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A Novel Concept on Fermatean Fuzzy Threshold Graph with its Application on Covid

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Abstract. In structuring and designing many problems, a graphical theory is significant. In real-life scenarios, a variety of structural designs can be found with crossings. Modelling vagueness and ambiguity in graphical network problems, numerous graph-theoretical extension ideas are provided. The threshold graph has received a lot of attention. However, the fuzzy threshold graph has recently been established and various properties have been explored. For each edge and vertex in a fuzzy threshold graph, only one threshold is considered. In a Fermatean fuzzy graph, each edge and node has a two-membership value. As a result, creating a threshold graph for a Fermatean fuzzy graph is difficult and requires some new features. In this paper, the Fermatean fuzzy threshold graph ($\boldsymbol{FFTG}$) and Fermatean fuzzy alternating 4-cycle are defined and certain properties are studied. Fermatean fuzzy threshold graph is an extension of a fuzzy threshold graph. There are also many interesting characteristics of$\boldsymbol{FFTG}$. The $\boldsymbol{FFTG}$ is also reconstructed interestingly. The important parameters viz. Fermatean threshold partition number and Fermatean threshold dimension are described and studied. It is demonstrated that every Fermatean fuzzy threshold graph can be treated as a Fermatean fuzzy split graph. Lastly, we have given a real-world application of $\boldsymbol{FFTG}$ on an oxygen supply from one state to another in a Covid situation.

Keywords: Fermatean fuzzy graphs, Fermatean fuzzy threshold graph, Fermatean fuzzy threshold dimension, Fermatean fuzzy threshold partition number.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

The idea of graph theory applies to many fields of computer science, data analysis, fragmentation of images, clusters, image capture, networking, and so on. A Fermatean fuzzy set (FFS) is a generalisation of the idea of a fuzzy set (FS). Compared with fuzzy models, Fermatean fuzzy models offer the device greater accuracy, flexibility and usability.

The definition of the Fermatean fuzzy set as a generalisation of FSs was introduced in 2019 by Senapati and Yager [41] and also they discussed some new operations over Fermatean fuzzy numbers in [40]. In the description of FFS, Senapati and Yager added a new dimension that defines the degree of a non-member. The FS indicates the extent of membership. In contrast, FFS suggests the degree of membership as well as the degree of non-membership are more or less independent of one another. Till today, several researches have been done which are purely based on FFSs. For instance, Liu et al. [16] originated the notion of the Fermatean fuzzy linguistic term set. The concept of decision-making analysis based on Fermatean fuzzy Yager aggregation operators (with application in COVID-19 testing facility) was studied by Garg et al. [12]. Yang et al. [43] discussed the differential Calculus of Fermatean fuzzy functions. Shahzadi and Akram [39] developed a novel decision-making concept to select an antivirus mask under Fermatean fuzzy soft information. In addition, Akram et al. [5] proposed a novel decision-making framework for the selection of an effective sanitizer to reduce COVID-19 in the Fermatean fuzzy environment. Liu et al. [16] proposed the concept of distance measure for Fermatean fuzzy linguistic term sets based on linguistic scale function which is illustrated by the TODIM and TOPSIS methods. The threshold graph was first introduced by Chvatal and Hammer [9]. These graphs are used in several applied areas, such as psychology, neuroscience, computer science, artificial intelligence and scheduling theory. Keshavarz et. al [15] were given a new decision-making approach based on Fermatean fuzzy sets. Ali et. al. [8] discussed a new concept on Fermatean fuzzy bipolar soft. Ordman [18] studied threshold coverings and resource allocation problems. Furthermore, Pelod and Mahadev [22] introduced threshold graphs and related topics on it. Notes on threshold graphs have been introduced by Andelic and Simic [1]. Letter on, Samanta and Pal[31] introduced a fuzzy threshold graph. Next, Pramanik et.al.[23] studied an undervalued fuzzy threshold graph. Recently, yang and Mao [42] presented intuitionistic fuzzy threshold graphs. Akram et.al. [6] studied complex Pythagorean fuzzy threshold graphs with the application of petroleum replenishment. The other interesting papers related to this work are [45-49].

1.1. Review of literature

Rosenfeld [30] explained the concept of a fuzzy graph (FG) in 1975, the basic knowledge of which was proposed by Kauffman [14] in 1973. Rosenfeld also considered the fuzzy relations between FSs, and he established the model of FGs, obtaining analogues of many theoretical concepts of graphs. The idea of intuitionistic fuzzy connection was proposed by Atanassov. In many papers, various types of intuitionistic fuzzy graphs ($IFGs$) and their implementations can be found. The idea of the intuitionistic fuzzy competition graph was discussed by Sahoo and Pal [33].

The FG theory increases with its different branch forms after Rosenfeld such as fuzzy threshold graph [31], bipolar FGs [27, 28], highly irregular interval FGs [24], interval-value isometry FGs [26], balanced interval-value FGs [19, 25], fuzzy k-competition-value isometry FGs [26]. In a network with imprecise edge weight, Nayeem and Pal [17] presented the shortest path problem. Till today, several researches have been done which are purely based on FFSs. For instance, Liu et al. [16] originated the notion of the Fermatean fuzzy linguistic term set. The concept of decision-making analysis based on Fermatean fuzzy Yager aggregation operators (with application in COVID-19 testing facility) was studied by Garg et al. [12]. Yang et al. [43] discussed the differential Calculus of Fermatean fuzzy functions. Shahzadi and Akram [39] developed a novel decision-making concept to select an antivirus mask under Fermatean fuzzy soft information. In addition, Akram et al. [5] proposed a novel decision-making framework for the selection of an effective sanitizer to reduce COVID-19 in the Fermatean fuzzy environment. Liu et al. [16] proposed the concept of distance measure for Fermatean fuzzy linguistic term sets based on linguistic scale function which is illustrated by the TODIM and TOPSIS methods. The threshold graph was first introduced by Chvatal and Hammer [9]. These graphs are used in several applied areas, such as psychology, neuroscience, computer science, artificial intelligence and scheduling theory. Keshavarz et. al [15] were given a new decision-making approach based on Fermatean fuzzy sets. Ali et al. [8] discussed a new concept on Fermatean fuzzy bipolar soft. Ordman [18] studied threshold coverings and resource allocation problems. Furthermore, Pelod and Mahadev [22] introduced threshold graphs and related topics on it. Notes on threshold graphs have been introduced by Andelic and Simic [1]. Letter on, Samanta and Pal[31] introduced a fuzzy threshold graph. Next, Pramanik et al. [23] studied an undervalued fuzzy threshold graph. Recently, yang and Mao [42] presented intuitionistic fuzzy threshold graphs. Akram et al. [6] studied complex Pythagorean fuzzy threshold graphs with the application of petroleum replenishment.

Strong IFG was specified by Akram and Davvaz[2]. IF hypergraphs are also addressed in [3]. Karunambigai et al. [13] discuss balanced IFG. The IF rivalry graph definition was explored by Sahoo and Pal [33]. They also addressed the IF tolerance graph with the application [34], various product styles with the $IFGs$ application [32], and the $IFGs$ product with the application [34]. The structures of IFG were described by Akram and Akmal [4]. Ghorai and Pal [11] have introduced m-polar FGs operations. The concepts of strength of connectedness in mPFG, mPF tree, and mPF cut node are established by Mandal et al. [36, 37]. Again covering problems on FG was introduced by Mandal et al. [38]. For more terminologies, theories and applications of FGs, $IFGs$ and other nodes of FG see [20].

Authors	Contributions
Chvatal and Hammer [9]	studied set packing on threshold graphs
Ordman [18]	Established threshold coverings
Peled and Mahadev [22]	Intuitionistic fuzzy graph
Andelic and Simic [1]	Given important notes on the threshold graph
Samanta and pal [31]	Defined fuzzy threshold graph
Pramanik et al.[23]	Studied interval-valued fuzzy threshold graph
Yang and Mao [42]	Studied intuitionistic fuzzy threshold graph
This Paper	Introduced Fermatean fuzzy threshold graph

1.2. Motivation

In the present-day situation, Fermatean fuzzy set theory is emerging as a novel mathematical tool to handle uncertainties in different domains of the real world. Fermatean fuzzy sets were presented so that uncertain information from quite general real-world decision-making situations could be mathematically tractable. To that purpose, these sets are more flexible and reliable than intuitionistic and Pythagorean fuzzy sets. The supreme tendency of FFS to address the exact human decision makes it more feasible and accurate to model dimensional (i.e. membership and non-membership) information in more wider space as compared to intuitionistic and Pythagorean fuzzy sets. In a fuzzy set, there exists only one membership value of an element belongs to $[0,1]$. But sometimes we have to handle a situation where we want one membership value and one non-membership value. For example, a country has contained some maximum manufacturing states and some minimum manufacturing states. The maximum manufacturing states supply a sufficient amount of oxygen to the poor or minimum oxygen manufacturing states in a country. Here we assign the membership value of a vertex depending on the following criteria:{ Demanding of medical oxygen, cost of buying oxygen from other states.}. These terms are uncertain and we need to use Fermatean fuzzy sets to represent this situation. This condition can not handled by using a fuzzy set. This type of problem can not be handled by using the concept of a fuzzy threshold graph. So we introduced the concept of the Fermatean fuzzy threshold graph.

