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A Study on *m*-polar Fuzzy Detour *g*-Boundary Node and *m*-polar Fuzzy Detour *g*-Interior Node of an *m*-polar Fuzzy and Application

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Abstract. In this article, we introduce the concepts of $\boldsymbol{m}$-polar fuzzy ($\boldsymbol{m}$PF) detour $\boldsymbol{g}$-boundary nodes and $\boldsymbol{m}$PF detour $\boldsymbol{g}$-interior nodes within $\boldsymbol{m}$-polar fuzzy graphs ($\boldsymbol{m}$PFGs). We explore their significance and examine their properties. Furthermore, we establish a relationship between $\boldsymbol{m}$PF detour $\boldsymbol{g}$-boundary nodes and $\boldsymbol{m}$PF cut vertices. Utilizing the notion of maximum $\boldsymbol{m}$PF spanning trees, we define $\boldsymbol{m}$PF detour $\boldsymbol{g}$-boundary nodes and $\boldsymbol{m}$PF detour $\boldsymbol{g}$-interior nodes in $\boldsymbol{m}$PF trees. Additionally, we investigate the characteristics of $\boldsymbol{m}$PF complete vertices, $\boldsymbol{m}$PF detour $\boldsymbol{g}$-interior nodes, and $\boldsymbol{m}$PF detour $\boldsymbol{g}$-boundary nodes. Finally, we provide applications of these concepts.

Keywords: mPFG, mPF detour g-distance, mPF detour g interior node, mPF detour g boundary node.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

In real life, graph theory is immensely utilized in various fields, including artificial intelligence, operations research, signal processing, network routing, robotics, electrical engineering, medical science, computer science, etc. In 1965, Zadeh [37] replaced the classical set with a fuzzy set which gives better exactness in both theory and application. In 1975, Rosenfeld [32] initiated the concept of a fuzzy graph and in various fields, it has various applications. The concept of mPF sets was established by Chen et al. [1] in 2014. Based on this concept, Ghorai and Pal [13] defined mPFG and Presented several new results. Ghorai and Pal [14, 16, 10, 15, 12] studied several new results, theories and applications on m-polar fuzzy graphs. Then Singh [21] defined m-polar fuzzy graph representation using the concept of a lattice. Several new results on m-polar fuzzy graphs are studied by Singh [22, 23] and he defined a new result on the bipolar fuzzy graph in the references [25, 26, 24]. The idea of a strong arc in a fuzzy graph was given by Bhutani and Rosenfeld [3] and Mathew and Sunitha [29] defined different types of arcs in a fuzzy graph. The notion of a bridge, trees, cycles, cut node, and end node was introduced by Rosenfeld [32]. The concepts of strength of connectedness in mPFG, mPF tree, and mPF cut node are established by Mandal et al. [28]. Different types of fuzzy graphs with operations and applications are explained in the references [27, 34]. Rashmanlou et al. [30, 31] presented some work on bipolar and interval-valued fuzzy graphs. Samanta and Pal defined fuzzy planar graphs [35]. Ghorai and Pal investigated the isomorphic properties of m-polar fuzzy graphs [16]. The other interesting papers related to this work are [40-44].

Linda and Sunitha [17] gave the concept of fuzzy detour g-distance. Rosenfeld and Bhutani [3] established the notion of g-distance in a fuzzy graph. Linda and Sunitha [18] founded the notation of g-boundary node, g-interior node, g-eccentric node. Sameena and Sunitha [33] gave a characterization of g-self centred fuzzy graph. The length of the longest $x-y$ path in a connected fuzzy graph G is the detour distance between two nodes x and y defined in [6]. Chartrand [9] defined the main concept of the detour centre of a graph. The notion of detour number, detour set, detour nodes, and detour basis in a graph was established by Chartrand et al. [8]. Interior nodes and boundary nodes are discussed in [7]. In this paper, we introduced mPF detour g-distance, mPF detour g-interior node, mPF detour g-boundary node and explained their relations. Also, some properties of these parameters are investigated. For more definitions, terminologies and applications of the fuzzy graph, the reader may consult the book [20].

2. Preliminaries

Firstly, we define mPFGs and other related terms.

