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Finite Difference Method to Solve Second Order Boundary Value Problem With Real Neutrosophic Coefficients

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Abstract. The finite difference method is a widely used numerical technique for solving differential equations, particularly in boundary value problems (BVPs). However, traditional approaches often face challenges in addressing the uncertainties present in real-world scenarios. This paper introduces an innovative method that combines the finite difference technique with real neutrosophic numbers, providing a more comprehensive framework for managing uncertainties. Real neutrosophic numbers enabling a more flexible representation of uncertain conditions in BVPs. By employing this approach, solutions that better capture the inherent uncertainties are achieved, leading to more reliable and accurate outcomes. The abstract covers the theoretical foundation, implementation, and potential applications of the proposed method across various scientific and engineering domains.

Keywords: Neutrofication, solution of system of equations, finite difference method, neutrosophic number

AMS Mathematics Subject Classification (2010): 90B05

Abbreviation	Meaning
NSs	Neutrosophic set
$\mathcal{M}_{mn}^{NFM}$	Set of all NFMs of order $m \times n$
$\mathcal{M}_{n}^{NFM}$	Set of all NFMs of order $n \times n$
$\mathcal{M}_{mn}^{FNM}$	Set of all FNMs of order $m \times n$
$\mathcal{M}_{n}^{FNM}$	Set of all FNMs of order $n \times n$
$\mathcal{M}_{mn}^{\mathbb{R}}$	Set of all real matrices of order $m \times n$
$\mathcal{M}_{n}^{\mathbb{R}}$	Set of all real matrices of order $n \times n$

1. Introduction

After the establishment of fuzzy set (FS) theory, it became evident that while FS effectively addressed many problems involving non-random uncertainty, there were instances where it fell short. In 1983 [4], Atanassov introduced intuitionistic fuzzy sets (IFS) as an extension of FS. IFS introduces two parameters: membership value and non-membership value, which must sum up to less than or equal to 1. When these values sum exactly to 1 for all members, IFS reduces back to FS. Essentially, IFS is designed to handle situations where FS fails due to insufficient information or inability to provide a complete solution.

However, even IFS may not suffice in cases involving incomplete, inconsistent, or indeterminate information. To address these more complex scenarios, Smarandache [40, 41, 42, 43, 44] introduced neutrosophic sets (NSs). NSs are characterized by three parameters: truth membership function (t), indeterminacy membership function (i), and falsity membership function (f). This concept expands upon IFS by accommodating situations where uncertainties extend beyond what IFS can effectively manage, offering a robust framework to handle such diverse and challenging problem.

This theory has found extensive application across various fields [51], addressing indeterminate and inconsistent information in real-world scenarios. Neutrosophic sets (NS) generalize classical fuzzy sets, interval-valued fuzzy sets (FS), intuitionistic fuzzy sets (IFS), and other related concepts. Researchers have further extended NS into various branches, such as interval neutrosophic sets [47, 49], generalized neutrosophic soft sets [7], among others.

Subsequently, there has been increased interest in NS, leading to the development of the Neutrosophic Fuzzy Number (NFN) along with corresponding matrices [11]. This extension builds upon FS and IFS frameworks. In 2010, Wang et al. [49] introduced the concept of single-valued neutrosophic sets (SVNS), where truth, indeterminacy, and falsity can be independently quantified. Such frameworks are particularly suited to model human reasoning processes, accommodating the imperfect knowledge individuals receive from external sources. For comprehensive details on neutrosophic sets, numbers, and matrices. This theory has been found extensive application in various fields $c10ye14$ for dealing with indeterminate and inconsistent information in the real world. The NS generalized the concept of classical fuzzy set, interval-valued FS, IFS and, so on. Taking into account the NS, several authors worked on its different branches, viz. interval neutrosophic set [47, 49], generalized neutrosophic soft set [7], etc. After that, some others became more interested in NS and developed NFN together with its corresponding matrices [11].

The neutrosophic fuzzy logic is used to solve many decision-making problems and it is also used to many interesting problems on social networks.