1.3. Our works

$FFTG$ and its various significant features are defined in this article. The Fermatean fuzzy threshold dimension ($FFTD$) and Fermatean fuzzy threshold partition number($FFTPN$) are introduced. It is established that there is a relationship between $FFTD$ and $FFPN$. Some essential characteristics of $FFTD$ and $FFPN$ on decomposed $FFTG$ are also investigated. The last section of the article also includes an application.

2. List of abbreviations

The following notations and abbreviations are used.

IF	Intuitionistic fuzzy
FG	Fuzzy graph
FS	fuzzy set
IFG	Intuitionistic fuzzy graph
FFS	Fermatean fuzzy set
FFG	Fermatean fuzzy graph
MV	membership value
NMV	non-membership value
UCG	underlying crisp graph
$FFTG$	Fermatean fuzzy threshold graph
$FFTD$	Fermatean fuzzy threshold dimension
$FFTPN$	Fermatean fuzzy threshold partition number
FFN	Fermatean fuzzy number
IFN	Intuitionistic fuzzy number
PFN	Pythagorean fuzzy number
$FFTS$	Fermatean fuzzy threshold subgraph

3. Preliminaries

An ordered pair $G = (V, E)$ is a graph, where E and Varea set of all arcs and nodes respectively.

Definition 1. *[44] On universal set X, a fuzzy set (FS) is an object of the format $A = \{(t, \mu_A(t)): t \in X\}$, where $\mu_A(t): X \to [0,1]$ denotes the membership function of A.*

Definition 2. *Let X be a universe of discourse. In X, an intuitionistic fuzzy set (IFS) I is an object having of the type $I = \{s \in X: < s, \alpha_I, \beta_I >\}$, where $\alpha_I: X \to [0.1]$ and $\beta_I: X \to$*

$[0.1]$, *denotes the membership value (MV) and non-membership value (NMV) of each elements* $\in X$ *respectively and* $0 \leq (\alpha_I(s)) + (\beta_I(s)) \leq 1 \forall s \in X$.

Definition 3. *Let* X *be a universe of discourse. In* X, *a Pythagorean fuzzy set* (PFS) P *is an object having of the type* $P = \{s \in X: < s, \alpha_P, \beta_P >\}$, *where* $\alpha_P: X \rightarrow [0.1]$ *and* $\beta_F: X \rightarrow [0.1]$, *denotes the* MV *and* NMV *of each elements* $\in X$ *respectively and* $0 \leq (\alpha_P(s))^2 + (\beta_P(s))^2 \leq 1 \forall s \in X$.

Definition 4. *[41] Let* X *be a universe of discourse. In* X, *a Femmatean fuzzy set* (FFS) F *is an object having of the type* $F = \{s \in X: < s, \alpha_F, \beta_F >\}$, *where* $\alpha_F: X \rightarrow [0.1]$ *and* $\beta_F: X \rightarrow [0.1]$, *denotes the* MV *and* NMV *of each element* $s \in X$ *respectively and* $0 \leq (\alpha_F(s))^3 + (\beta_F(s))^3 \leq 1 \forall s \in X$. *Moreover, for all* $s \in X$,

$\Pi_F(s) = \sqrt[3]{1 - \alpha_F^3(s) - \beta_F^3(s)}$ *is called Fermatean fuzzy index or degree of indeterminacy of* s *to* F.

For computational convenience, $F = (\alpha_F, \beta_F)$ is called a Fermatean fuzzy number (FFN), where $\alpha_F, \beta_F \in [0,1]$, $\alpha_F^3 + \beta_F^3 \leq 1$ and $\Pi_F = \sqrt[3]{1 - \alpha_F^3 - \beta_F^3}$.

For better understanding of FFS, we give an instance to illuminate the understanding of the FFS: The key difference between the intuitionistic fuzzy number (IFN), pythagorean fuzzy number (PFN) and FFN is their different constraint conditions, that is the constraint conditions of IFN, PFN and FFN are $\alpha_I + \beta_I \leq 1$, $\alpha_P^2 + \beta_P^2 \leq 1$ and $\alpha_F^3 + \beta_F^3 \leq 1$ respectively. In Fig. 1, only the orange space denotes the space of IFSs membership degree (MD). Then both Orange and rose colour spaces denote the space of PFSs MD. Again all orange, rose and light turquoise colour space denotes the space of FFSs MD. So the space of FFSs MD is larger than the space of PFSs MD and the space of PFSs MD is larger than the space of IFSs MD, as shown in Fig. 1.

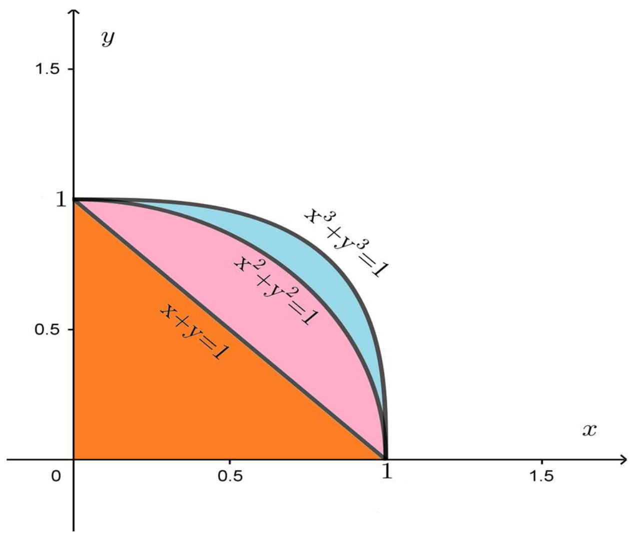

Figure 1: Comparison of spaces of $IFNs$, $PFNs$ and $FFNs$

Definition 5. *[41] Let $F_1 = (\alpha_{F_1}, \beta_{F_1})$ and $F_2 = (\alpha_{F_2}, \beta_{F_2})$ be two $FFSs$, then such set operations are defined as:*

i. $F_1 \cap F_2$=(min$\{\alpha_{F_1}, \alpha_{F_2}\}$, max$\{\beta_{F_1}, \beta_{F_2}\}$)

ii. $F_1 \cup F_2$=(max$\{\alpha_{F_1}, \alpha_{F_2}\}$, min$\{\beta_{F_1}, \beta_{F_2}\}$)

iii. $F_1^c = (\beta_{F_1}, \alpha_{F_1})$

4. Fermatean fuzzy graphs

The Fermatean fuzzy graph is now defined below.

Definition 6. *An Fermatean fuzzy graph (FFG) is of the form $G = (V, \sigma, \mu)$ where $\sigma = (\sigma_1, \sigma_2)$, $\mu = (\mu_1, \mu_2)$ and*

(i) $V = \{s_0, s_1, \ldots, s_n\}$ such that $\sigma_1: V \to [0,1]$ and $\sigma_2: V \to [0,1]$, indicate the MV and NMV of the node $s_i \in V$ respectively and $0 \le (\sigma_1(s_i))^3 + (\sigma_2(s_i))^3 \le 1$ for each $s_i \in V (i = 1,2, \ldots, n)$.

(ii) $\mu_1: V \times V \to [0,1]$ and $\mu_2: V \times V \to [0,1]$, where $\mu_1(s_i, s_j)$ and $\mu_2(s_i, s_j)$ denote the MV and NMV of the arc (s_i, s_j) respectively such that $\mu_1(s_i, s_j) \le \min\{\sigma_1(s_i), \sigma_1(s_j)\}$ and
$\mu_2(s_i, s_j) \le \max\{\sigma_2(s_i), \sigma_2(s_j)\}, 0 \le (\mu_1(s_i, s_j))^3 + (\mu_2(s_i, s_j))^3 \le 1$ for each (s_i, s_j).

Throughout the paper, we represent a FFG by $G = (V, \sigma, \mu)$.

Example 1. *Let G be a FFG. Let $V = \{s, t, u, v, w, x\}$ and $E = \{s, t), (t, x), (t, w), (x, w), (v, w), (s, v), (u, v), (s, u)\}$. The MV and NMV of each node are* $\sigma(s) = (0.8,0.9), \sigma(t) = (0.7,0.95), \sigma(u) = (0.9,0.9), \sigma(v) = (0.8,0.7), \sigma(w) = (0.8,0.8)$, $\sigma(x) = (0.7,0.8)$. *The MV and NMV of each edge are* $\mu(s, t) = (0.7,0.8), \mu(t, x) = (0.7,0.8), \mu(t, w) = (0.7,0.8), \mu(x, w) = (0.7,0.7), \mu(v, w) = (0.8,0.75), \mu(s, v) = (0.8,0.8), \mu(u, v) = (0.8,0.8), \mu(s, u) = (0.8,0.9)$.
The corresponding FFG is shown in Figure 2.