In this paper, for a natural number m, m-power of [0,1] or $[0,1]^m$ is considered as a poset with point-wise order $\leq$. "$\leq$" is defined by $x' \leq y' \Leftrightarrow p_i(x') \leq p_i(y')$ for each $i = 1,2,\ldots,m$, where $x', y' \in [0,1]^m$ and $p_i: [0,1]^m \to [0,1]$ be the ith projection mapping.

Definition 2.1. *[11] An m-polar fuzzy graph (mPFG) of a graph $G^* = (V, E)$ is a pair $G = (V, A, B)$ where $B: \widetilde{V^2} \to [0,1]^m$ and $A: V \to [0,1]^m$ are an mPF set in $\widetilde{V^2}$ and an mPF set in V respectively such that $p_i \circ B(a,b) \leq min\{p_i \circ A(a), p_i \circ A(b)\}$ for all $(a,b) \in \widetilde{V^2}$, for each $i = 1,2,\ldots,m$ and $B(a,b) = 0$ for all $(a,b) \in (\widetilde{V^2} - E)$, (The smallest element in $[0,1]^m$ is $0 = (0,0,\ldots,0)$).*

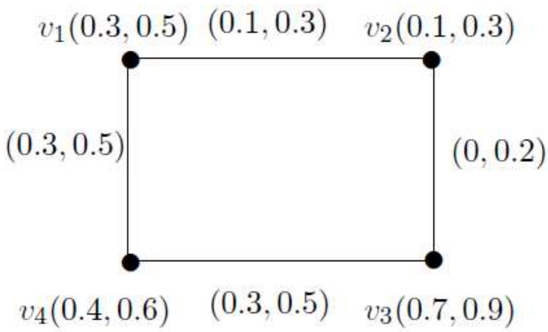

Figure 1: A 2PFG

Definition 2.2. *[10] If an mPFG $G = (V, A, B)$ satisfies the relation*

$$p_i \circ B(x,z) = min\{p_i \circ A(x), p_i \circ A(z)\}, for\ all\ x, z \in V, i = 1,2,3,\ldots,m.$$

Definition 2.3. *[28] A path $u' = v_1, v_2, \ldots, v_n = v'$ in mPFG G is said to be an mPF* *path if this path satisfies the relation* $p_i \circ B(v_j, v_{j+1}) > 0$, $(j = 1,2, \dots, n-1)$ *for at least one* i *and all the vertices are distinct except* v_1 *which may be the same as* v_n.

Definition 2.4. *[28] The strength of the* mPF *path* $P: u' = v_1, v_2, \dots, v_n = v'$ *in* $mPFG$ G *is defined as*

$$S(P) = (B_1^n(u', v'), B_2^n(u', v'), \dots, B_m^n(u', v')),$$

where, $B_k^n(u', v') = \min_{1 \leq i < j \leq n} (p_k \circ B(v_i, v_j)), k = 1,2, \dots, m.$

$CONN_G(u', v')$ is the strength of connectedness between u' and v' and is defined as

$$CONN_G(u', v') = ((\max_{n \in N}(B_1^n(u', v')), (\max_{n \in N}(B_2^n(u', v')), \dots (\max_{n \in N}(B_m^n(u', v'))).$$

Definition 2.5. *[28] An* $mPFG$ *is said to be* mPF *connected graph if* $(p_i \circ B(a', b'))^{\infty} > 0$, *for at least one* $i = 1,2,3, \dots, m$.

Definition 2.6. *[28] A* $u' - v'$ *path* $P: u' = v_1, v_2, \dots, v_n = v'$ *in* $mPFG$ G *is said to be a strongest* mPF $u' - v'$ *path if* $S(P) = CONN_G(u', v')$.

Definition 2.7. *[28] An edge* (a', b') *of an* $mPFG$ G *is said to be strong* mPF *arc if* $B(a', b') \geq CONN_{G-(a',b')}(a', b')$.

Definition 2.8. *[28] A path* $P: x = x_1, x_2, \dots, x_n = y$ *from* x *to* y *is called strong* mPF *path if* (x_i, x_{i+1}) *is strong* mPF *arc for all* $1 \leq i \leq n-1$.

