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An Inventory Model for Non-Instantaneous Deteriorating Items under Conditions of Permissible Delay in Payments for *n*-Cycles

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Abstract. This study develops an inventory model to find the minimum relevant inventory cost per unit time for non-instantaneous deteriorating goods over a finite time horizon with exponentially declining demand for n-cycles. The shortages are allowed and back ordered. Under the situation of delay in payments, the inventory model in this study is divided into four cases by the time of shortage, deadline of delay in payment. A numerical example is presented to illustrate the model and the sensitivity analysis is also studied.

Keywords: Inventory, non – instantaneous deteriorating items, exponentially declining demand, delay in payments

AMS Mathematics subject classification (2010): 90B05

1. Introduction

The controlling and regulating of deteriorating item is a measure problem in any inventory system. Certain products like vegetables, fruits, electronic components, chemicals deteriorate during their normal storage period. Hence when developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researches have continuously modified the deteriorating inventory models so as to more practicable and realistic. In the most inventory models for deteriorating items existed, almost all researches assume that the deterioration of the items in inventory starts from the instant of their arrival in stock. However, in real world, most of goods have a span of maintaining fresh quality of original condition. During that time interval, there is no deterioration. Those items are idled as non-instantaneous items. In the field of inventory control management, it is necessary to consider the inventory problems for non-instantaneous deteriorating items. Ouyang et al. [2] were the two earliest researches to consider inventory model for exponential declining demand and during the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. Later Chuang and Lin [1] presented an ordering Quantity model for Non-instantaneous deteriorating items under conditions of permissible delay in payments, they derived for ‘n’ cycles over a finite planning horizon. Liang and Zhou [6] developed a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment and Yu et al. [7] discussed an ordering policy for Two-phase deteriorating inventory system with changing deterioration rate. Ji [3] proposed deterministic EOQ inventory model for non-instantaneous deteriorating items with starting and ending without shortages. Sahoo and Sahoo [5] considered constant deteriorating items with price dependent demand, time varying holding cost. Recently, Misra et al. [4] provides a detailed review of deteriorating items. They indicated the assumption of Weibull distribution for deteriorating items with permissible delay in payments.

In the traditional inventory EOQ model, the retailer pays for his items as soon as it is received. However in real competitive business world, the supplier offers the retailer a delay period, known as trade credit period. Delay in payment to the supplier is an alternative way of price discount. Hence paying later in directly reduces the purchase cost which attracts the retailers to enhance their ordering quantity. Here Business would earn interest income during this period and pay interest charge after the trade credit period delay in opposite. For retailers, especially small businesses which tend to have a limited number of financing opportunities, rely on trade credit as a source of short-term funds.

In this paper, an inventory model with non-instantaneous deteriorating items is developed for ‘n’ cycles. We already developed this model for one cycle. We have a demand function which is exponentially decreasing and the backlogging rate is inversely proportional to the waiting time for the next replenishment. In this study, the inventory model is divided into four cases by the time of shortage and deadline of delay in payment. This study aimed to find the minimum relevant inventory cost per unit time.

2. Notations and Assumptions

2.1. Notations

$I(t)$ = The inventory level at time t

$R(t)$= The demand rate per unit time

A = The initial demand

θ = The deterioration rate

δ = The backlogging parameter

n = The number of replenishment cycles

H = The length of planning horizon

T = The length of replenishment cycle(H/n)

M = The permissible delay in settling account

t_d = The length of time with no deterioration ($0\leq t_d \leq T$)

k = The length of period with positive inventory ($t_d \leq k \leq T$)

OC = The fixed cost per order($)

c_h = The holding cost per unit per unit time ($)

c_d = The deterioration cost per unit per unit time ($)

c_s = The shortage cost per unit per unit time ($)

c_p = The purchasing cost per unit per unit time ($)

P = The selling price per unit ($)

I_e = The interest earned per \$ per year
I_c = The interest charged per \$ in stocks per year by the supplier

2.2. Assumptions

1. The demand rate is known and decreases exponentially,
   $R(t) = \begin{cases} Ae^{-\lambda t}, I(t) > 0 \\ D, I(t) \leq 0 \end{cases}$
   where A(>0) is initial demand and λ, $(0 < \lambda < \theta)$ is a constant.
2. The deteriorating rate θ, (0< $\theta < 1$) is constant and there is no replenishment or repair of deteriorated units during the period under consideration.
3. The inventory system involves n cycles and the planning horizon is finite.
4. Shortages are allowed except for the last cycle.
5. The backlogging rate is variable during the shortage period and is dependent on the length of the waiting time for the next replenishment. The proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T - t)$ waiting for the next replenishment. To take care of this situation, we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative, backlogging parameter δ is a positive constant.
6. Initializing the cycle, the goods would not deteriorate, but Starting to deteriorate in a constant rate after a fixed period.
7. During the credit period, business can earn the interest income. After this period, business starts paying for interest charges.