2. Neutrosophic number

Samrandche first proposed a concept of **neutrosophic number** which consists of the determinant part and the indeterminate part. It is usually denoted by $N = a + bI$, where a and b are real numbers and I is the indeterminacy such that $I^2 = I, I.0 = 0$ and $\frac{I}{I}$ is undefined. We call $N = a + bI$ as a pure neutrosophic number if $a = 0$.

For example, we consider a neutrosophic number $N = 5 + 3I$. If $I \in [0,0.02]$, then it is equivalent to $N \in [5,5.06]$ for $N \geq 5$. This means the determinant part is 5, whereas the indeterminacy part is $3I$ for $I \in [0,0.02]$, which means the possibility for number N to be a little bigger than 5.

Note that this number looks like a complex number, but, see that here $I^2 = I$, not -1 like a complex number.

The three basic operators defined on neutrosophic numbers $P = p_1 + q_1 I$ and $Q = p_2 + q_2 I$ are as follows:

(i) $P + Q = (p_1 + p_2) + (q_1 + q_2)I$

(ii) $P - Q = (p_1 - p_2) + (q_1 - q_2)I$

(iii) $P \times Q = p_1 p_2 + (p_1 q_2 + q_1 p_2 + q_1 q_2)I$

In real neutrosophic algebra, we denote K as the neutrosophic field over some neutrosophic vector spaces. We call the smallest field generated by $K \cup I$ or $K(I)$ to be the neutrosophic field for it involves the indeterminacy factor in it, where I has the special property that $I^n = I, I + I = I$ and if $t \in K$ be some scalar then $t.I = tI, 0.I = 0$. Thus, we generally denote neutrosophic field $K(I)$ generated by $K \cup I$, i.e. $K(I) = \langle K \cup I \rangle$.

3. Real neutrosophic matrix

Here we consider the neutrosophic matrix over real numbers based on the work of Smarandache [45]. So it is referred to as a real neutrosophic matrix and is abbreviated by RNM. For details of this matrix see [3].

The neutrosophic number over the field of real/complex numbers is defined in the form $a = a_1 + b_1 I$, where a_1, a_2 are real or complex numbers and I is the indeterminacy [21].

An RNM is defined as in FNM, i.e. of the form $M = M_1 + M_2 I$ where M_1 and M_2 are real matrices. The set of real matrices of order $m \times n$ is denoted by $\mathcal{M}_{mn}^{\mathbb{R}}$ and that of order $n \times n$ by $\mathcal{M}_n^{\mathbb{R}}$. The identity RNM of order $n \times n$ is denoted by U_n, all diagonal elements are 1 and all other elements are 0.

The null and identity matrices of order 3×3 are

$$O_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ and \ \ U_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The basic operations on RNMs $M = M_1 + M_2 I$ and $N = N_1 + N_2 I$ are

(i) $M + N = (M_1 + N_1) + (M_2 + N_2)I$

(ii) $M - N = (M_1 - N_1) + (M_2 - N_2)I$

(iii) $MN = (M_1 N_1) + (M_2 N_1 + M_1 N_2 + M_2 N_2)I$. In this case also, $I^n = I^2 = I$ for any positive integer n.

Assumed that the order of RNMs is compatible with the appropriate operations.

This matrix looks like a complex matrix, but see that here I represents indeterminacy, not complex $i = \sqrt{-1}$. Also, $I^n = I$ for all positive integer n, which is not true for complex numbers.

4. Solution of neutrosophic system of linear equations

Let us consider a system of linear equations:

$$PY = Q, \quad \text{i.e.,} \quad (P_1 + P_2 I)(Y_1 + Y_2 I) = (Q_1 + Q_2 I) \qquad (1)$$

where $P_1, P_2 \in \mathbb{R}^{n \times n}$, $Y_1, Y_2, Q_1, Q_2 \in \mathbb{R}^{n \times 1}$.