Definition 2.9. *[28] A vertex* y *is an* mPF *cut vertex of* G *if removing it from* G *reduces the connectedness strength between some other pair of nodes* G.

Definition 2.10. *[28] An* $mPFG$ G *is called an* mPF *tree if it has a spanning* mPF *subgraph* H' *which is an* m*-polar F-tree and such that for all* i, $p_i \circ B'(x, y) = 0$ *implies* $p_i \circ B(x, y) < p_i \circ CONN_{H'}(x, y)$.

Definition 2.11. *A maximum spanning* mPF *tree of a connected* $mPFG$ $G = (V, A, B)$ *is an* mPF *spanning subgraph* T *of* G, *which is a* m *polar F-tree, such that* $CONN_G(u, v)$ *is the strength of the unique strongest* uv mPF *path in* T *for all* $u, v \in G$.

3. mPF detour g-boundary node and mPF detour g-interior node of an mPFG

In this section, we defined mPF detour g boundary node and mPF detour g interior node of an mPFG G and discussed some results on these nodes.

Definition 3.1. *A node* k *in a connected* $mPFG$ G *is an* mPF *detour* g *boundary node of a node* l *if* $mPFD_g(l, k) \geq mPFD_g(l, j)$ *for each* j *in* G, *where* j *is a neighbor of* k. *The set of all* mPF *detour* g *boundary nodes of* l *symbolized as* $mPFD_gB(l)$. *The set of* *all mPF detour* g *boundary nodes of* G, *Symbolized as* $mPFD_gB(G)$.

Example 3.2. *For the connected* $mPFG$ G *shown in Figure 1,* $mPFD_gB(a) = \{c, g\}$, $mPFD_gB(b) = \{a, f, g\}$, $mPFD_gB(c) = \{a, g, f\}$, $mPFD_gB(d) = \{a, c, g, f\}$, $mPFD_gB(e) = \{a, f, g\}$, $mPFD_gB(g) = \{a, c, f\}$,$mPFD_gB(f) = \{a, g\}$. *Here* a, c, f, g *are the* m*-polar detour* g*- fuzzy boundary nodes of* G.

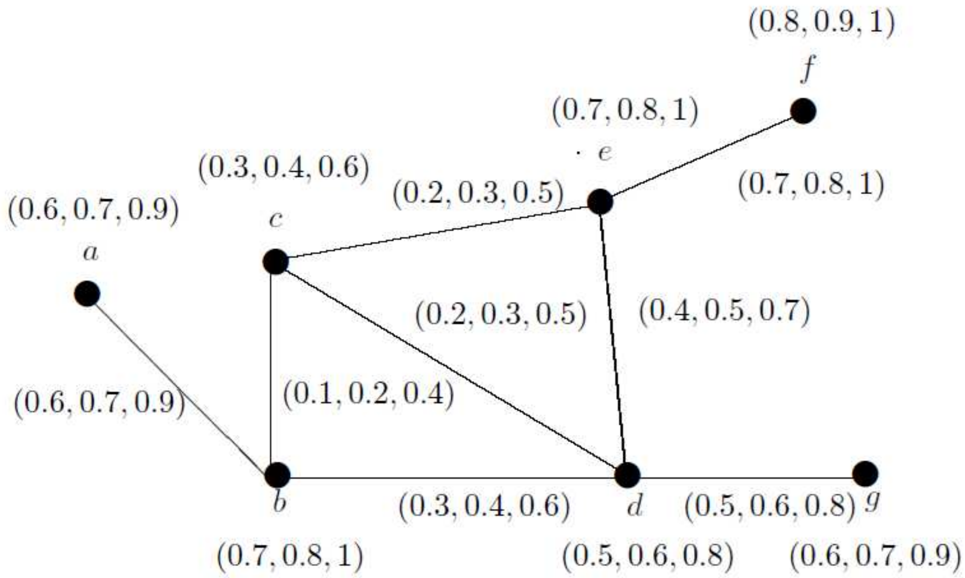

Figure 2: Connected 3PF graph G with boundary nodes $\{a, c, f, g\}$.