3. Model formulation

The objective of the inventory problem is to find the total relevant cost as low as possible. The behavior of inventory system at any time is depicted in Figure 1.

This inventory model has 'n' cycles over a finite planning horizon, and every cycle is beginning at $(j-1)T$, j=1,2,…,n. This study assumed that the goods would start to deteriorate in a constant rate after time $(j-1)T + t_d$ and shortage occurs during the period $(j-1)T + k$ to jT, j=1,2,…,n. We aimed to find the optimal value of k over the period $[0, H]$ for finding the optimal cost in model.

The change of inventory level can be described by the following equations

$$\frac{dI(t)}{dt} = -Ae^{-\lambda t}, (j-1)T < t < (j-1)T + t_d \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -Ae^{-\lambda t}, (j-1)T + t_d < t < (j-1)T + k \tag{2}$$

$$\frac{dI(t)}{dt} = \frac{-D}{1+\delta(jT-t)}, (j-1)T + k < t < jT \tag{3}$$

with boundary conditions

$$I[(j-1)T] = I_{max}(j), \quad I[(j-1)T + k] = 0 \tag{4}$$

The solutions of equations (1) – (3) are given by

$$I(t) = I_{\max(j)} + \frac{A}{\lambda}\left[e^{-\lambda t} - e^{-\lambda(j-1)T}\right], (j-1)T < t < (j-1)T + t_d \tag{5}$$

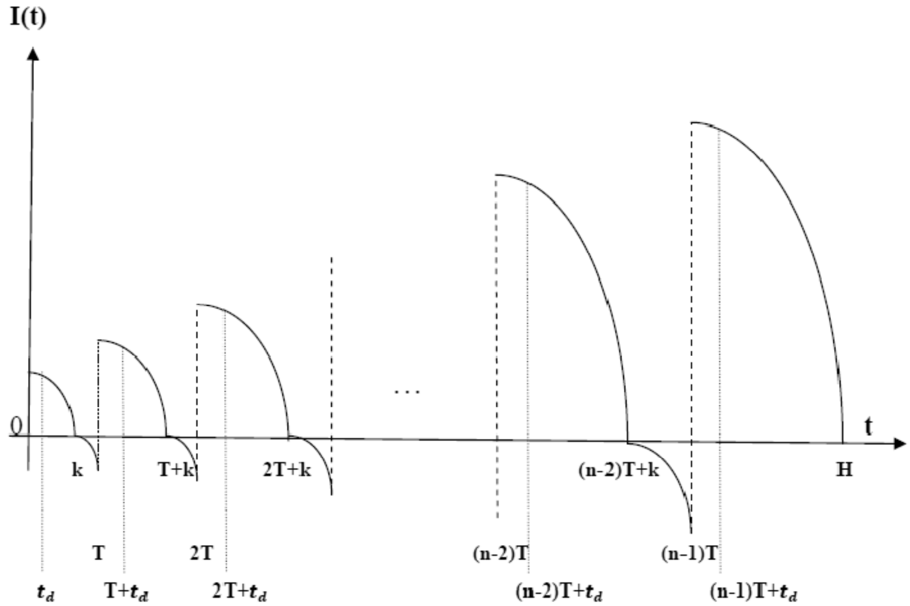

Figure 1: Inventory model for ‘n’ cycles

$$I(t) = \frac{A}{(\lambda-\theta)e^{\theta t}}\left[e^{-(\lambda-\theta)t} - e^{-(\lambda-\theta)[(j-1)T+k]}\right], (j-1)T + t_d < t < (j-1)T + k \tag{6}$$

$$I(t) = \frac{D}{\delta}\, log\left[\frac{1+\delta(jT-t)}{1+\delta(jT-(j-1)T-k}\right], (j-1)T + k < t < jT \tag{7}$$

At $t = (j-1)T + t_d$,

$$I_{max}(j) = \frac{A}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}}\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right] - \frac{A}{\lambda}\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda(j-1)T}\right] \tag{8}$$

The change of inventory level in the last cycle can be described by the following equation

$$I(t) = I_{max}(n) + \frac{A}{\lambda}\left[e^{-\lambda t} - e^{-\lambda(n-1)T}\right], (n-1)T < t < (n-1)T + t_d \tag{9}$$

$$I(t) = \frac{A}{e^{\theta t}(\lambda-\theta)}\left[e^{-(\lambda-\theta)t} - e^{-(\lambda-\theta)H}\right], (n-1)T + t_d < t < H \tag{10}$$

