

Type-2 Fuzzy Shortest Path

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Abstract. This paper addresses a Type – 2 fuzzy shortest path on a network using ranking function. A proposed algorithm gives the shortest path and shortest path length from source node to destination node. Here each arc length is assigned to type-2 fuzzy number. An illustrative example is also included to demonstrate our proposed algorithm.

Keywords: Type-2 fuzzy number, Ranking function , Interval type-2 fuzzy number, Footprint of Uncertainty

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1. Introduction

The concept of a type-2 fuzzy set was introduced by Zadeh [14] as an extension of the concept of an ordinary fuzzy set. A type-2 fuzzy set is characterized by a membership function where the membership value for each element of this set is a fuzzy set in $[0,1]$, unlike an ordinary fuzzy set where the membership value is a crisp number in $[0,1]$. They are useful for incorporating linguistic uncertainties, for example the words that are used in linguistic knowledge can mean different things to different people [5,6].

A fuzzy relation of higher type (Eg: type-2) has been regarded as one way to increase the fuzziness of a relation, and according to Hisdal [2], “increased fuzziness in a description means increased ability to handle in exact information in a logically correct manner”. According to John, “Type-2 fuzzy sets allow for linguistic grades of membership, thus assisting in knowledge representation and they also offer improvement on inferencing with type-1 sets [3]”. By using Extension principle, fuzzy number arithmetic has been studied by Dinagar and Anbalagan [12] and they have also presented new ranking function and arithmetic operations on generalized type-2 trapezoidal fuzzy numbers in [11]. Basic operations on type-2 fuzzy sets was introduced by Nilesh and Mendal [9]. Type-2 triangular fuzzy matrices were studied by Dinagar and Latha [13]. Ranking of type-2 fuzzy numbers is developed by Mitchell [7].

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The fuzzy Shortest path problem was first analyzed by Dubois and Prade [1]. Okada and Soper [10] developed an algorithm based on the multiple labeling approach, by which a number of non dominated paths can be generated. Nagoorgani and Anusuya [8] have studied on Fuzzy shortest path by linear multiple objective programming.

In conventional shortest path problem it is assumed that decision maker is certain about the parameters (distance, time etc.) between different nodes. But in real life situations, there always exist uncertainty about the parameters between different nodes. In such cases the parameters are requested by fuzzy numbers. Lee and Lee [4] have introduced “Shortest path problem in a Type-2 Weighted Graph”.

This paper is organized as follows: In section 2, some basic concepts are given. Section 3, gives an algorithm to find out the Type-2 fuzzy shortest path and shortest path length from source node to destination node. Section 4 gives the network terminology. To illustrate the proposed algorithm the numerical example is solved in Section 5.

2. Concepts

2.1. Type-2 Fuzzy Set

A Type-2 fuzzy set denoted \tilde{A} , is characterized by a Type-2 membership function $\mu_{\tilde{A}}(x, u)$ where $x \in X$ and $u \in J_x \subseteq [0, 1]$.

i.e., $\tilde{A} = \{ ((x, u), \mu_{\tilde{A}}(x, u)) / \forall x \in X, \forall u \in J_x \subseteq [0, 1] \}$ in which $0 \leq \mu_{\tilde{A}}(x, u) \leq 1$.

\tilde{A} can be expressed as $\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad J_x \subseteq [0, 1]$, where \int denotes

union over all admissible x and u . For discrete universe of discourse \int is replaced by \sum .

2.2. Interval Type-2 Fuzzy Set

Interval type-2 fuzzy set is defined to be a T2FS where all its secondary grade are of unity for all $f_x(u) = 1$.

2.3. Footprint of Uncertainty

Uncertainty in the primary membership of a type-2 fuzzy set, \tilde{A} , consists of a bounded region that we call the footprint of uncertainty (FOU). It is the union of all primary membership.

$$\text{i.e., } \text{FOU}(\tilde{A}) = \int_{x \in X} J_x$$

The FOU can be described in terms of its upper and lower membership function.

$$\text{FOU}(\tilde{A}) = \int_{x \in X} [\mu_{\tilde{A}}^l(x), \mu_{\tilde{A}}^u(x)].$$

2.4. Principal Membership Function

The principal membership function defined as the union of all the primary membership having secondary grades equal to 1.

$$\text{i.e., } P_r(\tilde{A}) = \int_{x \in X} u/x | f_x(u) = 1.$$

2.5. Type-2 Fuzzy Number

Let \tilde{A} be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied:

1. \tilde{A} is normal,
2. \tilde{A} is a convex set,
3. The support of \tilde{A} is closed and bounded, then \tilde{A} is called a type-2 fuzzy number.

2.6. Normal Type-2 Fuzzy Number

A T2FS, \tilde{A} is said to be normal if its FOU is normal IT2FS and it has a primary membership function.