The i,j-th element of P is $p_{ij} = p_{ij}^{(1)} + p_{ij}^{(2)} I$, the j-th element of Y and Q are

$y_j = y_j^{(1)} + y_j^{(2)}I$ and $q_j = q_j^{(1)} + q_j^{(2)}I$ respectively.

Then the matrices P_1 and P_2 are:

$$P_1 = \left(p_{ij}^{(1)}\right)_{n\times n}, \quad P_2 = (pij^{(2)})_{n\times n}$$

and the vectors Y_1, Y_2, Q_1, Q_2 are:

$$Y_1 = \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \\ \vdots \\ y_n^{(1)} \end{pmatrix}, \quad Y_2 = \begin{pmatrix} y_1^{(2)} \\ y_2^{(2)} \\ \vdots \\ y_n^{(2)} \end{pmatrix}, \quad Q_1 = \begin{pmatrix} q_1^{(1)} \\ q_2^{(1)} \\ \vdots \\ q_n^{(1)} \end{pmatrix}, \quad Q_2 = \begin{pmatrix} q_1^{(2)} \\ q_2^{(2)} \\ \vdots \\ q_n^{(2)} \end{pmatrix}$$

From Eq. (1),

$$P_1Y_1 + [(P_1 + P_2)(Y_1 + Y_2) - P_1Y_1]I = Q_1 + Q_2I \tag{2}$$

That is,

$$P_1Y_1 = Q_1 \tag{3}$$

$$(P_1 + P_2)(Y_1 + Y_2) - P_1Y_1 = Q_2 \quad \text{or} \quad (P_1 + P_2)(Y_1 + Y_2) = Q_1 + Q_2 \tag{4}$$

Eq. (3) gives the vector Y_1 and Eq. (4) gives $Y_1 + Y_2$. The final solution of Eq. (1) is:

$$Y = Y_1 + Y_2I \tag{5}$$

Notice that Eqs. (3) and (4) are systems of real equations.

Example 1. *Let us consider the neutrosophic system of equations:*

$$\begin{pmatrix} 2+4I & 3-I \\ 4-2I & -5+7I \end{pmatrix} \begin{pmatrix} y_1^{(1)} + y_1^{(2)}I \\ y_2^{(1)} + y_2^{(2)}I \end{pmatrix} = \begin{pmatrix} 8+2I \\ -6+12I \end{pmatrix}$$

In this problem,

$$P_1 = \begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 4 & -1 \\ -2 & 7 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 8 \\ -6 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 4 \\ 14 \end{pmatrix}$$

$$Y_1 = \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \end{pmatrix}, \quad Y_2 = \begin{pmatrix} y_1^{(2)} \\ y_2^{(2)} \end{pmatrix}$$

The real system of equations are:

$$P_1Y_1 = Q_1, \quad \text{i.e.,} \quad \begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix} \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$$

and

$$(P_1 + P_2)(Y_1 + Y_2) = Q_1 + Q_2, \quad \text{i.e.,} \quad \begin{pmatrix} 6 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} y_1^{(1)} + y_1^{(2)} \\ y_2^{(1)} + y_2^{(2)} \end{pmatrix} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$$

The solution of these equations is:

$$Y_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \text{and} \quad Y_1 + Y_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Thus, the final solution is:

$$Y = \begin{pmatrix} 1 + 0I \\ 2 + I \end{pmatrix}$$

5. Neutrofication of finite difference method

The finite difference method is a numerical technique used for solving differential equations by approximating derivatives with differences. When combined with a real neutrosophic matrix, it can provide a more flexible framework to handle uncertainties and indeterminacies in boundary value problems (BVPs).

5.1. Finite difference in real setup

In a real sense i.e., here we take independent variable(x) and the dependent variable (y) also a real. In this method,the derivatives y' and y'' are replaced by finite differences (either by forward or central) and generates a system of linear algebraic equations. the central difference formula are used to replace derivatives

$$y'(x_i) = \frac{y_{i+1} - y_{i_1}}{2h} + O(h^2) and \;\; y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + O(h^2) \tag{6}$$

the method to solve first order differential equation using finite difference method is nothing but the Euler's method. The finite difference method is commonly used method to solve second order initial and boundary value problem.Here we solve the boundary value problem.