Definition 3.3. *The set of all neighbors of* u *is symbolized as* $N_{mPF}(u)$ *and the set of all strong* mPF *neighbors of* u *is symbolized as* $N_{mPFS}(u)$.

Definition 3.4. *If an* mPF *subgraph formed by a strong* m*-polar neighbor of a node* a *in an* $mPFG$ G, *form a complete* $mPFG$ *then the node* b *is said to be a complete node of* G.

Theorem 3.5. *A node in a complete* $mPFG$ *is* mPF *detour* g *boundary node of every other node if and only if the node is complete.*

Proof: Let a node l be a node in a connected mPFG G and l be a complete node. Let k be another node of G. Each arc in G is strong, because of the completeness of G. So $mPFD_g(k, l) = n - 1 = mPFD_g(k, s), \forall s \in N(l)$. Then $l \in mPFD_GB(a)$.

Conversely, let l be an mPF detour g boundary node of every other node. Then each arc in G is strong, because of the completeness of G. Then $mPFD_g(k, l) = n - 1, \forall k \in G$. So all neighbors of l are strong. Hence by Definition 4.3, the node l is complete.

Theorem 3.6. *If a node in a connected* $mPFG$ G *is a complete vertex, then the vertex is an* mPF *detour* g *boundary node of every other node.*

Proof: Here a node l is a complete node in a connected mPFG G. If k is another node of G. Assume that $k = l_0, l_1, \ldots, l_{k-1}, l_k = l$ be a $k - l$ mPF g-detour and $c \in N_m PFS(b)$. Here two cases will arise:

Case 1: If $c = l_{k-1}$, then $mPFD_g(k, c) \leq mPFD_g(k, l)$. Hence, l is a m-polar detour g-fuzzy boundary node of k.
Case 2: If $c \neq l_{k-1}$, since c is a strong neighbor of l, an arc (c, l_{k-1}) is a strong mPF arc and also $c \neq l_{k-1}$. So the length of a path $k = l_0, l_1, \ldots, l_{k-1}, c, l_k = l$ is greater than than the length of a path $k = l_0, l_1, \ldots l_{k-1}, l_k = l$. That is $mPFD_g(k, c) \leq mPFD_g(k, l)$. Hence, $l \in mPFD_g B(k)$.

Remark 3.7. *The converse of the above theorem may not be true. For example, consider the $mPFG$ of Figure 3. We see that s is an mPF detour g boundary node of every other node, but s is not a complete node.*

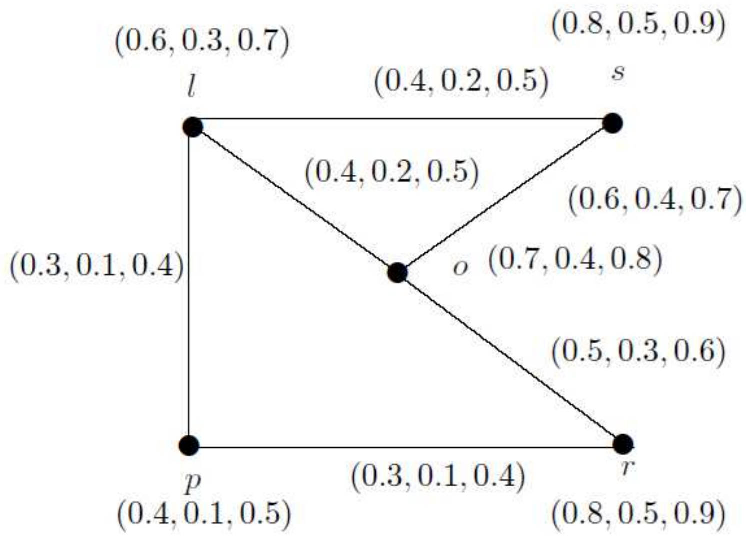

Figure 3: Connected mPFG G.

Theorem 3.8. *A connected $mPFG$ G is an mPF tree iff G is mPF detour g graph.*
Proof: Between any two nodes in mPF tree G, there is exactly one strong mPF path. So $mPFD_g(l, k) = mPFD_g(l, k)$ for any two vertices l, k in G. Hence, G is mPF g-detour graph.

