Intern. J. Fuzzy Mathematical Archive
Vol. 20, No. 2, 2022, 71-79
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 17 September 2022
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v20n2a05239

International Journal of Fuzzy Mathematical Archive

Solution of System of Fuzzy Linear Equations with Polynomial Parametric Form

Arnab Bag

Department of Applied Mathematics with Oceanology and Computer Programming
Vidyasagar University, Midnapore-721102, West Bengal, India
Email: arnabbag1495@gmail.com

Received 31 July 2022; accepted 15 September 2022

Abstract. In this paper, we have discussed a new and simple solution method to solve a fuzzy system of linear equations having fuzzy coefficients and crisp variables using a polynomial parametric form of fuzzy numbers. Here related theorems are stated and discussed and the proposed methods are used to solve example problems. The results of the examples obtained are also compared with the known solutions and are found to be in good agreement.

Keywords: Fuzzy number, Polynomial parametric fuzzy number, Fuzzy centre, System of fuzzy linear equations.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

The system of linear equations plays a vital role in real-life problems, i.e. optimization, engineering, etc. A standard real system may be written as AX= B, where A and B are real crisp matrices and X be the unknown real vector. For easy and simple computation, the variables are generally taken as crisp numbers. But, in the real situation, the variables may be uncertain. So these variables may be considered as a fuzzy number [4]. So, to represent such vagueness or uncertainty, one may use fuzzy numbers in place of crisp numbers. Thus the system of linear equations becomes a system of Fuzzy Linear Equations (FSLE). It is an important area of research in recent years.

Fuzzy sets were introduced by Zadeh (1965)[4] as an extension of crisp sets (classical sets) or non-fuzzy sets. A fuzzy set is a class of objects with a continuum of grades of membership function which assigns to each object a grade of membership ranging between zero and one.

A fuzzy real system of linear equations has been investigated by various authors. Also, different authors discovered different procedures to solve systems of fuzzy linear equations. The solution methods depend upon the coefficient matrix, fuzziness of the variables, right-hand side vector, etc. Cong-Xing and Ming (1991)[2] used an embedding approach for fuzzy number space. Friedman et al. (1991)[2] proposed a general model for solving a fuzzy system of linear equations by using this embedding concept. Wang et al. (2001)[16] discovered an iterative method for solving a system of linear equations of the form X = AX + B. Asady et al. (2005)[6] also developed different methods of a general fuzzy system using an embedding approach.

Vroman et al. (2007)[14] solved the fuzzy general linear systems using the parametric form of fuzzy numbers. Recently, Li et al. (2010) presented a new procedure to solve a fuzzy system of linear equations. Sevastjanov and Dymova (2009) developed a new method for both the interval and fuzzy systems. Garg and Singh (2008)[10] solved the fuzzy system of linear equations with the Gaussian fuzzy membership function using a numerical approach. Behera and Chakraverty (2012)[8] recently developed a new solution method which can handle both fuzzy real and complex systems of linear equations. Also recently Amirfakhrian (2012)[1] developed one solution method for solving a system of fuzzy linear equations using a fuzzy distance approach. Chakraverty and Behera (2012)[8] developed a centre and width-based approach for solving fuzzy systems of linear equations. Chakraverty and Behera (2015)[9] also proposed a new approach to solving a fully fuzzy system of linear equations using single and double parametric forms of fuzzy numbers. Senthilkumara and Rajendran (2011)[13] proposed an algorithmic approach for solving fuzzy linear systems. Das and Chakraverty (2012)[8], and Senthilkumara and Rajendran (2011)[13] also investigated a fully fuzzy system of linear equations. Akram et al. [7] proposed methods for solving LR-bipolar fuzzy linear systems. Jun [11] discovered a relaxation technique for solving fuzzy linear systems of linear fuzzy real numbers. Abbasi, Allahviranloo [3] solve a fully fuzzy linear system, a new solution concept. Saqib et al. [12] proposed certain efficient iterative methods for bipolar fuzzy systems of linear equations.