5.2. Second order boundary value problem (BVP)

Let us consider the linear second order differential equation

$$y'' + p(x)y' + q(x)y = r(x), \quad a < x < b \tag{7}$$

with boundary conditions y(a)=λ_1 and y(b)=λ_2.

Let the interval [a,b] divided into n subintervals with spacing h. That is, $x_i = x_{i-1} + h, i = 1,2,\ldots,n-1$.

The equation (3.2) is satisfied by $x = x_i$. Then

$$y''_i + p(x_i)y'_i + q(x_i)y_i = r(x_i). \tag{8}$$

Now, y''_i and y'_i are replaced by finite difference expressions

$$y'(x_i) = \frac{y_{i+1} - y_{i_1}}{2h} + O(h^2) \quad and \quad y''(x_i) = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + O(h^2) \tag{9}$$

Using these substitutions and drooping $O(h^2)$, equation (8) becomes,

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + p(x_i)\frac{y_{i+1} - y_{i_1}}{2h} + q(x_i)y_i = r(x_i); \tag{10}$$

that is,

$$y_{i-1}[2 - hp(x_i)] + y_i[2h^2q(x_i) - 4] + y_{i+1}[2 + hp(x_i)] = 2h^2r(x_i) \tag{11}$$

6. Finite difference of second order BVP in neutrosophic setup

In the above equation (11) we apply neutrosophic number,

Let, $p(x_i) = (p_1 + p_2 I)x_i$, $q(x_i) = q_1 + q_2 I + x_i$, $r(x_i) = r_1 + r_2 I x_i$.

where p_1,p_2,q_1,q_2,r_1,r_2 are real or complex number and I represent the indeterminacy.

Then,

$$y_{i-1}[2 - h(p_1 + p_2 I)x_i] + y_i[2h^2(q_1 + q_2 I + x_i) - 4] + y_{i+1}[2 + h(p_1 + p_2 I)x_i] = 2h^2(r_1 + r_2 I x_i) \quad (12)$$

Again let,

$$2 - h(p_1 + p_2 I)x_i = C_i;$$
$$2h^2(q_1 + q_2 I + x_i) - 4 = A_i;$$
$$2 + h(p_1 + p_2 I)x_i = B_i;$$
$$2h^2(r_1 + r_2 I x_i) = D_i,$$

With these notations the equations (12) is simplified to

$$C_i y_{i-1} + A_i y_i + B_i yi + 1 = D_i, \quad (13)$$

for $i = 1,2,\dots,n-1$.

The boundary conditions then are y(a)=λ_1 and y(b)=λ_2.

For $i = 1,2,\dots,n-1$ equation (13) reduces to

$C_1 y_0 + A_1 y_1 + B_1 y_2 = D_1$, $or, A_1 y_1 + B_1 y_2 = D_1$ (as $y_0 = \lambda_1$)

and $C_{n-1} y_{n-2} + A_{n-1} y_{n-1} = D_{n-1} - B_{n-1}\lambda_2$. (as $y_n = \lambda_2$)

The equation (13) can be written as

$$Ay = b \quad (14)$$

where $y = [y_1, y_2, \dots, y_{n-1}]^t$

$$b = 2h^2[(r_1 + r_2 x_1 I) - C_1\lambda_1/(2h^2), (r_1 + r_2 x_2 I), \dots, (r_1 + r_2 x_{n-2}, (r_1 + r_2 x_{n-1} - B_{n-1}\lambda_2/(2h^2)]^t$$

and

$$\mathbf{A} = \begin{pmatrix} A_1 & B_1 & 0 & 0 & \cdots & 0 & 0 \\ C_2 & A_2 & B_2 & 0 & \cdots & 0 & 0 \\ 0 & C_3 & A_3 & B_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & C_{n-1} & A_{n-1} \end{pmatrix} \quad (15)$$

Equation (14) is a system of equation which can be solved by the neutrosophic algebraic equations . The solution of this system i.e., the values of y_i for $i = 1,2,\dots,n-1$ constitutes the the solution of the BVP. The solution like of the form,

$$y_i = \underline{y_i} + \overline{y}_i I$$

The above result is illustrated by the following example.