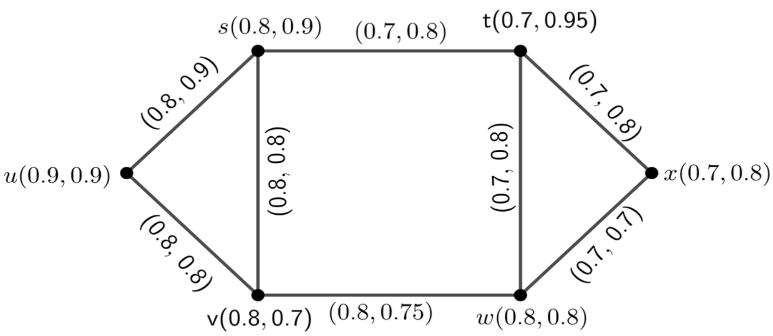

Figure 2: A FFG

Definition 7. *A $FFG G = (V, \sigma, \mu)$ is called a complete FFG if $\mu_1(s_i, s_j) = \sigma_1(s_i) \wedge \sigma_1(s_j)$ and $\mu_2(s_i, s_j) = \sigma_2(s_i) \vee \sigma_2(s_j) \forall (s_i, s_j) \in E$.*

Definition 8. *A support of an* $FFGF = (X, \alpha_F, \beta_F)$ *is determined as*

$$Supp(F) = \{x \in X: \alpha_F(x) \geq 0 \; and \beta_F(x) \leq 1\}.$$

Also, the support length is $SL(F) = |Supp(F)|$.

Definition 9. *The core of an* $FFSF = (X, \alpha_F, \beta_F)$ *is determined as*

$$Core(F) = \{x \in X: \alpha_F(x) = 1 \; and \beta_F(x) = 0\}.$$

Also, the core length is $CL(F) = |core(F)|$.

Definition 10. *The* α*-cut* $(0 \leq \alpha \leq 1)$ *of a* $FFGG = (V, \sigma, \mu)$, *where* $\sigma = (\sigma_1, \sigma_2)$, $\mu = (\mu_1, \mu_2)$ *is* $G_\alpha = (\sigma_\alpha, \mu_\alpha)$ *such that* $\sigma_\alpha = \{s \in V: \sigma_1(s) \geq \alpha, \sigma_2(s) \leq \alpha\}$ *and* $\mu_\alpha = \{(s,t): \mu_1(s,t) \geq \alpha, \mu_2(s,t) \leq \alpha\}$.

Definition 11. *A* $FFGG$ *is said to be bipartite if the node set* V *can be partitions into two* V_1 *and* V_2 *sets such that* $\mu_2(s,t) = 0$ *if* $s, t \in V_1$ *or* $s, t \in V_2$ *and* $\mu_2(s,t) > 0$ *if* $s \in V_1 (or V_2)$ *and* $t \in V_2 (or V_1)$.

Definition 12. *A* $FFGG$ *is a complete* FFG *if* $\mu_1(s,t) = min\{\sigma_1(s), \sigma_1(t)\}$ *and* $\mu_2(s,t) = max\{\sigma_2(s), \sigma_2(t)\}$ *for all* $s, t \in V$.

Definition 13. *A* $FFGG$ *is a strong* FFG *if* $\mu_1(s,t) = min\{\sigma_1(s), \sigma_1(t)\}$ *and* $\mu_2(s,t) = max\{\sigma_2(s), \sigma_2(t)\}$ *for all* $(s,t) \in E$.

Definition 14. *The Underlying crisp graph* $G^* = (V^*, E^*)$ *of a* $FFGG = (V, \sigma, \mu)$ *is such that* $V^* = u \in V: \sigma(u) > 0$ *and* $E^* = (u,v): \mu(u,v) > 0$.

Definition 15. *Let* G *be a* FFG *having an underlying crisp graph* (UCG) $G^* = (V, E)$. *Then* (σ, μ) *is called a Fermatean fuzzy cycle if* $(supp(\sigma), supp(\mu))$ *is a cycle and* $\nexists$ *unique* $(s,t) \in supp(\mu)$ *such that* $\mu(s,t) = \wedge \; \{\mu(w,x): (w,x) \in supp(\mu)\} = \{\wedge \; \mu_1(w,x), \wedge \; \mu_2(w,x): (w,x) \in supp(\mu)\}$.

Illustration of decomposition of $\boldsymbol{FFG}$**:**
Here, we take the $FFGG$ shown in Fig. 3.

We can decompose $FFGG$ into one FGG_1 shown in Fig. 4 in such a way that G_1 is contracted by selecting first components of the MV of nodes and edges of G.

Throughout this paper $G = (V, \sigma, \mu)$ represents the FFG.

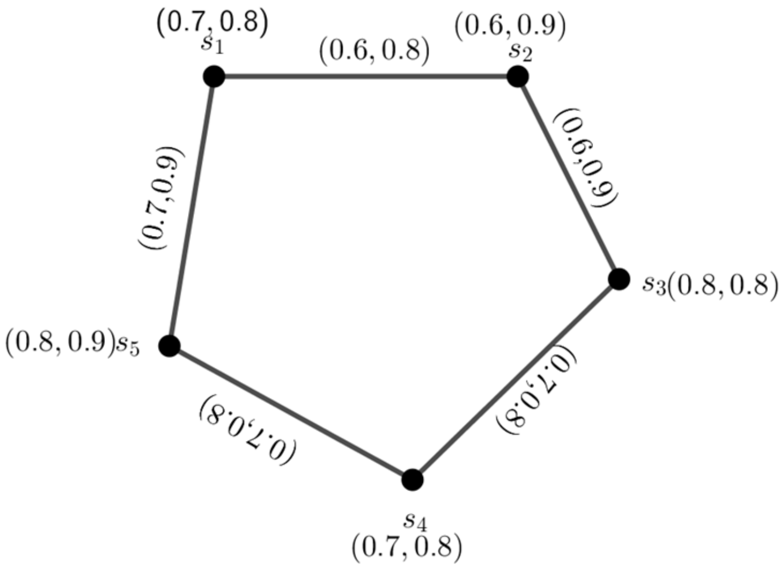

Figure 3: A $FFGG$

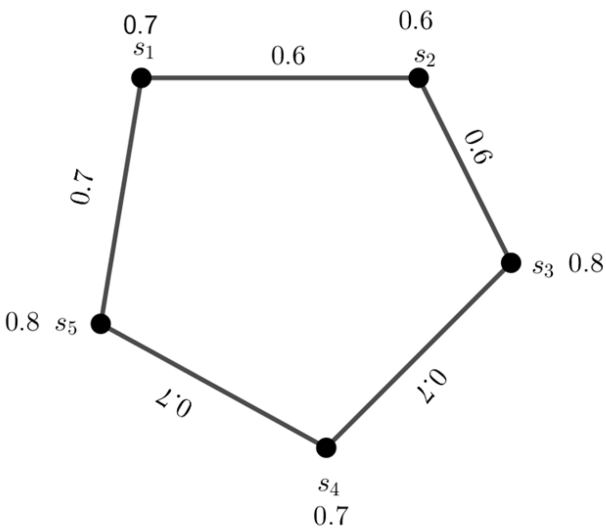

Figure 4: A fuzzy graph G_1

5. Fermatean fuzzy threshold graph

In this section, we defined the Fermatean fuzzy threshold graph ($FFTG$) and studied some of its interesting properties.

Definition 16. *Let G be a FFG with the $UCGG^* = (V, E)$. Let $H = (V', \sigma', \mu')$ be a subgraph of G having $UCGH^* = (V', E')$. Then H is said to be a Fermatean fuzzy clique* *if* $(supp(V'), supp(E'))$ *is a clique and every cycle of* H *is a Fermatean fuzzy cycle.*

Definition 17. *A* $FFGG$ *is called a* $FFTG$ *if there exists a non negative number* $t = [t_1, t_2] \in \mathbb{R}^2$ *such that* $\sum_{s \in U} \sigma_1(s) \le t_1$ *and* $\sum_{s \in U} \sigma_2(s) \le t_2$ *iff* $U(\subseteq V)$ *is a stable set in* G.

Example 2. *Here, we consider a* FFG *shown in Fig. 5.*

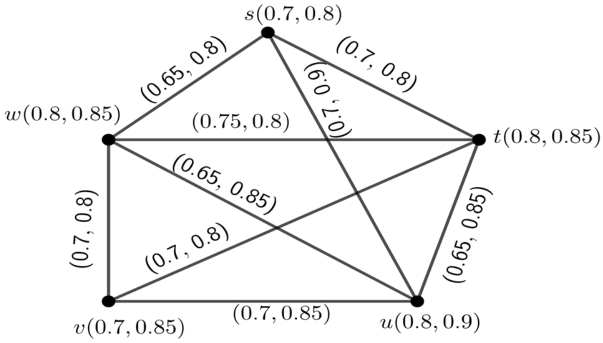

Figure 5: A $FFTG$

The stable set is $S = \{s, v\}$, as seen in the above figure. For the given S, we have

$$\sum_{x \in S} \sigma_1(x) = 0.7 + 0.7 = 1.4 \le 1.4$$

$$\sum_{x \in S} \sigma_2(x) = 0.8 + 0.75 = 1.55 \le 1.6$$

Again, we have two sets of non-stable set and they are $S_1 = \{u, v, t, w\}$ and $S_2 = \{u, w, t, s\}$. Now for S_1, we have

$$\sum_{x \in S_1} \sigma_1(x) = 0.8 + 0.7 + 0.8 + 0.8 = 3.1 \ge 1.4$$

$$\sum_{x \in S_1} \sigma_2(x) = 0.9 + 0.75 + 0.85 + 0.85 = 3.35 \ge 1.6$$

Now for S_2, we have

$$\sum_{x \in S_2} \sigma_1(x) = 0.8 + 0.8 + 0.8 + 0.7 = 3.1 \ge 1.4$$

$$\sum_{x \in S_2} \sigma_2(x) = 0.9 + 0.85 + 0.85 + 0.8 = 3.4 \ge 1.6$$

So, $(1.4, 1.6)$ is the threshold value of G.