The coefficient matrix is considered as real crisp whereas the unknown variable vector and right-hand side vector are considered as fuzzy or whole as fuzzy in general. But, in this paper, we have considered the fuzzy system of linear equations with fuzzy coefficients and crisp variables using a polynomial parametric form of fuzzy numbers. The same type of problem is investigated by Amirfakhrian (2007) [1] excellently. Here fuzzy real system of linear equations is taken as

$$\tilde{A}X = \tilde{b}$$

where the coefficient matrix $\tilde{A}$ is a real fuzzy matrix, $\tilde{b}$ is a column vector of fuzzy numbers and X is the vector of crisp variables.

This paper aims to propose two new methods which can handle the real fuzzy linear systems. Accordingly, Section 2, introduces the preliminaries with fuzzy arithmetic. A fuzzy system of linear equations with the proposed methodologies is explained in Section 3. In the Section 4, numerical examples are discussed. Advantages are discussed in Section 5. The last section includes the conclusion.

2. Preliminaries

An interval number is defined as an ordered pair of finite real numbers $[a, b]$, where $a \leq b$. When $a = b$ the interval number $[a, b]$ degenerates to the scalar real number â€˜aâ€™. An interval number can be thought of as

(i) an extension of the concept of a real number and also as a subset of the real line,

(ii) the simplest form of tolerance-type uncertainty with no information about the probabilities within this tolerance range,

(iii) a grey number whose exact value is unknown but a range within which the value lies is known.

Thus an interval number represents a set of possible values that a particular entity or variable may assume without any prior assumption about exact value and probability measure. In other words, interval numbers should be used whenever decision variables can assume different values, but a probability measure on these values is not available or justifiable.

Let F(R) be the set of all normal and convex fuzzy numbers on the real line [1].

2.1. Membership function

A generalized LR fuzzy number $\tilde{A}$ with the membership function $\mu_{\tilde{A}}(x),\ x \in R$ can be defined as[1]

$$\mu_{\tilde{A}}(x) = \begin{cases} l_{\tilde{A}}(x), & a \le x \le b \\ 1, & b \le x \le c \\ r_{\tilde{A}}(x), & c \le x \le d \\ 0, & otherwize \end{cases} \tag{2.1}$$

where $l_{\tilde{A}}(x)$ is the left membership function which is an increasing function in the interval $[a, b]$ and $r_{\tilde{A}}(x)$ is the right membership function that is a decreasing function in the interval $[c, d]$ such that $l_{\tilde{A}}(a) = r_{\tilde{A}}(d) = 0$ and $l_{\tilde{A}}(b) = r_{\tilde{A}}(c) = 1$.

If $l_{\tilde{A}}(x)$ and $r_{\tilde{A}}(x)$ are linear, then $\tilde{A}$ is a trapezoidal fuzzy number. It is denoted by (a, b, c, d).

If $b = c$, then we may be written as (a, c, d) or (a, b, d), which is a triangular fuzzy number.

The parametric form of a fuzzy number is given by $\tilde{v} = (\underline{v(x)}, \overline{v(x)})$, where functions $\underline{v(x)}$ and $\overline{v(x)}$; $0 \le x \le 1$ satisfy the following requirements:

$\underline{v(x)}$ is a bounded left continuous non-decreasing function over [0,1]

$\overline{v(x)}$ is a bounded right continuous non-increasing function over [0,1]

$\underline{v(x)} \ \le \ \overline{v(x)},\ 0 \le x \le 1$

Fuzzy centre of an arbitrary fuzzy number $\tilde{v} = (\underline{v(x)}, \overline{v(x)})$ is defined as $\tilde{v}^c = (\underline{v(x)} + \overline{v(x)})/2$, for all $0 \le x \le 1$

2.2. Some basic properties

The fuzzy numbers may be transformed into an interval through parametric form. So, for any arbitrary fuzzy number $\tilde{x} = (\ \underline{x(\alpha)}, \overline{x(\alpha)})$ and $\tilde{y} = (\ \underline{y(\alpha)},\ \overline{y(\alpha)})$ and scalar k, the interval based fuzzy arithmetic is defined as

