2,3,4,5,6$$

The solution of this problem (under achievement) is
$\underline{f_{max}} = 12.3$ for $y_1 = 1, y_2 = 0.3$ and $y_3 = 0$.
For the over achievement we choose $I = 1$ so the NLPP is

$$Max\ f(Y, I=1) = 10y_1 + 10y_2 + 7y_3 \quad (8)$$
$$subject\ to, y_1 + y_2 + 2y_3 + y_4 \quad = 4$$
$$2y_1 + 3y_2 + 4y_3 \quad + y_5 \quad = 6$$
$$6y_1 + 6y_2 + 2y_3 \quad + y_6 = 13$$
$$y_i \geq 0, i = 1,2,3,4,5,6$$

The solution of this problem is $\overline{f_{max}} = 23.5$ for $y_1 = 2, y_2 = 0$ and $y_3 = \frac{1}{2}$.
Thus, we conclude that the optimal solution is $f_{max} = \frac{123+112I}{10}$, for
$y_1 = \frac{13+7I}{10}, y_2 = 0, y_3 = \frac{1+4I}{10}$.

5.3. An application

Three metals, viz. iron, copper, and zinc are used to produce three commodities: A, B, and C. For one unit of A, the requirements are $40 + 5I$ kg of iron, 32 kg of copper, and $6 + 4I$ kg of zinc. For one unit of B, the needs are 70 kg of iron, $14 + 6I$ kg of copper, and $9 + I$ kg of zinc. To produce one unit of C, $40 + I$ kg of iron, $18 + 2I$ kg of copper, and $8 + 2I$ kg of zinc are required. The total available amounts are 1 metric ton of iron, 5 quintals of copper, and 2 quintals of zinc. The profits per unit for A, B, and C are Rs. $300 + 50I$, Rs. 200, and Rs. $100 + 20I$, respectively, with the indeterminacy I ranging within [0,1]. The goal is to maximize the total profit z, which represents the objective function. The details of the required quantities are summarized in the following table:

	Iron	Copper	Zinc
Total	1000kg	500kg	200kg
A	$(40 + 5I)$kg	32 kg	$(6 + 4I)$kg
B	70 kg	$(14 + 6I)$kg	$(9 + I)$kg
C	$(40 + I)$kg	$(18 + 2I)$kg	$(8 + 2I)$kg

To get the maximum profit, let y_1 units of A, y_2 units of B and y_3 units of C are to be produced.

Then total quantity of iron needed is

$$\{(40 + 5I)y_1 + 70y_2 + (40 + I)y_3\} \text{ kg.}$$

Similarly, total quantity of copper needed is

$$\{32y_1 + (14 + 6I)y_2 + (18 + 2I)y_3\} \text{ kg.}$$

Zinc

$$\{(6+4I)y_1+(9+I)y_2+(8+2I)y_3\} \text{ kg.}$$

and from the condition of the NLPP

$$(40+5I)y_1+70y_2+(40+I)y_3 \leq 1000$$
$$32y_1+(14+6I)y_2+(18+2I)y_3 \leq 500$$
$$(6+4I)y_1+(9+I)y_2+(8+2I)y_3 \leq 200$$

The objective function

$$z=(300+50I)y_1+200y_2+(100+20I)y_3$$

which is to be maximizes. Hence, the NLPP can be formulated as,

$$\begin{aligned}&Maximize,\ z=(300+50I)y_1+200y_2+(100+20I)y_3\\&subject\ to, (40+5I)y_1+70y_2+(40+I)y_3 \leq 1000\\&32y_1+(14+6I)y_2+(18+2I)y_3 \leq 500\\&(6+4I)y_1+(9+I)y_2+(8+2I)y_3 \leq 200\\&y_j \geq 0, j=1,2,3, I \in [0,1]\end{aligned}$$

Introducing three slack variables y_4, y_5 and y_6 to the L.H.S. of the first, second, and third constraints respectively, we get the standered converted NLPP form as

$$\begin{aligned}&Maximize,\ z=(300+50I)y_1+200y_2+(100+20I)y_3\\&subject\ to, (40+5I)y_1+70y_2+(40+I)y_3+y_4=1000\\&32y_1+(14+6I)y_2+(18+2I)y_3+y_5=500\\&(6+4I)y_1+(9+I)y_2+(8+2I)y_3+y_6=200\\&y_j \geq 0, j=1,2,3,4,5,6, I \in [0,1]\end{aligned}$$

By similar solution method of Example-1 and Example-2, we can solve the above NLPP problem.

Clearly, the basic solution in the NLPP contain the indeterminate result (usually NNs, but not always). Theoretically, I is any subinterval of the unit interval [0,1], but practically I is considered as $I=[0,1]$.

The optimal value for the under achievement is $\underline{z_{max}}=5178.571$, for $y_1=12.5, y_2=7.142857$ and $y_3=0$.

The optimal value for over achievement is $\overline{z_{max}}=5468.75$, for $y_1=15.625, y_2=0$ and $y_3=0$.

Thus, the solution is $z(Y,I)=[5178.57,5468.75]$ for $I \in [0,1]$.

When the indeterminacy I is set to $I=0$, the NLPP simplifies to a traditional LPP. In this scenario, the optimal solution is $y_1^*=12.5$, $y_2^*=7.14$, and $y_3^*=0$, with the maximum objective function value being $x(Y^*,I)=5178.57$. This shows that traditional linear programming is a special case of NLPP. Thus, the NLPP method is more versatile and applicable than traditional linear programming methods, particularly for problems involving indeterminate conditions.

In terms of NNs, the optimal solution is $z_{max}=5178.57+290.18\theta, \theta \in [0,1]$.

6. Conclusion

This paper introduces basic operations of NNs and the concept of NFs, and then develops a method for Neutrosophic Linear Programming Problems (NLPP) to address optimization issues under indeterminate conditions. A numerical example demonstrates the application of this NLPP method, which is subsequently used to solve a production planning problem.

The key benefits of the proposed NLPP method include: (1) It provides neutrosophic optimal solutions (often represented by NNs, though not exclusively), which can indicate possible ranges for decision variables and the neutrosophic objective function when the indeterminacy I is defined within a specific interval range corresponding to practical requirements. This capability overcomes the limitations of traditional uncertain linear programming methods, which typically yield only unique crisp optimal solutions; and (2) The NLPP method extends traditional linear programming techniques, offering a more general and practical approach for solving problems in indeterminate environments.

Thus, this study enriches existing methods in uncertain linear programming and offers a novel approach to handle indeterminate optimization challenges. The paper's contributions highlight that traditional uncertain linear programming methods fall short in managing NLPPs under indeterminate conditions. Future research will aim to expand the NLPP framework to address neutrosophic nonlinear programming problems and explore its applications in diverse fields such as engineering, management, and design.

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