Definition 18. *Let* G *be an* FFG. *The four nodes say* s, t, y, z *of* V *consists a Fermatean fuzzy alternating* 4*- cycle if* $\mu(s,t) > [0,0]$, $\mu(y,z)$, $\mu(s,y) = [0,0]$ *and* $\mu(t,z) = [0,0]$.

Now, Fermatean fuzzy alternating 4- cycle induces a P_4 if $\mu(s,z) = [0,0]$, $\mu(t,y) > [0,0]$. Again depending on the MVs of the edges (s,z) and (t,y) then the four nodes induce C_4 if $\mu(s,z) > [0,0]$ and $\mu(t,y) > [0,0]$.Two edges $e_1 = \{s,z\}$ and $e_2 = \{t,y\}$ are said to be induce a matching $2K_2$ if $\mu(s,z) = [0,0]$, $\mu(t,y) = [0,0]$.

Example 3. *Through an example, We illustrated $\mathcal{P}_4$, $\mathcal{C}_4$ and $2\mathcal{K}_2$ in Fig. 6.*

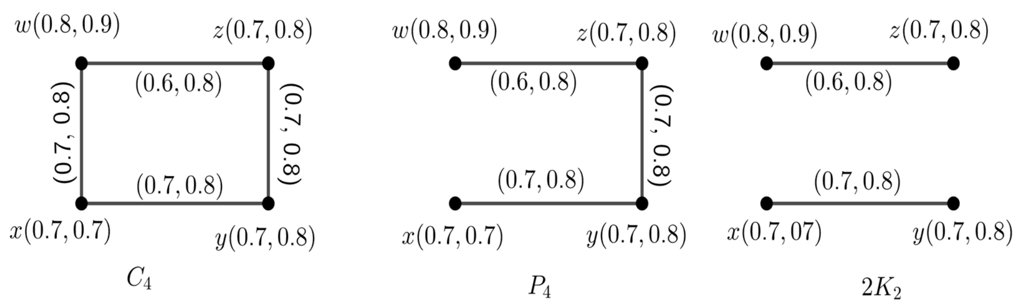

Figure 6: $\mathcal{C}_4$,$\mathcal{P}_4$ and $2\mathcal{K}_2$

Here we have taken the graph $\mathcal{C}_4$ and its MV of the edges are $\mu(w,y) = [0,0]$, $\mu(x,z) = [0,0]$,$\mu(w,z) > [0,0]$, $\mu(x,y) > [0,0]$, $\mu(w,x) > [0,0]$ and $\mu(z,y) > [0,0]$. So, it is satisfies the four condition of Fermatean fuzzy alternating 4 cycle. So $\mathcal{C}_4$ is a Fermatean fuzzy alternating 4 cycle. This implies, a Fermatean fuzzy alternating 4 cycle induces $\mathcal{C}_4$ graph if extra two condition $\mu(w,x) > [0,0]$ and $\mu(z,y) > [0,0]$ are satisfied. Similarly, a Fermatean fuzzy alternating 4 cycle induce $\mathcal{P}_4$ graph if $\mu(z,y) > [0,0]$ and $\mu(w,x) = [0,0]$ [or, $\mu[z,y] = [0,0]$ and $\mu(w,x) > [0,0]$. And a Fermatean fuzzy alternating 4 cycle induces $2\mathcal{K}_2$ graph if $\mu[w,x] = [0,0]$ and $\mu(z,y) = [0,0]$.

Definition 19. *A strong Fermatean fuzzy alternating 4-cycle is a Fermatean fuzzy alternating 4-cycle if Fermatean fuzzy $\mathcal{C}_4$ can be induced from it.*

Definition 20. *A FFG is called a Fermatean fuzzy split if the node set can be partitioned into a stable set and a Fermatean fuzzy clique.*

Theorem 1. *A $FFTG$ does not contain any Fermatean fuzzy alternating 4-cycle.*
Proof: Let G be a $FFTG$ with threshold $t = [t_1, t_2]$. Let, G contain a Fermatean fuzzy alternating 4-cycle. Thus $\exists s,t,w,x \in V$ such that $\mu(s,t) > [0,0]$ and $\mu(w,x) > [0,0]$ and $\mu(s,w) = \mu(t,x) = [0,0]$.
Since G is a $FFTG$ with a non-negative threshold number $t = [t_1, t_2]$ such that,

$$\left.\begin{array}{ll} \sigma_1(s) + \sigma_1(t) & > t_1, \\ \sigma_2(s) + \sigma_2(t) & > t_2 \end{array}\right\} \quad (1)$$

$$\left.\begin{array}{ll} \sigma_1(w) + \sigma_1(x) & > t_1, \\ \sigma_2(w) + \sigma_2(x) & > t_2 \end{array}\right\} \quad (2)$$

Again,

$$\left.\begin{aligned}\sigma_1(s)+\sigma_1(w) &\le t_1,\\ \sigma_2(s)+\sigma_2(w) &\le t_2\end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned}\sigma_1(t)+\sigma_1(x) &\le t_1,\\ \sigma_2(t)+\sigma_2(x) &\le t_2\end{aligned}\right\} \quad (4)$$

Adding equation 1 and 2, we get

$$\left.\begin{aligned}\sigma_1(s)+\sigma_1(t)+\sigma_1(w)+\sigma_1(x) &> 2t_1,\\ \sigma_2(s)+\sigma_2(t)+\sigma_2(w)+\sigma_2(x) &> 2t_2\end{aligned}\right\} \quad (5)$$

Adding equations 3 and 4, we get

$$\left.\begin{aligned}\sigma_1(s)+\sigma_1(t)+\sigma_1(w)+\sigma_1(x) &\le 2t_1,\\ \sigma_2(s)+\sigma_2(t)+\sigma_2(w)+\sigma_2(x) &\le 2t_2\end{aligned}\right\} \quad (6)$$

But the equations 5 and 6 are inconsistent. Therefore, no Fermatean fuzzy alternating 4-cycleÂ can be found in $FFTG$.

Theorem 2. *The α −cut of a $FFTGG = (V, \sigma, \mu)$ is also a $FFTG$.*

Proof: Consider, $G = (V, \sigma, \mu)$ to be $FFTG$ with threshold $t = [t_1, t_2]$ and G_α be its $\alpha -$ cut. As, G be a $FFTG$ with threshold $t = [t_1, t_2]$, then $\sum_{s\in S} \sigma_1(s) \le t_1$ and $\sum_{s\in S} \sigma_2(s) \le t_2$ iff $S \subseteq V$ is stable set.

Hence, G_α is a α −cut of G that means $G_\alpha = (\sigma_\alpha, \mu_\alpha)$ such that $\sigma_\alpha = \{w \in V: \sigma_1(w) \ge \alpha, \sigma_2(w) \le \alpha\}$ and $\mu_\alpha = \{(w, x): \mu_1(w, x) \le \alpha, \mu_2(w, x) \ge \alpha\}$. Let S_1 be the stable set of G_α. Two cases are possible here.

Case 1: At first we consider, If the number of nodes stays equal and the number of arcs reduces in G_α. As the number of nodes in G and G_α is same then the cardinality of the stable set S_1 never decrease which means either S_1 contains all of the nodes of S or may contains more nodes from S. When, S_1 and S contains same number of nodes then $\sum_{s\in S_1} \sigma_1(s) = \sum_{s\in S} \sigma_1(s) \le t_1$ and $\sum_{s\in S_1} \sigma_2(S) = \sum_{s\in S} \sigma_2(s) \le t_2$ iff $S_1 \subseteq V$ is a stable set in G_α.

So, G_α is a $FFTG$ with threshold $[t_1, t_2]$.

Next, if S_1 contains all those nodes of S as well as may contains more nodes of G. Suppose S contains n nodes and S_1 contains $l + n$ nodes. Then, $S_1 \subseteq V$ is a stable set iff

$$\begin{aligned}&\textstyle\sum_{s\in S_1} \sigma_1(s) = \sum_{s\in S} \sigma_1(s) + \sum_{s\in(S_1-S)} \sigma_1(s)\\ &\le t_1 + l\sqrt[3]{1-\alpha^3}[As, \sigma_1^3(x) + \sigma_2^3(x) \le 1\, and \sigma_2 \le \alpha]\\ &= T_1 where T = t_1 + l\sqrt[3]{1-\alpha^3}.\end{aligned}$$

And,

$$\begin{aligned}&\textstyle\sum_{s\in S_1} \sigma_2(s) = \sum_{s\in S} \sigma_2(s) + \sum_{s\in(S_1-S)} \sigma_2(s)\\ &\le t_2 + l\alpha \quad [As, \sigma_2 \le \alpha]\\ &= T_2 where T_2 = t_2 + l\alpha.\end{aligned}$$

So, G_α is a $FFTG$ with threshold $[T_1, T_2]$.