Conversely, let G be an mPF g-detour graph, which has n nodes. Then $mPFD_g(l, k) = mPFD_g(l, k)$ for any two nodes l, k in G. If $n = 2$ then G is an mPF tree.

Let $n \geq 3$. If G is not an mPF tree. So two nodes a, b are exist in G for which there is at least two strong mPF paths between a and b. Let B_1 and B_2 be two $a - b$ strong mPF paths. So, $B_1 \cup B_2$ has a cycle $C(say)$ in G. If the nodes p and q are adjacent nodes in G, then we have $mPFD_g(q, p) = 1$ and $mPFD_g(q, p) > 1$. This contradicts the fact that $mPFD_g(q,p) = mPFD_g(q,p)$. So, G is an mPF tree.

Theorem 3.9. *In an mPF tree G, a vertex l is an mPF detour g boundary node of G iff l cannot be an mPF cut vertex of G.*

Proof: Suppose a node l in mPF tree G is an mPF detour g boundary node of a node g in G. If l is an mPF cut node of G.

Let E be an mPF maximum spanning tree ($MST_{mPF}(G)$) in G and this tree is unique in G. Again since l is an mPF cut node that means l cannot be an internal node of E. Let $p \in N_{mP.F.S}(l)$ such that p does not lie on the mPF detour in E. Therefore, $mPFD_g(q,z)$ is the same when q,z be any two nodes of E. But $mPFD_g(g,p) = mPFD_g(g,l) + mPFD_g(l,p) > mPFD_g(g,l)$. This contradicts the fact that $l \in mPFD_gB(G)$. Therefore the node l cannot be an mPF cut node of G.

Conversely, suppose l be not an mPF cut vertex of the mPFG G. So l is the end vertex of $MST_{mPF}(G)$, which is unique. Then l has a strong neighbor which is also unique [28]. So there does not exist any extension of any mPF g-detour for a node p to l. Hence, $l \in mPFD_g(G)$.

Definition 3.10. *A node l in an $mPFG$ G is an mPF end vertex of G if h is only a strong mPF neighbour of l, where $h \in G$.*

Example 3.11. *For the $mPFG$ G in Fig. 1, the nodes a, f, g are mPF end vertex of G.*

Theorem 3.12. *A vertex a in an mPF tree G is an mPF detour g boundary node then b is an mPF end node. Again if b is an mPF end node then a is an mPF detour g boundary node.*

Proof: Suppose a is an belonging to $mPFD_gB(b)$ in an mPF tree G. Let E be a $MST_{mPF}(G)$ in G, which is unique in G [28]. By Theorem 4.9, each node of G is an mPF cut vertex of G or an mPF end node of G. So by Theorem 4.9, a must be an mPF end node of G.

Conversely, let a be an mPF end vertex of an mPF tree G. Let E be the $MST_{mPF}(G)$ of G. Then a is an mPF end node of E. Hence, a is not an mPF cut node. Therefore, by Theorem 4.9, $a \in mPFD_gB(G)$.

In a connected mPFG G, a node b lies between the nodes a and c in the sense of mPF detour g-distance if $mPFD_g(a,c) = mPFD_g(a,b) + mPFD_g(b,c)$.

Definition 3.13. *In a connected $mPFG$ G, a node b is an m-polar detour g $-$fuzzy interior nodes if for each node a in G different from b, there is a node c in G for which $mPFD_g(a,c) = mPFD_g(a,b) + mPFD_g(b,c)$.*

Definition 3.14. *The set of all mPF detour g-interior node of G, Symbolized as $Int_{mPFD_g}(G)$, form an mPF subgraph of G.*

Example 3.15. *For the $mPFG$ in Figure 1, $Int_{mP.F.D_g}(G) = \{b, e, d\}$.*

Theorem 3.16. *A node in a connected $mPFG$ G is an mPF detour g boundary node of G iff the node cannot be an mPF detour g interior node of G.*

Proof: Let $b \in mPFD_gB(a)$ in a connected mPFG G. If possible, let $b \in Int_{mPFD_g}(G)$. So there exists a node c different from a and b such that b lies between a and c. Let $U: a = b_1, b_2, \ldots, b = b_k, b_{k+1}, \ldots, b_l = c$ be a $a - c$ mPF $g -$detour and $1 < k < l$. Then $b_{k+1} \in N_{mP.F.S}(b)$, and this implies $mPFD_g(a, b_{k+1}) > mPFD_g(a, b)$, so contradiction arise. Hence $b \notin G$.