$\tilde{x} = \tilde{y}$ if and only if $\underline{x(\alpha)} = \underline{y(\alpha)}$ and $\overline{x(\alpha)} = \overline{y(\alpha)}$

$\tilde{x} + \tilde{y} = (\underline{x(\alpha)} + \underline{y(\alpha)}, \overline{x(\alpha)} + \overline{y(\alpha)})$

$\tilde{x} - \tilde{y} = (\ \underline{x(\alpha)} - \overline{y(\alpha)}, \overline{x(\alpha)} - \underline{y(\alpha)})$

$$\tilde{x} * \tilde{y} = [\min((\underline{x(\alpha)})(\underline{y(\alpha)}), (\underline{x(\alpha)})(\overline{y(\alpha)}), (\overline{x(\alpha)})(\underline{y(\alpha)}), (\overline{x(\alpha)})(\overline{y(\alpha)})),$$
$$max((\underline{x(\alpha)})(\underline{y(\alpha)}), (\underline{x(\alpha)})(\overline{y(\alpha)}), (\overline{x(\alpha)})(\underline{y(\alpha)}), (\overline{x(\alpha)})(\overline{y(\alpha)}))]$$

$$\tilde{x}/\tilde{y} = ((\underline{x(\alpha)}), (\overline{x(\alpha)}))/(\underline{y(\alpha)}), (\overline{y(\alpha)}) = (\underline{x(\alpha)})/(\overline{y(\alpha)}), (\overline{x(\alpha)})/(\underline{y(\alpha)})$$

provided

$$(\underline{y(\alpha)}) = (\overline{y(\alpha)}) \neq 0$$

$$k\tilde{x} = \begin{cases} [k\underline{x}(\alpha), k]\overline{x}(\alpha), & if \quad k < 0 \\ [k\overline{x}(\alpha), k\underline{x}(\alpha)], & if \quad k \geq 0. \end{cases}$$

Some well-known facts about fuzzy arithmetic are expressed below

$\tilde{x} - \tilde{y}$ can be represented as $\tilde{x} + (-1)\tilde{y}$

$\tilde{x} + \tilde{0} = \tilde{x}$

$0 \times \tilde{x} = \tilde{0} = 0$

In the above expression, $\tilde{0}$ is the zero fuzzy number. For triangular and trapezoidal zero fuzzy numbers $\tilde{0}$ may be represented as (0,0,0) and (0,0,0,0) respectively.

Polynomial representation of fuzzy numbers

We say a fuzzy number $\tilde{v}$ has m-degree polynomial form if there exist two polynomials $p_m(\alpha)$ and $q_m(\alpha)$, of degree at most m; such that $\tilde{v} = (p_m(\alpha), q_m(\alpha))$, which is also called polynomial parametric fuzzy number [1].

2.3. Types of fuzzy sets

Now we proceed to define certain standard set-theoretic operations for fuzzy sets.

Empty fuzzy set

A fuzzy set $\tilde{A}$ defined over the universe X is said to be empty if its membership function is identically zero, i.e. if $\mu_{\tilde{A}}(x) = 0$ for all x in X.

Subset

A fuzzy set $\tilde{A}$ is said to be a subset of a fuzzy set if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ for all x in X. This is denoted by $\tilde{A} \subseteq \tilde{B}$.

Equality of fuzzy set

Two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are said to be equal if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$ i.e. if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ for all x in X.

Complement

The complement of a fuzzy set $\tilde{A}$ defined over the universal set X is another fuzzy $\tilde{A}'$ defined by the membership function $\mu_{\tilde{A}'}(x) = 1 - \mu_{\tilde{A}}(x)$ for all x in X.

Union

The union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is another fuzzy set $\tilde{C}$ defined by the membership function $\mu_{\tilde{C}}(x) = max[\mu_{\tilde{B}}(x), \mu_{\tilde{B}}(x)]$ for all x in X.