At $t = (n-1)T + t_d$,

$$I_{max}(n) = \frac{A}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}}\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right] - \frac{A}{\lambda}\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda(n-1)T}\right] \tag{11}$$

The holding cost (HC) is given by

$$= c_h\left[\sum_{j=1}^{n-1}\left\{\int_{(j-1)T}^{(j-1)T+t_d} I(t)dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I(t)dt\right\} + \int_{(n-1)T}^{(n-1)T+t_d} I(t)dt + \int_{(n-1)T+t_d}^{H} I(t)dt\right]$$

$$= c_h\left[\sum_{j=1}^{n-1}\left\{\frac{At_d\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{At_d e^{-\lambda[(j-1)T+t_d]}}{\lambda}\right.\right.$$
$$- \frac{A\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda[(j-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+t_d]}\right]}{\lambda(\lambda-\theta)}$$
$$\left.+ \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\left[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+t_d]\right]}\right]}{\theta(\lambda-\theta)}\right\}$$
$$+ \frac{At_d\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{At_d e^{-\lambda[(n-1)T+t_d]}}{\lambda}$$
$$- \frac{A\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda[(n-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+t_d]}\right]}{\lambda(\lambda-\theta)}$$
$$\left.+ \frac{A\left[e^{-\lambda H} - e^{-\left[(\lambda-\theta)H+\theta[(n-1)T+t_d]\right]}\right]}{\theta(\lambda-\theta)}\right] \quad (12)$$

The deterioration cost (DC) is given by

$$= c_d\left[\sum_{j=1}^{n-1}\left\{I_{max}(j) - \int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}\,dt\right\} + I_{max}(n) - \int_{(n-1)T+t_d}^{H} I(t)dt\right]$$

$$= c_d\left[\sum_{j=1}^{n-1}\left\{\frac{A\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{Ae^{-\lambda[(j-1)T+t_d]}}{\lambda}\right.\right.$$
$$\left.+ \frac{Ae^{-\lambda[(j-1)T+k]}}{\lambda}\right\} + \frac{A\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}}$$
$$\left.- \frac{Ae^{-\lambda[(n-1)T+t_d]}}{\lambda} + \frac{Ae^{-\lambda H}}{\lambda}\right] \quad (13)$$

The shortage cost (SC) is given by

$$= c_s\left[\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I(t)dt\right]$$

$$= -\frac{c_s D(n-1)}{\delta}\Big[1 + \big(1+\delta(T-k)\big)\big\{log\big(1+\delta(T-k)\big) - 1\big\}$$
$$+ (T-k)log\big(1+\delta(T-k)\big)\Big] \quad (14)$$

Case I. $(\mathbf{0 \leq M \leq t_d})$

The length of delay in payment (M) is absolutely is less than the length with no deterioration (t_d) in this case. Business can earn the interest income during credit period. However, business starts paying for interest charges after the credit period.

The total interest expenditure (IC)

$$= c_p I_c \left[\sum_{j=1}^{n-1} \left\{ \int_{(j-1)T+M}^{(j-1)T+t_d} I(t)dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I(t)dt \right\} + \int_{(n-1)T+M}^{(n-1)T+t_d} I(t)dt + \int_{(n-1)T+t_d}^{H} I(t)dt \right]$$

$$I_c \left[\sum_{j=1}^{n-1} \left\{ \frac{A(t_d - M)\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{A(t_d - M)e^{-\lambda[(j-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda[(j-1)T+M]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\left[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+t_d]\right]}\right]}{\theta(\lambda-\theta)} \right\} + \frac{A(t_d - M)\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{A(t_d - M)e^{-\lambda[(n-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda[(n-1)T+M]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-\left[(\lambda-\theta)H+\theta[(n-1)T+t_d]\right]}\right]}{\theta(\lambda-\theta)} \right] \tag{15}$$

The total interest revenue (IE)

$$= PI_e \sum_{j=1}^{n} \int_{(j-1)T}^{(j-1)T+M} Ae^{-\lambda t}\,[t-(j-1)T]dt$$

$$= PI_e \sum_{j=1}^{n} \frac{Ae^{-\lambda(j-1)T}}{\lambda^2}\left[1 - \lambda M e^{-\lambda M} - e^{-\lambda M}\right] \tag{16}$$

Therefore, here

$$TC = HC + DC - SC + IC - IE + n\,OC \tag{17}$$

Case II. $(\boldsymbol{t_d \leq M \leq k})$

Here the period of delay in payment (M) is more length than the period with no deterioration (t_d) but less than length of period with positive inventory (k)