Conversely, let the node $b \notin Int_{mPFD_g}(G)$. Then a node a exists in G for which any node c different from b and a, $mPFD_g(a,c) \neq mPFD_g(a,b) + mPFD_g(b,c)$. Therefore, $mPFD_g(a,q) \leq mPFD_g(a,b)$ where $q \in N_{mP.F.S}(b)$. This implies that b is a m-polar detour g-fuzzy boundary node of a.

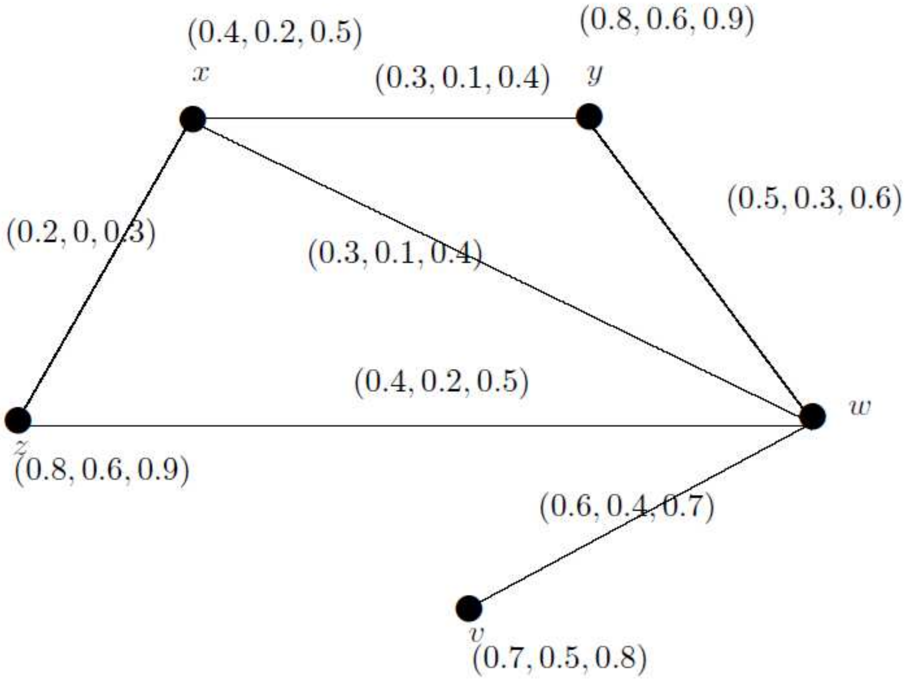

Figure 4: Connected mPFG G.

Example 3.17. For the Connected mPFG G shown in Figure 3, $mPFD_gB(z) = \{u\}$, $mPFD_gB(x) = \{u\}$, $mPFD_gB(y) = \{u\}$, $mPFD_gB(w) = \{x, u\}$, $mPFD_gB(u) = \{x\}$. Here x, u are the m-polar detour g- fuzzy boundary nodes of G, but x, u are not m-polar detour g- fuzzy interior nodes of G. Again z, w, y are m-polar detour g- fuzzy interior nodes of G but they are not m-polar detour g- fuzzy boundary nodes of G. So if we consider any Connected mPFG, we can easily show that the above theorem is true.

Theorem 3.18. *A mPF end vertex of a connected $mPFG$ G cannot be an mPF detour g interior node.*

Proof: Let q be an mPF end node of an mPFG G. Then there is only one mPF strong neighbor of q. So there is no strong mPF g-detour for which b lies between a and c, where a and c are two nodes of G and also different from b. Hence, $b \notin Int_{mPFD_g}(G)$.