6.1. Illustration

Let, $p_1 = 3$,$p_2 = 2$,$q_1 = -5$,$q_2 = 3$,$r_1 = 4$,$r_2 = 1.5$ i.e.,

$p(x_i) = (3 + 2I)x_i$, $\quad q(x_i) = -5 + 3I + x_i$, $\quad r(x_i) = 4 + 1.5 I x_i$.

with the boundary condition $y(0) = 0, y(1) = 0$.
Here $nh = 1$. The difference scheme is

$$y_{i-1}[2 - h(3 + 2I)x_i] + y_i[2h^2(-5 + 3I + x_i) - 4] + y_{i+1}[2 + h(3 + 2I)x_i] = 2h^2(4 + 1.5Ix_i) \quad (16)$$

For $i = 1,2,3$.

$$A_1 = 2h^2(-5 + 3I + x_1) - 4,$$
$$A_2 = 2h^2(-5 + 3I + x_2) - 4,$$
$$A_3 = 2h^2(-5 + 3I + x_3) - 4,$$
$$B_1 = 2 + h(3 + 2I)x_1,$$
$$B_2 = 2 + h(3 + 2I)x_2,$$
$$C_1 = 2 - h(3 + 2I)x_1,$$
$$C_2 = 2 - h(3 + 2I)x_2,$$
$$C_3 = 2 - h(3 + 2I)x_3,$$
$$D_1 = 2h^2(4 + 1.5Ix_1),$$
$$D_2 = 2h^2(4 + 1.5Ix_2),$$
$$D_3 = 2h^2(4 + 1.5Ix_3).$$

Let $\underline{n = 2}$. Then $h = 0.5,\ x_0 = 0,\ x_1 = 0.5,\ x_2 = 1,\ y_0 = 0,\ y_2 = 0$.
The difference scheme is

$$y_0[2 - h(3 + 2I)x_1] + y_1[2h^2(-5 + 3I + x_1) - 4] + y_2[2 + h(3 + 2I)x_1] = 2h^2(4 + 1.5Ix_1)$$

or, $\quad y_1[2(0.5)^2(-5 + 3I + 0.5) - 4] = 2(0.5)^2(4 + (1.5)I(0.5))$

or, $\quad y_1 = -0.64 + (-0.2809375)I$

i. e., $\quad y(0.5) = -0.64 + (-0.2809375)I$

Let n=4.Then $h = 0.25,\ x_0 = 0,\ x_1 = 0.25,\ x_2 = 0.50,$ $x_3 = 0.75, x_4 = 1.0, y_0 = 0, y_4 = 0$.
This system of equations becomes

$$y_0[2 - h(3 + 2I)x_1] + y_1[2h^2(-5 + 3I + x_1) - 4] + y_2[2 + h(3 + 2I)x_1] = 2h^2(4 + 1.5Ix_1)$$
$$y_1[2 - h(3 + 2I)x_2] + y_2[2h^2(-5 + 3I + x_2) - 4] + y_3[2 + h(3 + 2I)x_2] = 2h^2(4 + 1.5Ix_2)$$
$$y_2[2 - h(3 + 2I)x_3] + y_3[2h^2(-5 + 3I + x_3) - 4] + y_4[2 + h(3 + 2I)x_3] = 2h^2(4 + 1.5Ix_3)$$

This system is finally simplified to

$$y_1(-4.59375 + 0.375I) + y_2(2.1875 + 0.125I) = 0.5 + 0.046875I$$
$$y_1(1.625 - 0.25I) + y_2(-4.5625 + 0.375I) + y_3(2.375 + 0.25I) = 0.5 + 0.09375I$$
$$y_2(1.4375 + 0.375I) + y_3(-4.53125 + 0.375I) = 0.5 + 0.1406I$$