The total interest expenditure (IC)

$$= c_p I_c \left[\sum_{j=1}^{n-1} \left\{ \int_{(j-1)T+M}^{(j-1)T+k} I(t)dt \right\} + \int_{(n-1)T+M}^{H} I(t)dt \right]$$

$$= c_p I_c \left[\sum_{j=1}^{n-1} \left\{ -\frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+M]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\left[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+M]\right]}\right]}{\theta(\lambda-\theta)} \right\} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+M]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-\left[(\lambda-\theta)H+\theta[(n-1)T+M]\right]}\right]}{\theta(\lambda-\theta)} \right] \quad (18)$$

The total interest revenue (IE)

$$= PI_e \sum_{j=1}^{n} \int_{(j-1)T}^{(j-1)T+M} Ae^{-\lambda t}\,[t-(j-1)T]dt$$

$$= PI_e \sum_{j=1}^{n} \frac{Ae^{-\lambda(j-1)T}}{\lambda^2}\left[1-\lambda M e^{-\lambda M} - e^{-\lambda M}\right] \quad (19)$$

Therefore here

$$TC = HC + DC - SC + IC - IE + n\,OC \quad (20)$$

Case III. ($\boldsymbol{k \le M \le T}$)

The period of delay in payment(M) is more length than period with positive inventory (k) except for the last cycle. Therefore, the interest charge occurs in the last cycle only.

The total interest expenditure (IC)

$$= c_p I_c \left[\int_{(n-1)T+M}^{H} I(t)dt \right]$$

$$= c_p I_c \left[-\frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+M]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-\left[(\lambda-\theta)H+\theta[(n-1)T+M]\right]}\right]}{\theta(\lambda-\theta)} \right] \quad (21)$$

The total interest revenue (IE)

$$= PI_e\left[\sum_{j=1}^{n}\left\{\int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}[t-(j-1)T]dt + (M-k)\int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}\,dt\right\} + \int_{(n-1)T}^{(n-1)T+M} Ae^{-\lambda t}(t-(n-1)T)dt\right]$$

$$= PI_e\left[\sum_{j=1}^{n}\frac{Ae^{-\lambda(j-1)T}}{\lambda^2}\left[1-e^{-\lambda k}-\lambda Me^{-\lambda k}+(M-k)\lambda\right] + \frac{Ae^{-\lambda(n-1)T}}{\lambda^2}\left[1-\lambda Me^{-\lambda M}-e^{-\lambda M}\right]\right] \tag{22}$$

Therefore here

$$TC = HC + DC - SC + IC - IE + n\,OC \tag{23}$$

Case IV. $(\boldsymbol{T \leq M})$

The period of delay in payment(M) is absolutely more length than a cycle(T). To compare with case III, there is no interest charge in this case.

The total interest revenue (IE)

$$= PI_e\left[\sum_{j=1}^{n}\left\{\int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}[t-(j-1)T]dt + (M-k)\int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}\,dt\right\} + \int_{(n-1)T}^{H} Ae^{-\lambda t}(t-(n-1)T)dt + (M-k)\int_{(n-1)T}^{H} Ae^{-\lambda t}\,dt\right]$$

$$= PI_e\left[\sum_{j=1}^{n}\frac{Ae^{-\lambda(j-1)T}}{\lambda^2}\left[1-e^{-\lambda k}-\lambda Me^{-\lambda k}+(M-k)\lambda\right] + \frac{Ae^{-\lambda(n-1)T}}{\lambda^2}[1+\lambda(M-T)]\frac{Ae^{-\lambda H}}{\lambda^2}[1+\lambda[H-(n-1)T] + \lambda(M-T)]\right] \tag{24}$$

Therefore here $TC = HC + DC - SC + IC - IE + n\,OC$ (25)

For a minimum TC, $\frac{\partial TC}{\partial k} = 0 \; and \; \frac{\partial^2 TC}{\partial k^2} > 0$ in all the cases.

4. Numerical Example

Consider the following data A=2, $\lambda = 0.01, \theta = 0.1$, $t_d = \frac{60}{365}$, δ =1, H=3, n=3, D=5, OC=10,c_h =3, c_d =7, $c_s = 6, c_p$ =8, P=12, $I_e = 0.03$, $I_c = 0.08$

Case	M	k	Total Cost
Case I. $(0 \le M \le t_d)$	30/365	0.925287	$43.79
Case II. $(t_d \le M \le k)$	90/365	0.926305	$41.95
Case III. $(k \le M \le T)$	360/365	0.935610	$40.34
Case IV. $(T \le M)$	375/365	0.935646	$40.26

5. Conclusion

In this paper, we have an inventory model for non – instantaneous deteriorating items with permissible delay in payment over a finite horizon. With the reason of permissible delay in payments and non-instantaneous deterioration, the total is minimized compared to existing models. The proposed model can be extended in several situations. We could extend the constant deterioration to exponential distribution.

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