4. Application

Many problems in the real world involve multipolar information or multi-agents or multi-objects. Here, we present an application of mPFG about how a person can reach his destination in a short time using the strong path. Compared to a fuzzy graph, mPFG gives more accurate and exact results for real problems. In modern days, if we go from one town to another, we usually use a car, train, bus, etc. The availability of buses or trains is not the same everywhere. When a person makes the same trip every day to work or school, this type of travelling is usually called commuting. Some people travel on their vacation to visit other states, cities or countries. If the communication system is good, then the journey will be good. This communication system depends not only on the economic condition but also on many other things, for example, infrastructure, environment, fire safety, security, etc. Again, if the economic system of a city is good, then the road condition is generally good.

Here, we present a model of 3PFG, which is used to find the shortest strong path between two cities. Fig. 3 shows a model of the road network which is represented by a 3PFG $G = (V, A, B)$. Here the vertices stand for cities and each edge of G stands for the roads between two cities. Here six cities are considered and they are denoted as $V = \{V_6, V_5, V_4, V_3, V_2, V_1\}$. Then the membership value of every vertex depended on three criteria namely $\{\text{environment, economic system,}$

$\text{infrastructure}\}$ and the membership value of each road depended on three criteria namely $\{\text{Transportation availability, traffic, roadlength}\}$ and these characteristics are uncertain. Using the relation $B(u,v) \leq min\{A(u), A(v)\}$ for all $(u,v) \in E$, we calculated Edge membership value and edge membership value represent the relation between two cities.

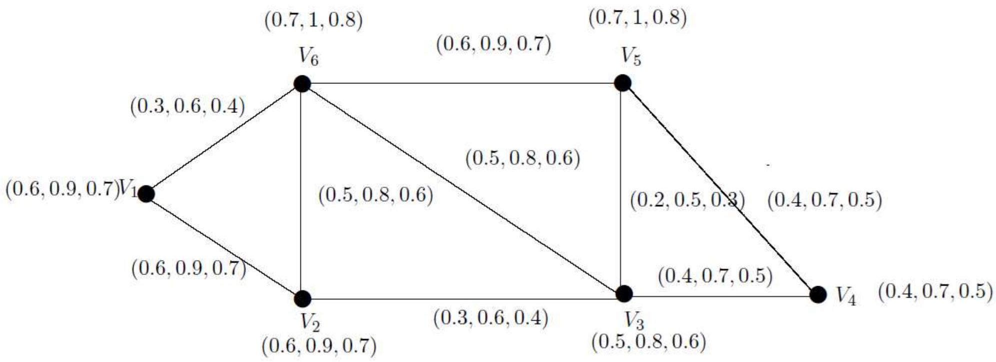

Figure 5: 3PFG G corresponding to the communication between some towns.

Suppose a person has started his/her journey from V_1 and he/she wants to go the place V_5. Then his first goal is to find the strong path between V_1 and V_5. And then he/she wants to find out the shortest path between those strong paths. So, he tries to find out the shortest strong path between V_1 and V_5 for his safe journey. For the 3PFG G in Figure 4, the arcs $(V_5, V_4), (V_4, V_3), (V_3, V_6), (V_5, V_6), (V_6, V_2), (V_2, V_1)$ are strong arcs. The paths

$V_1 - V_2 - V_6 - V_3 - V_4 - V_5$ and $V_1 - V_2 - V_6 - V_5$ are only two strong paths from V_1 to V_5. So $mPFD_g(V_1, V_5) = 5$ and $B.F.d_g(V_1, V_5) = 3$. So the path $V_1 - V_2 - V_6 - V_5$ is the shortest strong path from V_1 to V_5. If a person wants to go from V_1 to V_5 in the shortest path with the best communication system, then for him the path $V_1 - V_2 - V_6 - V_5$ will be the best route to go for his safe journey.

5. Conclusion

In this article, we have introduced mPF detour g-distance, mPF detour g-boundary nodes, mPF detour g-interior nodes in mPFGs and properties of these. We initiated theorems on mPF detour g-interior node, mPF detour g-boundary node, mPF cut node in mPFG, using maximum mPF spanning tree. We are extending our research work to define the connectivity index on the m-polar fuzzy graph and its properties and its applications on real-life problems etc.

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