Intersection

The intersection of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is another fuzzy set $\tilde{C}$ defined by the membership function $\mu_{\tilde{C}}(x) = min[\mu_{\tilde{B}}(x), \mu_{\tilde{B}}(x)]$ for all x in X.

3. System of fuzzy linear equations

The $n \times n$ fuzzy system of linear equations with m-degree polynomial parametric form may be written as

$$\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 + \ldots + \tilde{a}_{1n}x_n = \tilde{b}_1 \quad (3.1)$$
$$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 + \ldots + \tilde{a}_{2n}x_n = \tilde{b}_2 \quad (3.2)$$
$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \quad (3.3)$$
$$\tilde{a}_{n1}x_1 + \tilde{a}_{n2}x_2 + \ldots + \tilde{a}_{nn}x_n = \tilde{b}_n \quad (3.4)$$

In matrix notation, the above system may be written as

$$\tilde{A}X = \tilde{b}$$

where the coefficient matrix

$$\tilde{A} = (\tilde{a}_{kj}) = (\underline{a}_{kj}(\alpha), \overline{a}_{kj}(\alpha)) = (\underline{A}, \overline{A}), 1 \leq k, j \leq n \quad (3.5)$$

is a fuzzy $n \times n$ matrix,

$$\tilde{b} = (\tilde{b}_k) = (\underline{b}_k(\alpha), \overline{b}_k(\alpha)) = (\underline{b}, \overline{b}), 1 \leq k \leq n \quad (3.6)$$

is a column vector of fuzzy number and $X = x_j$ is the vector of crisp unknown. For positive integer m, all $\tilde{a}_{kj}$ and $\tilde{b}_k$ are fuzzy numbers with m-degree polynomial form.

The above system $\tilde{A}X = \tilde{b}$, can be written as

$$\sum_{j=1}^{n} \tilde{a}_{kj}x_j = \tilde{b}_k, \; k = 1,2, \ldots, n \quad (3.7)$$

As per the parametric form, we may write Equation (3.7) as

$$\sum_{j=1}^{n} (\underline{a}_{kj}(\alpha), \overline{a}_{kj}(\alpha))x_j = (\underline{b}_k(\alpha), \overline{b}_k(\alpha)), k = 1,2, \ldots, n \quad (3.8)$$
$$where \;\; 0 \leq \alpha \leq 1, \quad (3.9)$$

Equation (3.8) can equivalently be written as the following two Equations (3.10) and (3.11)

$$\sum_{x_j \geq 0} \underline{a}_{kj}(\alpha)x_j + \sum_{x_j < 0} \overline{a}_{kj}(\alpha)x_j = \underline{b}_k(\alpha) \quad (3.10)$$

and

$$\sum_{x_j \geq 0} \overline{a}_{kj}(\alpha)x_j + \sum_{x_j < 0} \underline{a}_{kj}(\alpha)x_j = \overline{b}_k(\alpha) \quad (3.11)$$

3.1. Proposed methods for solving system of fuzzy linear equations

In this section, we propose a new method to solve a fuzzy system of linear equations. One related theorem is proved related to the present procedure below.

Theorem 1. If X is the solution vector of the fuzzy system $\tilde{A}X = \tilde{b}$, then X is the solution vector of the crisp system of linear equation$(\underline{A} + \overline{A})X = \underline{b} + \overline{b}$.

Proof: Let us now first consider the left-hand side of the system

$$(\underline{A} + \overline{A})X = \underline{b} + \overline{b}. \quad (3.12)$$

Hence one may write $(\underline{A} + \overline{A})X$ as

$$\sum_{j=1}^{n} (\underline{a}_{kj}(\alpha) + \overline{a}_{kj}(\alpha))x_j, for \ k = 1,2,\ldots,n \tag{3.13}$$

This can be written as $\sum_{j=1}^{n} \underline{a}_{kj}x_j + \sum_{j=1}^{n} \overline{a}_{kj}(\alpha)x_j$ which is equivalent to

$$\sum_{x_j \geq 0} \underline{a}_{kj}(\alpha)x_j + \sum_{x_j < 0} \underline{a}_{kj}(\alpha)x_j + \sum_{x_j \geq 0} \overline{a}_{kj}(\alpha)x_j + \sum_{x_j < 0} \overline{a}_{kj}(\alpha)x_j \tag{3.14}$$