Case 2: Now, if the number of nodes and arcs may decreases in G_α from G. So, the cardinality of S_1 decreases from cardinality of S. Therefore $\sum_{s\in S_1} \sigma_1(s) \le \sum_{s\in S} \sigma_1(s) \le t_1$ and $\sum_{s\in S_1} \sigma_2(s) \le \sum_{s\in S} \sigma_2(s) \le t_2$ iff $S_1 \subseteq V$ is a stable set in G_α.

Hence, in both cases, $G_\alpha (0 \le \alpha \le 1)$ is a $FFTG$.

Definition 21. *A FFG is called a Fermatean fuzzy split graph if the node set can be separated into a stable set and Fermatean fuzzy clique.*

Theorem 3. *Every $FFTG$ is a Fermatean fuzzy split graph.*

Proof: Let G be a $FFTG$. Let D be the largest Fermatean fuzzy clique D and the stable set $V - D$.

Assume that $V - D$ is not a stable set, if possible. Then $\exists$ an arc (s, t) in $V - D$ such that $\mu_1(s,t) > 0$ and $\mu_2(s,t) > 0$. Since, D is the largest Fermatean fuzzy clique in G then for $(u, v) \in D$, $\mu_1(s,u) = \mu_1(t,v) = 0$ and $\mu_2(s,u) = \mu_2(t,v) = 0$.

This implies that s, t, u, v from a Fermatean fuzzy alternating 4-cycle, this contradicts the statement that G is $FFTG$. Next, $V - D$ is a fuzzy independent set. So, G is a $FFTG$.

Theorem 4. *Every Fermatean fuzzy split graph is a $FFTG$ or it can be converted to an $FFTG$ after some to the MV of the nodes.*

Proof: It is shown in the above theorem that every $FFTG$ is a Fermatean fuzzy split graph.

Now, if $G = (V, \sigma, \mu)$ be a Fermatean fuzzy split graph with Fermatean fuzzy clique D and stable set S is not a $FFTG$ then changes can be made to the MV of nodes so the next condition holds for some t_1, t_2 i.e. $\sum_{s\in S} \sigma_1(s) \le t_1$ and $\sum_{s\in S} \sigma_2(s) \le t_2$ iff $S \subseteq V$ will be stable set in G.

$$\sigma_1(s) + \sigma_1(t) > t_1$$
$$\sigma_1(s) + \sigma_1(t) > t_2, \ \ s, t \in D.$$

The inequations are consistent since no $s \in D$ is adjacent to $x \in S$, there is no interference in thoseÂ inequations. Thus a Fermatean fuzzy split graph can be made into a $FFTG$.

Theorem 5. *Let G be a FFG having $UCGG^* = (V, E)$. If G is a $FFTG$ then G^* is a split graph.*

Proof: Here, we are going to verify that G^* is a split graph, when $G = (V, \sigma, \mu)$ is a $FFTG$. First we prove that V can be partitioned into a clique and stable set. Let D be greatest clique in G^*. Then it will only be shown that $V - D$ is a stable set. Let $V - D$ to be a non-stable set if at all possible. Then $\exists$ an arc $(w, z) \in V - D$ such that

$$\mu(w, z) > 0$$
$$i.e. (\mu_1(w,z), \mu_2(w,z)) = [0,0].$$

Since, D is the greatest clique, then $\exists$ distinct nodes s, t in D then

$$\mu(w, s) = [0,0] \quad (7)$$
$$\mu(t, z) = [0,0] \quad (8)$$

from equations (7) and (8), we get a Fermatean fuzzy alternating 4-cycle. This is really a contradiction. Hence G^* is a split graph.

Example 4. *(Illustration of the Theorem 5) Here we consider the $FFTGG$ of Fig.5. The* *underlying crisp graph of the* $FFTGG$ *is* G^* *and which is shown in Fig.7.* G^* *is a split graph because its node set separated into a stable set* $\{s, v\}$ *and a clique* G_2 *(see Fig.8).*

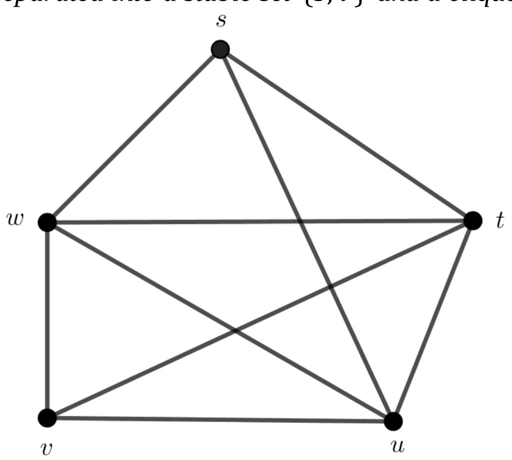

Figure 7: The Corresponding Underlying crisp graph G^*.

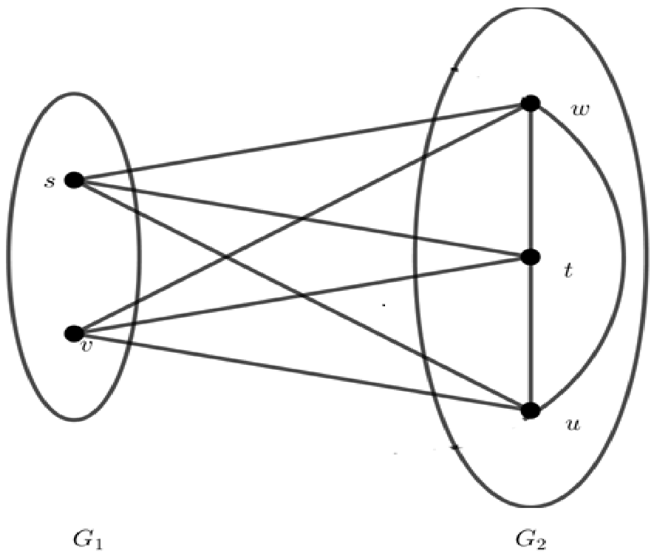

Figure 8: G^* divided a stable set and a clique G_2

Theorem 6. *The threshold value of complete* $FFTG$ *is* $(0,0)$.

Proof: Let $G = (V, \sigma, \mu)$ be a complete $FFTG$. Any two nodes are adjacent in a complete $FFTG$. In G, only ϕ is the stable set i.e. $S = \phi$. As $S = \phi$ then,

$$\sum_{t \in S} \sigma(t) = [0,0]$$

$$i.e. \sum_{t\in S} \sigma_1(s) = 0 \leq 0 \; and \sum_{t\in S} \sigma_2(t) = 0 \leq 0$$

where, $S = \phi$ is a stable set of complete $FFTGG$.

Hence, a complete $FFTG$ has a threshold value $(0,0)$.

Theorem 7. *Every single vertex FFG can be made $FFTG$ by adding a Fermatean fuzzy isolated node or a Fermatean dominating node one after the other.*

Proof: Let G be a FFG and which has only a single node $\{s_0\}$. To demonstrate this, we utilize Theorem 4, which states that if Fermatean fuzzy isolated or dominating node is added to G, then G is a Fermatean split graph.

Since $FFGG$ has only one node, it is possible to think of it as a split graph with a stable set $I = \{s_0\}$ and a Fermatean clique C. Next, we take a node s_1. s_1 can be added in two ways. By adding either an dominating node or a isolated node. If s_1 is a dominating node or an isolated node then $s_1 \in C$ or $s_1 \in I$ respectively. Therefore the resulting new graph is similarly a Fermatean fuzzyÂ split graph. Now, we know from theorem 4 that every Fermatean split graph is either a $FFTG$ or it can be converted to $FFTG$ after a few modification of the MV of nodes.

As a result, the theorem is established.

Definition 22. *The Fermatean fuzzy threshold dimension $(FFTD)$ $\tilde{t}(G)$ of $FFGG$ is the smallest +ve integer l for which $\exists l$ number of Fermatean fuzzy threshold subgraphs $(FFTS)$ $T_1, T_2, \dots, T_l$ which cover the whole arc set of G.*

Example 5. *We used the idea of threshold dimension in this example. Here we taken a $FFTGG$ (See Fig. 9) with threshold* $[2.5,3]$ *because the graph G has the stable set $S = \{a, a', e, e'\}$ and for the stable set S we have,*

$$\sum_{s\in S} \sigma_1(s) = 0.6 + 0.7 + 0.6 + 0.6 = 2.5 \leq 2.5$$

$$\sum_{s\in S} \sigma_2(s) = 0.8 + 0.8 + 0.7 + 0.7 = 3 \leq 3$$

And for all adjacent vertices in G, the threshold condition is also hold.

Now we construct two $FFTGT_1$ and T_2 shown in Fig. 10 and Fig. 11 such that they cover the arc set of G.