These equations can be written as

$$MX = b \quad (17)$$

or, $(M_1 + M_2I)(X_1 + X_2I) = (b_1 + b_2I)$.

$$\text{Here, } M_1 = \begin{pmatrix} -4.59375 & 2.1875 & 0 \\ 1.625 & -4.5625 & 2.375 \\ 0 & 1.4375 & -4.53125 \end{pmatrix};$$

$$M_2 = \begin{pmatrix} 0.375 & 0.125 & 0 \\ -0.25 & 0.375 & 0.25 \\ 0 & 0.375 & 0.375 \end{pmatrix}$$

$$X_1 = \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_3^{(1)} \end{pmatrix}; \quad X_2 = \begin{pmatrix} y_1^{(2)} \\ y_2^{(2)} \\ y_3^{(2)} \end{pmatrix}; \quad b_1 = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix}; \quad b_2 = \begin{pmatrix} 0.046875 \\ 0.09375 \\ 0.1406 \end{pmatrix};$$

And $y_1 = y_1^{(1)} + y_1^{(2)}I$; $y_2 = y_2^{(1)} + y_2^{(2)}I$; $y_3 = y_3^{(1)} + y_3^{(2)}I$;

The real system of equation are

$$M_1X_1 = b_1, \text{ i.e. } \begin{pmatrix} -4.59375 & 2.1875 & 0 \\ 1.625 & -4.5625 & 2.375 \\ 0 & 1.4375 & -4.53125 \end{pmatrix} \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_3^{(1)} \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix};$$

and

$$(M_1 + M_2)(X_1 + X_2) = (b_1 + b_2).$$

$$\text{i.e., } \begin{pmatrix} -4.21875 & 2.3125 & 0 \\ 1.375 & -4.1875 & 2.625 \\ 0 & 1.333 & -4.15625 \end{pmatrix} \begin{pmatrix} y_1^{(1)} + y_1^{(2)} \\ y_2^{(1)} + y_2^{(2)} \\ y_3^{(1)} + y_3^{(2)} \end{pmatrix} = \begin{pmatrix} 0.546875 \\ 0.59375 \\ 0.6406 \end{pmatrix}.$$

The solution of these equation are

$$X_1 = \begin{pmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_3^{(1)} \end{pmatrix} = \begin{pmatrix} -0.2257618 \\ -0.27009623 \\ -0.179543 \end{pmatrix},$$

$$\text{and } \quad X_1 + X_2 = \begin{pmatrix} y_1^{(1)} + y_1^{(2)} \\ y_2^{(1)} + y_2^{(2)} \\ y_3^{(1)} + y_3^{(2)} \end{pmatrix} = \begin{pmatrix} -0.32487991 \\ -0.40267961 \\ -0.28954742 \end{pmatrix}.$$

The solution are

$$y_1 = (-0.2257618) + (-0.09911811)I,$$
$$y_2 = (-0.27009623) + (-0.13258338)I,$$
$$y_3 = (-0.179543) + (-0.11000442)I.$$

The solution of the system is

$$y_1 = y(0.25) = (-0.2257618) + (-0.09911811)I.$$
$$y_2 = y(0.50) = (-0.27009623) + (-0.13258338)I.$$
$$y_3 = y(0.75) = (-0.179543) + (-0.11000442)I.$$

This is also the solution of the differential equation.

7. Conclusion

In this project study, finite difference method has been real neutrosophified.In the present study efforts have been made to neutrosophif the finite difference method using netrosophic number and real neutrosophic matrix and also solutions are made for neutrosophic method. The combination of the finite difference method with a real neutrosophic matrix provides a powerful tool for solving BVPs under uncertainty. This approach allows for a more nuanced representation of the problem, accommodating degrees of truth, indeterminacy, and falsity. By applying these methods, one can obtain solutions that better reflect the inherent uncertainties in real-world problems.

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Authors' Contributions. It is the author's full contribution.

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