Using Equations (3.10) and (3.11), the above expression can be written as combining first with fourth term and second with third term respectively in the above equation we get,

$$\underline{b}_k(\alpha) + \overline{b}_k(\alpha) = [\underline{b} + \overline{b}]$$

Thus, we have

$$(\underline{A} + \overline{A})X = \underline{b} + \overline{b}. \tag{3.15}$$

This proves that X is the solution vector of the system

$$(\underline{A} + \overline{A})X = \underline{b} + \overline{b}. \tag{3.16}$$

Similarly to find the crisp solution of a system of fuzzy linear equations as discussed in the previous discussion, here one related theorem is stated below.

Theorem 2. If X is the solution vector of fuzzy system $\tilde{A}X = \tilde{b}$, then X is the solution vector of the crisp system of linear equation $\tilde{C}X = \tilde{d}$ where $\tilde{C} = (\underline{a}_{kj}(\alpha) + \overline{a}_{kj}(\alpha))/2$ and $\tilde{d} = (\underline{b}_k(\alpha) + \overline{b}_k(\alpha))/2$

4. Numerical examples

Some examples are here

4.1. Example 1

Now let us consider a 2×2 system of fuzzy linear equations and the equations are represented as a linear polynomial form.

$$(-1 + \alpha, 3 - \alpha)x_1 + (1 + 2\alpha, 4 - \alpha)x_2 = (-12 + 11\alpha, 17 - 8\alpha) \tag{4.1}$$

$$(-1 + 2\alpha, 3 - 2\alpha)x_1 + (3\alpha, 6 - 2\alpha)x_2 = (-15 + 19\alpha, 23 - 16\alpha) \tag{4.2}$$

Using Theorem 1. the above system is now converted to the following system.

$$(-1 + \alpha + 3 - \alpha)x_1 + (1 + 2\alpha + 4 - \alpha)x_2 = (-12 + 11\alpha + 17 - 8\alpha) \tag{4.3}$$

$$(-1 + 2\alpha + 3 - 2\alpha)x_1 + (3\alpha + 6 - 2\alpha)x_2 = (-15 + 19\alpha + 23 - 16\alpha) \tag{4.4}$$

Implies,

$$2x_1 + (\alpha + 5)x_2 = 3\alpha + 5 \tag{4.5}$$
$$2x_1 + (\alpha + 6)x_2 = 3\alpha + 8 \tag{4.6}$$

Solving these equations, we get $x_1 = -5,\ x_2 = 3$ for any α.

4.2. Example 2

Again consider another 2×2 fuzzy system of linear equation and representing the system of equation using second degree polynomial.

$$\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 = \tilde{b}_1 \tag{4.7}$$
$$\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 = \tilde{b}_2 \tag{4.8}$$

where it is assumed that

$$\tilde{a}_{11} = (3\alpha + \alpha^2, 7 - 3\alpha + 2\alpha^2) \tag{4.9}$$
$$\tilde{a}_{12} = (2\alpha + \alpha^2, 4 - 2\alpha + 2\alpha^2) \tag{4.10}$$
$$\tilde{a}_{21} = (1 + 2\alpha + \alpha^2, 8 - 3\alpha + \alpha^2) \tag{4.11}$$
$$\tilde{a}_{22} = (1 + 2\alpha + \alpha^2, 6 - 3\alpha + 2\alpha^2) \tag{4.12}$$
$$\tilde{b}_1 = (48.45\alpha + 17.1\alpha^2, 111.15 - 48.45\alpha + 34.2\alpha^2) \tag{4.13}$$
$$\tilde{b}_2 = (17.1 + 34.2\alpha + 17.1\alpha^2, 131.1 - 51.3\alpha + 19.95\alpha^2) \tag{4.14}$$