For the graph T_1, we see that the stable set is $S' = \{a, e\}$ and for the stable set S' we have, $\sum_{s\in S} \sigma_1(s) = 0.6 + 0.6 = 1.2 \leq 1.2$

$$\sum_{s\in S} \sigma_2(s) = 0.8 + 0.8 = 1.6 \leq 1.6$$

Again, T_1 have 6 set of non-stable set which are $S_1' = \{a, b, c, d\}$, $S_2' = \{b, c, d, e\}$, $S_3' = \{b, c\}$, $S_4' = \{b, c, d\}$, $S_5' = \{b, d\}$ and $S_6' = \{c, d\}$. For all S_i' where i=1,2,3,4,5,6, we have $\sum_{s\in S_i'} \sigma_1(s) > 1.2$

$$\sum_{s\in S_i'} \sigma_2(s) > 1.6$$

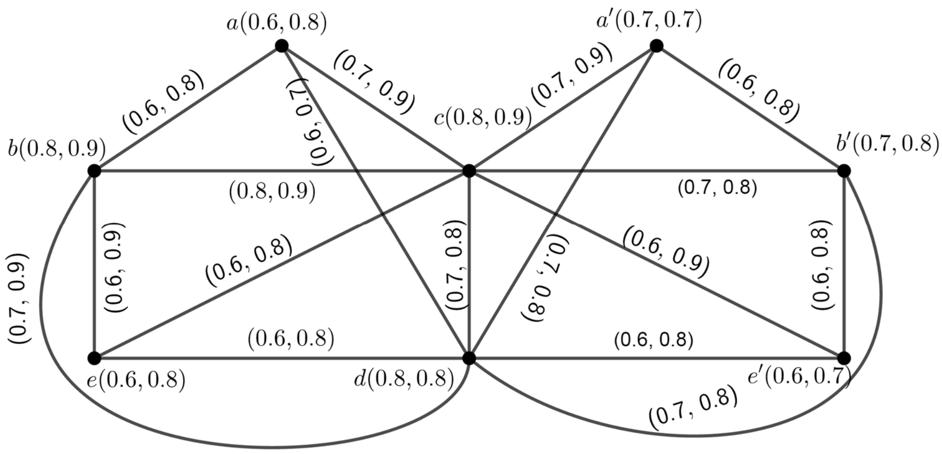

Figure 9: $FFTGG$

So the threshold value of T_1 is [1.2,1.6]. Similarly, For the graph T_2, the stable set is $S''=\{a',e'\}$ and for the stable set S' we have, $\sum_{s\in S}\sigma_1(s)=0.6+0.6=1.2\leq 1.3$

$$\sum_{s\in S}\sigma_2(s)=0.8+0.8=1.6\leq 1.4$$

Again, T_1 have 6 set of non-stable set which are $S_1''=\{a',b',c,d\}$, $S_2''=\{b',c,d,e'\}$, $S_3''=\{b',c\}$, $S_4''=\{b',c,d\}$, $S_5''=\{b',d\}$ and $S_6''=\{c,d\}$. For all S_i'' where i=1,2,3,4,5,6, we have $\sum_{s\in S_i''}\sigma_1(s)>1.3$

$$\sum_{s\in S_i''}\sigma_2(s)>1.4$$

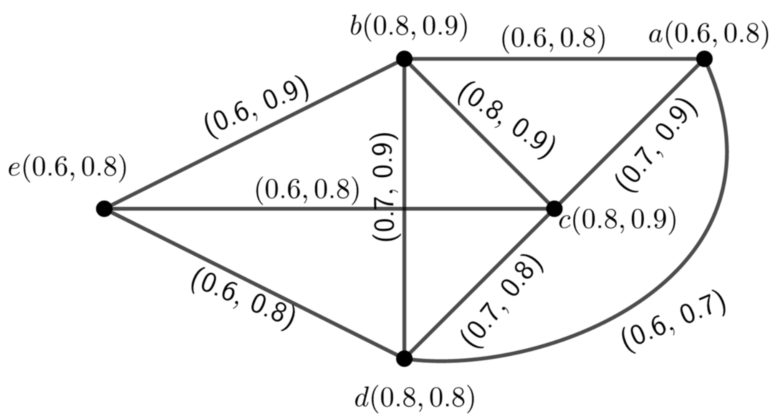

Figure 10: $FFTGT_1$

So the threshold value of T_1 is [1.3,1.4]. Hence, the $FFTGT_1$ andT_2 are having threshold value [1.2,1.6] and [1.3,1.4] respectively. Those are shown in Fig. 10 and Fig. 11. So the $FFTSsT_1$ and T_2 are cover all the whole arc set of G, which implies $\tilde{t}(G) = 2$.

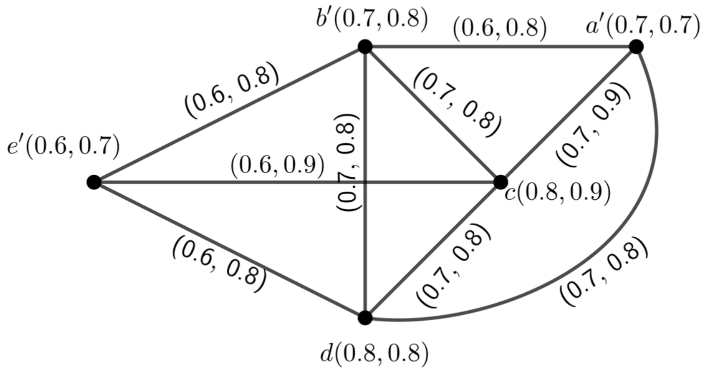

Figure 11: $FFTGT_2$

Definition 23. *The Fermatean fuzzy threshold partition number ($FFTPN$) of a FFG is the smallest positive integer P for which there exists P number of $FFTSs$ which covers the edge set FFG and there are no common arcs between them. It is represented by the symbol $\tilde{t}_P(G)$.*

Theorem 8. *A FFG has the same partition number as a decomposed FG.*

Proof: Let G be a FFG with threshold $t = (t_1, t_2)$ and G_1 be the decomposed fuzzy graph with threshold value t_1. Let G have partition number P, that means, $\exists$ a P number of $FFTS$s which covers the arc set of G with no common arcs.

Now, the decomposed fuzzy graph G_1, have the arc set same as the arc set of G. So the arc set of G_1 can be covered by the same number P of $FFTS$ of G_1 having no common edge.

Hence, The partition number for G_1 is P.

Theorem 9. *A FFG has the same threshold dimension as a decomposed FG.*

Proof: Let G be a FFG with threshold $t = [t_1, t_2]$ and G_1 be the decomposed fuzzy graph with threshold value t_1. Let K be the threshold dimension of G, which means K number of $FFTS$s cover the arc set of G.

Now, the decomposed fuzzy graph G_1, have the arc set same as the arc set of G. So the arc set of G_1 can be covered by the same number K of $FFTS$s of G_1.

Hence K be the threshold dimension of G_1.

Remark 1. *Clearly, $|E(G)| \geq \tilde{t}_p(G) \geq \tilde{t}(G)$, where $|E(G)|$ signifies the number of arcs of G.*

Theorem 10. *The α −cut of a FFG has threshold dimension at least one.*

Proof: Let G be a FFG. Let G_α be the $\alpha-$ cut of G. her, we have to show that $\exists$ at least one $FFTS$ which cover edge set of G_α.

If G_α is a $FFTG$ then G_α itself the condition. Therefore, G_α has threshold dimension 1. If G_α is not a $FFTG$ then We know from theorem 7 that every single node can be made $FFTG$. Hence, every FFG must have at least one subgraph and this is a FFG and cover the arc set G_α. There $\alpha-$ cut of a FFG has threshold dimension at least one.

Theorem 11. *For every FFG, $G=(V,\sigma,\mu)$ on n nodes we have, $\tilde{t}(G) \le n-\alpha(G)$, where $\alpha(G)$, where $\alpha(G)$ is the stability number of a $FFGG$ that means which is the order of the largest stable set of G. Furthermore, If G is a FFG without triangle, then $\tilde{t}(G)=(n-|Supp(S)|)$, where S is the stable set with largest number of nodes.*

Proof: Let G be a FFG with n nodes and S be the largest stable set. For each vertex $a \in V-S$, we consider the star centered at a. Each such star is a $FFTG$. If we add one or more weak fuzzy arc of stable set to the stars then they satisfy the condition of $FFTG$. As a result, all of these stars along with weak arcs of stable sets cover the arc set of G. Thus $\tilde{t}(G) \le (n-\alpha(G))$ as S being a crisp sets.

It is correct that $|S|=|Supp(S)|$. So $\tilde{t}(G) \le (n-|Supp(S)|)$. Again if G is FFG with out triangle, then each FFG is a star or star with weak arcs. So, $\tilde{t}(G) \ge (n-|Supp(S)|)$. Hence, $\tilde{t}(G)=(n-|Supp(S)|)$.

The converse of the theorem in general may not be true that is, if $\tilde{t}(G)=|V|-\alpha(G)$, then G may not be triangle-free. This can be explained by an example.

Example 6. *Here, we consider a FFG which is shown in Fig. 12.*

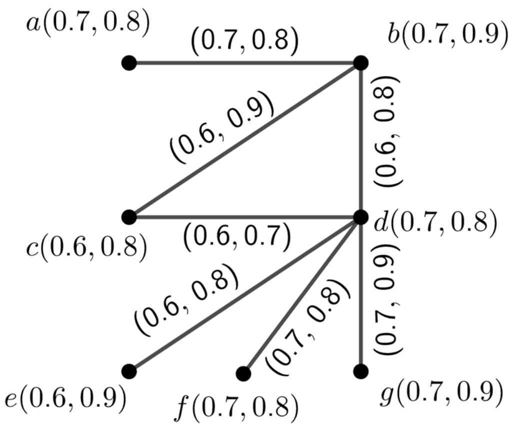

Figure 12: A $FFGG$.

The maximum independent set is $\{a,c,e,f,g\}$ in this case and $\alpha(G)=5$. Here we found two $FFTS G_1$ and G_2 which cover the whole arc set of G. The two $FFTS$ of G are shown in Fig. 13. So the $FFTD$ of G is $\tilde{t}=2=7-5=|V|-\alpha(G)$. The $FFGG$ have a triangle (nodes b,c,d forms a triangle) but $FFTD$ of G satisfies the condition $\tilde{t}=|V|-\alpha(G)$. So that is proved that the converse of the theorem 11 may not be true in general.

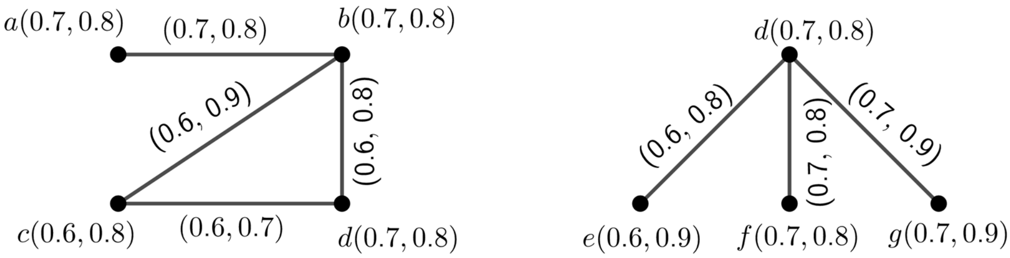

Figure 13: Two $FFTSG_1,\ G_2$.

Theorem 12. *Let G be a FFG without triangle, then $\tilde{t}_p(G) = \tilde{t}(G)$.*

Proof: Let G be a FFG without triangle. We know that the minimum number of $FFTS$s which are required to cover the edge set of G is equal to $\tilde{t}(G)$. If a strong edge belongs to more than one $FFTS$, then we delete it from all $FFTS$ except one $FFTS$. Therefore, this $FFTS$ cover the edge set of G having no common edge. Hence $\tilde{t}_p(G) = \tilde{t}(G)$.

Example 7. *Here, we consider a FFG having 14 nodes which shown in Fig. 14.*

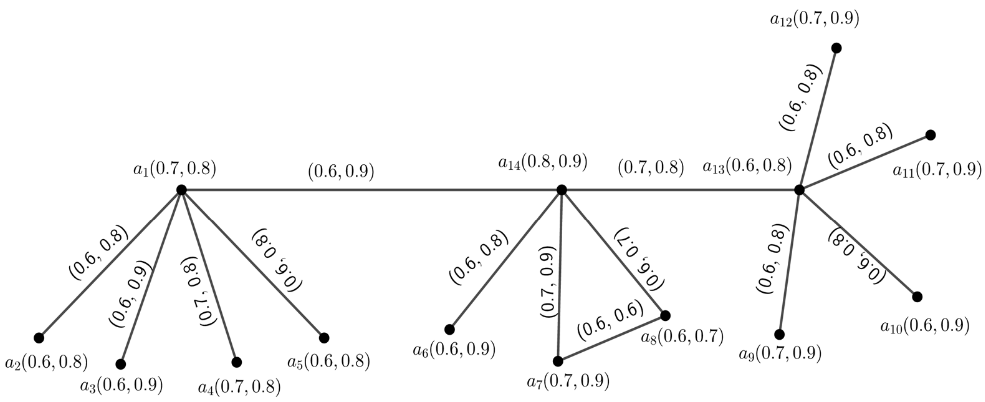

Figure 14: $FFGG$.

From Fig. 14, we can construct three $FFTS$ which cover the edge set of G and those three $FFTS$s does not have any common edge. Hence $\bar{t}_p(G) = \bar{t}(G) = 3$. Two $FFTS$ are shown in Fig. 15.

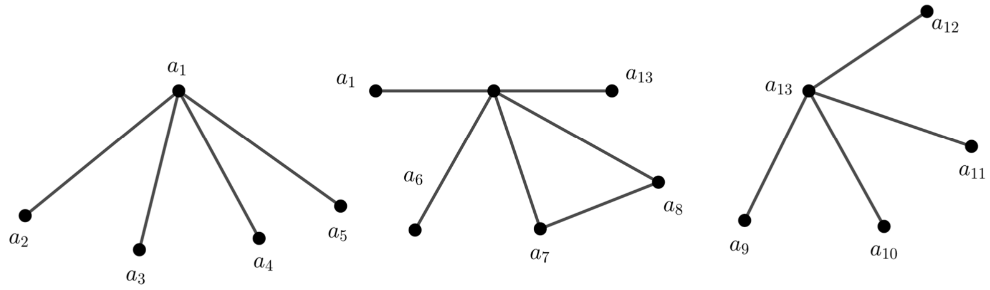

Figure 15: Three Fermatean fuzzy threshold subgraph G_1, G_2 and G_3.

From Fig. 14, we see that $\tilde{t}(G) = \tilde{t}_p(G) = 3$ but G contains a triangle which is created by the nodes a_{14}, a_7, a_8. Hence the converse part of the Theorem 12 is not generally true.

6. Application

The $FFTG$ is an important mathematical structure to represent the information in many real connected graphical system in which the vertices and edges both lie in an Fermatean fuzzy information. In this section, using $FFTG$ we investigated to find the major city which supply the oxygen to the minor cities on this covid situation.

6.1. Model construction

As you may be aware, COVID-19, a new breathing disease, is sweeping the world. India has also reported cases from states and the government is attempting to stem the spread of the disease. COVID-19 is transmitted when humans breathe in polluted air including droplets and small airborne particles. When shortness of breath escalates to a more severe condition, a small number of Covid-19 patients require oxygen support.The Covid-19 effects the patient's lungs due to shortages of breathing. Oxygen is essential for many individuals impacted by COVID-19.

The delivery of oxygen to patients in low and middle-income nations has significant challenges. Many nations have ignored appropriate oxygen delivery infrastructure for decades, even though pneumonia was the leading cause of hospitalization in poor and medium-income countries even before the epidemic. So, maximum oxygen manufacturing states provide a sufficient amount of oxygen to the poor or medium oxygen manufacturing states in a country.

Here we consider 4 medium oxygen manufacturing states namely a_1, a_2, a_3,a_4 and 2 maximum oxygen manufacturing states namely b_1, b_2 as modes. There will be an edge between two nodes if a vertex can provide oxygen to another vertex. There is an edge between 2 maximum oxygen manufacturing states if they transfer the oxygen between them. Here we use $FFTG$ to find the minimum amount of oxygen from maximum oxygen manufacturing states, so that the oxygen supply could fulfil the actual demands of oxygen of the medium oxygen manufacturing states. The membership value of two vertices b_1 and b_2 are calculated depending on the capability for storing medical oxygen, and cost per unit. The membership value of 4 medium oxygen manufacturing states a_1, a_2,a_3 and a_4 are calculated depending on the demand of medical oxygen and, the cost of buying oxygen from other states. The edge membership value between two maximum oxygen manufacturing states is calculated depending on road condition, oxygen transfer cost between those vertices. The edge membership value between two minimum oxygen manufacturing states is calculated depending on travel time, transformation cost between the end vertices.The edge membership value between minimum oxygen manufacturing states and maximum oxygen manufacturing states is calculated depending on oxygen transfer capability, and oxygen transfer cost between vertices. The model $FFTG$ is shown in Fig. 16.

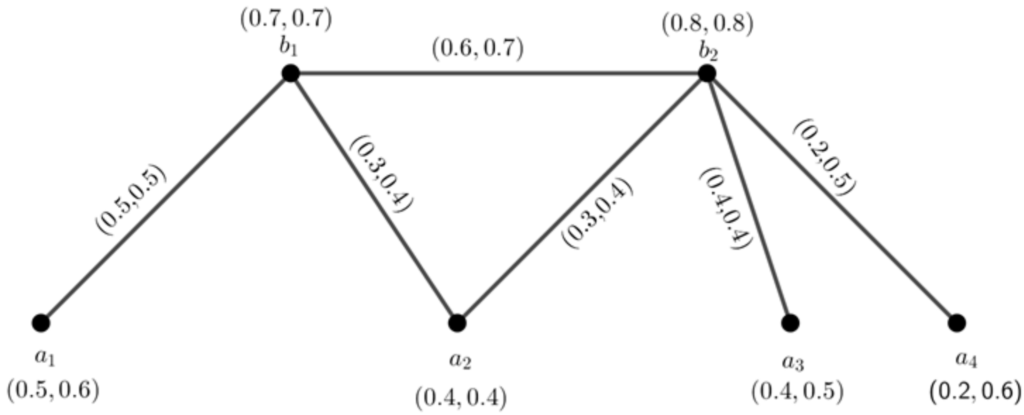

Figure 16: Fermatean fuzzy graph G.

6.2. Illustration of membership values

We see that a_1 and a_2 state needed 0.5 and 0.3 amount of medical oxygen respectively. b_1 vertices is connected with the two minimum oxygen manufacturing states (a_1 and a_2). So, b_1 state send 0.5 amount oxygen to a_1 state and also send 0.2 amount oxygen to a_2 state because b_1 have 0.7 amount of oxygen for distribution. Again a_2 also connected with b_2 vertex. b_2 have 0.2 amount of medical oxygen after distribution of oxygen in the states a_3, a_4. The cost of oxygen per unit in a state b_1 is less than the cost of oxygen per unit in a state b_2. So, in minimum oxygen manufacturing states there are a lot of COVID-19patientswho must want medical oxygen at minimum cost. Therefore firstly the state $a_2$has taken its demand for oxygen from b_1 and then from b_2.