Using Theorem (1) the above system is now converted to the following system

$$(3\alpha + \alpha^2 + 7 - 3\alpha + 2\alpha^2)x_1 + (2\alpha + \alpha^2 + 4 - 2\alpha + 2\alpha^2)x_2 = (48.45\alpha + 17.1\alpha^2 + 111.15 - 48.45\alpha + 34.2\alpha^2)$$
$$(1 + 2\alpha + \alpha^2 + 8 - 3\alpha + \alpha^2)x_1 + (1 + 2\alpha + \alpha^2 + 6 - 3\alpha + 2\alpha^2)x_2 = (17.1 + 34.2\alpha + 17.1\alpha^2 + 131.1 - 51.3\alpha + 19.95\alpha^2)$$

Implies,

$$(3\alpha^2 + 7)x_1 + (3\alpha^2 + 4)x_2 = (51.3\alpha^2 + 111.15) \tag{4.15}$$
$$(2\alpha^2 - \alpha + 9)x_1 + (3\alpha^2 - \alpha + 7)x_2 = (37.05\alpha^2 - 17.1\alpha + 148.2) \tag{4.16}$$

From equation(4.15) we have,

$$x_1 = \frac{(51.3\alpha^2 + 111.15) - (3\alpha^2 + 4)x_2}{3\alpha^2 + 7} \tag{4.17}$$

Putting the value of x_1 in equation (4.16), we get,

$$(2\alpha^2 - \alpha + 9)[(51.3\alpha^2 + 111.15) - (3\alpha^2 + 4)x_2] + (3\alpha^2 - \alpha + 7)(3\alpha^2 + 7)x_2 = (37.05\alpha^2 - 17.1\alpha + 148.2)(3\alpha^2 + 7)$$

or, $[(3\alpha^2 - \alpha + 7)(3\alpha^2 + 7) - (2\alpha^2 - \alpha + 9)(3\alpha^2 + 4)x_2 = (37.05\alpha^2 - 17.1\alpha + 148.2)(3\alpha^2 + 7) - (2\alpha^2 - \alpha + 9)(51.3\alpha^2 + 111.15)$

or,

$$(9\alpha^4 - 3\alpha^3 + 21\alpha^2 + 21\alpha^2 - 7\alpha + 49 - 6\alpha^4 + 3\alpha^3 - 27\alpha^2 - 8\alpha^2 + 4\alpha - 36)x_2 = 111.15\alpha^4 - 51.3\alpha^3 + 444.6\alpha^2 + 259.35\alpha^2 - 119.7\alpha + 1037.4 - 102.6\alpha^4 + 51.3\alpha^3 - 461.7\alpha^2 - 222.3\alpha^2 + 111.15\alpha - 1000.35$$

or,

$$(3\alpha^4 + 7\alpha^2 - 3\alpha + 13)x_2 = 8.55\alpha^4 + 19.95\alpha^2 - 8.55\alpha + 37.05$$

or,

$$(3\alpha^4 + 7\alpha^2 - 3\alpha + 13)x_2 = 2.85(3\alpha^4 + 7\alpha^2 - 3\alpha + 13)$$

Finally,

$$x_2 = 2.85$$

Putting the value of x_2 in (4.17) we get,

$$x_1 = \frac{(51.3\alpha^2 + 111.15) - (3\alpha^2 + 4) \times 2.85}{3\alpha^2 + 7} = 14.25$$

The solution is $x_1 = 14.25,\ x_2 = 2.85$.

Now we try to compare the obtained results by the present method with the solution obtained by Amirfakhrian (2007)[1] and are tabulated in Table 1 for all the examples as discussed in Section 4. The results obtained by the proposed method are the same as that of Amirfakhrian (2007) for Example 1. However, the results of Example 2 are quite different from Amirfakhrian (2007). It is worth mentioning that the results obtained by the present method exactly satisfy the corresponding system whereas Amirfakhrian (2007) it is not. The solution by Amirfakhrian (2007) may have some typographical errors.