The state a_1 will get its actual demand oxygen from the state b_1 and b_2. Here we investigated that these two maximum oxygen manufacturing states have sufficient amount of medical oxygen for supply to the other low oxygen manufacturing states. $FFTD$ of G is two. Since the maximum stable set is $\{a_1, a_2, a_3, a_4\}$ in G. and therefore $FFTPN$ is at most 2 by using theorem 11. The two $FFTSG_1$, G_2 are shown in next Fig. 17.

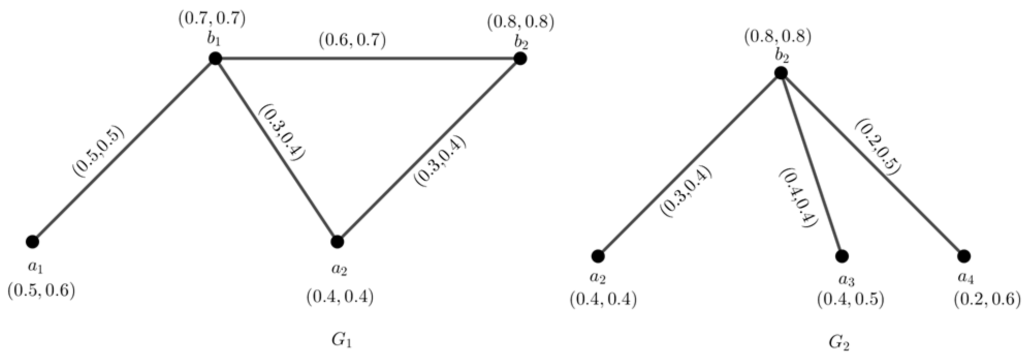

Figure 17: Two Fermatean fuzzy threshold subgraphsG_1 and G_2.

In G_1, we see that the independent set is $S_1 = \{a_1, a_2\}$ and we see that $\sum_{a \in S_1} \sigma_1(a) = 0.5 + 0.4 = 0.9 \leq 0.9$

$$\sum_{a \in S_1} \sigma_2(a) = 0.6 + 0.4 = 1 \leq 1$$

and for all adjacent vertices in G_1, the threshold condition is also hold. Therefore the threshold value for G_1 is $(0.9,1)$.

In G_2, we see that the independent set is $S_1 = \{a_2, a_3, a_4\}$ and we see that $\sum_{a \in S_2} \sigma_1(a) = 0.4 + 0.4 + 0.2 = 1 \leq 1$

$$\sum_{a \in S_2} \sigma_2(a) = 0.4 + 0.5 + 0.6 = 1.5 \leq 1.5$$

and for all adjacent vertices in G_2, the threshold condition is also hold. Therefore the threshold value for G_2 is $(1,1.5)$.

6.3. Decision making

The MV of each vertex of a_1,a_2,a_3 and a_4 represent the demand of medical oxygen from the threshold value of G_1, we see that the states b_1 and b_2 need at least 0.9 unit amount of medical oxygen for distribution to the states a_1 and a_2. Here, we see that any one of maximum oxygen manufacturing states can not provide total oxygen in between a_1 and a_2 as because they need at least 0.9 unit amount of oxygen for their covid patient and the MV of b_1 (or b_2) is not greater than 0.9.

From the threshold value of G_2, we see that the maximum oxygen manufacturing state b_2 demands at least 1 unit amount of oxygen.

Hence we can conclude that the high oxygen manufacturing states b_1 and b_2 can supply sufficient amount of oxygen for the low oxygen manufacturing states in which the state a_2 must get the needed oxygen with minimum cost from the state b_1.

Through the above discussion, we know that $FFTG$ really plays an important rule in controlling oxygen supply. Moreover, we also recognize that $FFTG$ is more applicable than FTG in controlling oxygen supply.

6.4. Real life problem

Here we discuss about the Fig.18. Here the nodes represent states in India and the corresponding edges denote the national highway between two states. We consider 6 states and the states are {Bihar, Jharkhand, West Bengal, Assam, Meghalaya, Mizoram}. The state Jharkhand and west bengal represent using red colored nodes because those states produces maximum liquid oxygen in India. The name of the national highway between the states Jharkhand and Bihar, Jharkhand and West Bengal, Bihar and West Bengal, West Bengal and Assam, Assam and Maghalaya, Assam and Mizoram are National Highway No. 19, National Highway No. 19, National Highway No. 27, National Highway No. 27, National Highway No. 6, National Highway No. 6 respectively. Because Jharkhand and West Bengal producesÂ the most liquid oxygen compared to the other states in the provided Fermatean fuzzy threshold graph G_1, those states will deliver oxygen to the other states in this covid situation. For this reason, we take the membership value of those two states Jharkhand and West Bengal are characterised by two criteria by: $\{capability for storing madical oxygen, Cost per unit\}$. The membership value of each state except Jharkhand and West Bengal are characterised by two criteria: $\{Demand\ of\ medical\ oxygen, Cost\ of\ buying\ Oxygen\ from\ other\ state\}$.T he membership value of each road is characterised by two criteria: $\{Travel\ time, Trans\ formation\ Cost\}$.

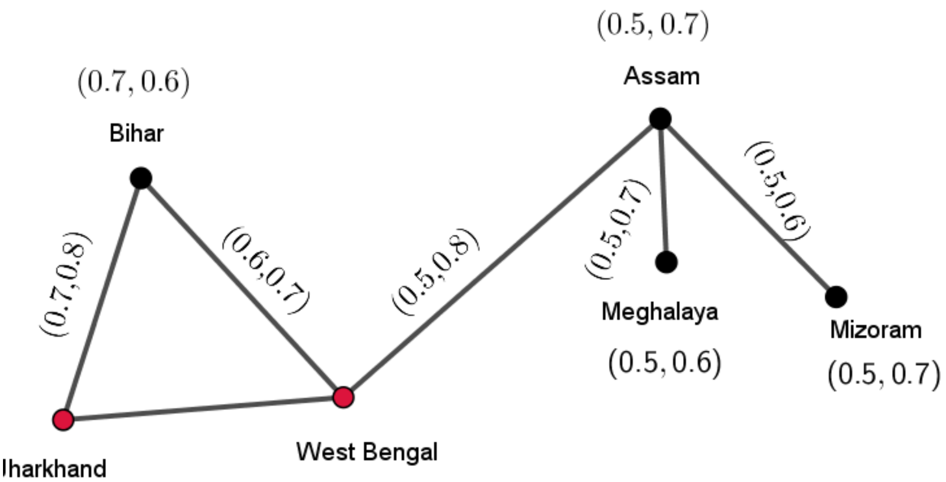

Figure 18: Fermatean fuzzy threshold graph G_1.

In G_1, we see that the independent set are $S_1 = \{Bihar, Assam\}$ and $S_2 = \{Bihar, Meghalaya, Mizoram\}$. For S_1, we see that $\sum_{x\in S_1} \sigma_1(x) = 0.7 + 0.5 = 1.2 \leq 1.2$

$$\sum_{x\in S_1} \sigma_2(x) = 0.6 + 0.7 = 1.3 \leq 1.3$$

and For S_2 we see that $\sum_{x\in S_2} \sigma_1(x) = 0.7 + 0.5 + 0.5 = 1.7 \leq 1.7$

$$\sum_{x\in S_2} \sigma_2(x) = 0.6 + 0.6 + 0.7 = 1.9 \leq 1.9$$

and for all adjacent nodes in G_1, the threshold condition is also hold for the threshold value $(1.7,1.9)$. Therefore, the threshold value for G_1 is $(1.7,1.9)$. we easily see that, the two states Jharkhand and West bengal will produced the $\sigma_1(Jharkhand) + \sigma_1(West\ Bengal) = 0.9 + 0.8 = 1.7$ amount of medical oxygen then those states helps to the low oxygen manufacturing states (Bihar, Meghalaya, Mizoram) in Covid situation. Again using the concept of Fermatean fuzzy threshold graph, We may conclude that second component of the threshold value is 1.9 and this indicates that S_1 or S_2 states will pay total 1.9 unit cost for the required oxygen. we can simply determine which states will send how much oxygen to which states. Using this concept, all low oxygen manufacturing states will get their needed medical oxygen from maximum oxygen producing states in India.

7. Conclusion

We defined the $FFTG$ as an extension of a fuzzy threshold graph in this work. These graphs helps in the solution of resource allocation issues in a fuzzy system, and they help in the management of information flow in a fuzzy system. We also looked into some relation between $FFTD$ and $FFTPN$.On decomposed $FFTG$, we investigated several of their properties. At the end of this paper an application is also provided. Our study will be expanded based on $FFTG$ in order to discover some more features as well as some applications. We plan to extend ourwork to:

(1) Single-valued neutrosophic soft threshold graphs.;
(2) Rough fuzzy threshold graphs;
(3) Pythagorean fuzzy soft threshold graphs, and
(4) Fuzzy soft threshold graphs.

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