Examples	**Amirfakhrian (2007)**	**Present method**
1	$x_1 = -5$ and $x_2 = 3$	$x_1 = -5$ and $x_2 = 3$
2	$x_1 = 14.85$ and $x_2 = 2.25$	$x_1 = 14.25$ and $x_2 = 2.85$

Table 1: Comparison of results between Amirfakhrian and the present method

5. Advantages

This paper deals with a new and simple solution to solve a fuzzy system of linear equations having fuzzy coefficients and crisp variables using a polynomial parametric form of fuzzy numbers. The main advantage of the previous proposed methods is that, in the solution procedure the order of original fuzzy systems does not change. But in the other methods, the order of the fuzzy system changes just by double its original order order. So, the present procedures have less effort to solve a system of fuzzy linear equations. Therefore it is computationally efficient.

6. Conclusions

In this paper, a general system of fuzzy linear equations having fuzzy coefficients and crisp variables using a polynomial parametric form of fuzzy numbers is solved by new and simple procedures. Here we have to choose m depending on the shape of left and right spread functions L and R, and their derivation order. The proposed methods can be applied to any system of equations with LR fuzzy number coefficients.

Acknowledgements. The author is also thankful to the referees for their valuable comments for improvement of the comments.

REFERENCES

1. Amirfakhrian, Analyzing the solution of a system of fuzzy linear equations by a fuzzy distance *Springer*,16(6)(2012) 1035-1041.
2. Friedman, M Ming and A Kandel, Fuzzy linear systems, Fuzzy Sets and Systems, 96(2)(1998)201-209.

3. Abbasi and Allahviranloo, Solving fully fuzzy linear system: a new solution concept, *Information Sciences*, 589(2022)608-635.
4. A. Zadeh, Fu and Tanaka, Fuzzy sets and their applications to cognitive and decision processes: Proceedings of the us–japan seminar on fuzzy sets and their applications, held at the university of california, berkeley, california, july 1-4, 1974 *Academic press*,2014.
5. Amirfakhrian, Numerical solution of fuzzy system of linear equations with parametric polynimial form, *International Journal of Computer Mathematics*, 84(2007)1089-1097.
6. B Asady, Abbasbandy and M. Alavi, Fuzzy general linear systems, *Applied Mathematics and Computations*, 169(1)(2005)34-40.
7. Akram, Allahviranloo, Pedrycz and Ali, Methods for solving linear-bipolar fuzzy linear systems, *Soft Computing*, 25(1)(2021)85-108.
8. Behera and S. Chakraverty, Solution of fuzzy system of linear equations with polynomial parametric form, *Applications and Applied Mathematics: An International Journal (AAM)*, 7(2)(2012)12.
9. Behra and S. Chakraverty, New approach to solve fully fuzzy system of linear equations using single and double parametric form of fuzzy numbers, *Sadhana*, 40(2015)35-49.
10. Garg and S.R. Singh, Solving systems of linear fuzzy equations by parametric functionsâ€"an improved algorithm, *Fuzzy sets and systems*, 7(4)(2009)25-30.
11. Jun, Relaxation technique for solving fuzzy linear systems of linear fuzzy real numbers, *J. Math. Comput. Sci.*, 11(6)(2021)8211-8220.
12. Saqib, Akram and Bashir, Certain efficient iterative methods for bipolar fuzzy system of linear equations, *Journal of Intelligent & Fuzzy Systems*, 39(3)(2020)3971-3985.
13. P. Senthilkumara and G. Rajendran, New approach to solve symmetric fully fuzzy linear systems, *Sadhana*, 36(2011)933-940.
14. Vromen, Deschrijver and E. Kerre, Solving systems of linear fuzzy equations by parametric functions: an improved algorithm, *Fuzzy sets and systems*, 158(14)(2007)1515-1534.
15. Zimmermann, Fuzzy set theory and its applications, *Springer Science & Business Media*,2011.
16. Wang, Zhong and Ha, Iteration algorithms for solving a system of fuzzy linear equations, *Fuzzy Sets and Systems*, 119(1)(2001)121-128.