2.7. Extension Principle

Let A_1, A_2, \dots, A_r be type-1 fuzzy sets in X_1, X_2, \dots, X_r , respectively. Then, Zadeh's Extension Principle allows us to induce from the type-1 fuzzy sets A_1, A_2, \dots, A_r a type-1 fuzzy set B on Y, through f, i.e., $B = f(A_1, \dots, A_r)$, such that

$$\mu_B(y) = \begin{cases} \sup_{x_1, x_2, \dots, x_n \in f^{-1}(y)} \min\{\mu_{A_1}(x_1), \dots, \mu_{A_n}(x_n)\} & \text{iff } f^{-1}(y) \neq \phi \\ 0, & f^{-1}(y) = \phi \end{cases}$$

2.8. Addition on Type-2 Fuzzy Numbers

$$\begin{aligned} \text{Let } \tilde{A} &= \bigcup_{\text{forall } \tilde{\alpha}} \tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}}) = (A^L, A^M, A^N, A^U) \\ &= ((a^L_1, a^L_2, a^L_3, a^L_4; \lambda_A), (a^M_1, a^M_2, a^M_3, a^M_4), (a^N_1, a^N_2, a^N_3, a^N_4), (a^U_1, a^U_2, a^U_3, \\ &\quad a^U_4)) \end{aligned}$$

$$\begin{aligned} \text{and Let } \tilde{B} &= \bigcup_{\text{forall } \tilde{\alpha}} \tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}}) = (B^L, B^M, B^N, B^U) \\ &= ((b^L_1, b^L_2, b^L_3, b^L_4; \lambda_B), (b^M_1, b^M_2, b^M_3, b^M_4), (b^N_1, b^N_2, b^N_3, b^N_4), (b^U_1, b^U_2, b^U_3, b^U_4)) \end{aligned}$$

be two normal type-2 fuzzy numbers. By using extension principle we have

$$\begin{aligned} \tilde{A} + \tilde{B} &= \left[\bigcup_{\text{forall } \tilde{\alpha}} \tilde{\alpha}FOU(\tilde{A}_{\tilde{\alpha}}) \right] + \left[\bigcup_{\text{forall } \tilde{\alpha}} \tilde{\alpha}FOU(\tilde{B}_{\tilde{\alpha}}) \right] \\ &= (A^L + B^L, A^M + B^M, A^N + B^N, A^U + B^U) \\ \tilde{A} + \tilde{B} &= ((a^L_1 + b^L_1, a^L_2 + b^L_2, a^L_3 + b^L_3, a^L_4 + b^L_4; \min\{\lambda_A, \lambda_B\}), (a^M_1 + b^M_1, a^M_2 + \\ &\quad b^M_2, a^M_3 + b^M_3, a^M_4 + b^M_4), (a^N_1 + b^N_1, a^N_2 + b^N_2, a^N_3 + b^N_3, a^N_4 + b^N_4), (a^U_1 + b^U_1, a^U_2 + \\ &\quad b^U_2, a^U_3 + b^U_3, a^U_4 + b^U_4)). \end{aligned}$$

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2.9. Ranking Function on Type-2 Fuzzy Number

Let $\tilde{A} = ((a^L_1, a^L_2, a^L_3, a^L_4; \lambda_A), (a^M_1, a^M_2, a^M_3, a^M_4), (a^N_1, a^N_2, a^N_3, a^N_4), (a^U_1, a^U_2, a^U_3, a^U_4))$ be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as

$$R(\tilde{A}) = (a^L_1 + 2a^L_2 + 2a^L_3 + a^L_4 + 2a^M_1 + 4a^M_2 + 4a^M_3 + 2a^M_4 + 2a^N_1 + 4a^N_2 + 4a^N_3 + 2a^N_4 + a^U_1 + 2a^U_2 + 2a^U_3 + a^U_4) / 36.$$

3. Algorithm

Step 1: Form the possible paths from starting node to destination node and compute the corresponding path lengths, \tilde{L}_i $i = 1, 2, \dots, n$ for possible n paths.

Step 2: Compute the ranking function for all possible lengths

$$R(\tilde{L}_i) = (a^L_1 + 2a^L_2 + 2a^L_3 + a^L_4 + 2a^M_1 + 4a^M_2 + 4a^M_3 + 2a^M_4 + 2a^N_1 + 4a^N_2 + 4a^N_3 + 2a^N_4 + a^U_1 + 2a^U_2 + 2a^U_3 + a^U_4) / 36.$$

Step 3: Decide the shortest path length and corresponding path with the minimum rank.

4. Network Terminology

Consider a directed network $G(V,E)$ consisting of a finite set of nodes $V = \{1, 2, \dots, n\}$ and a set of m directed edges $E \subseteq VXV$. Each edge is denoted by an ordered pair (i,j) , where $i, j \in V$ and $i \neq j$. In this network, we specify two nodes, denoted by s and t , which are the source node and the destination node, respectively. We define a path P_{ij} as a sequence $P_{ij} = \{i = i_1, (i_1, i_2), i_2, \dots, i_{l-1}, (i_{l-1}, i_l), i_l = j\}$ of alternating nodes and edges. The existence of at least one path P_{si} in $G(V,E)$ is assumed for every node $i \in V - \{s\}$.

\tilde{d}_{ij} denotes a Type-2 Fuzzy Number associated with the edge (i,j) , corresponding to the length necessary to transverse (i,j) from i to j . The fuzzy distance along the path P is denoted as $\tilde{d}(P)$ is defined as $\tilde{d}(P) = \sum_{(i,j \in P)} \tilde{d}_{ij}$

5. Numerical Example

The problem is to find the shortest path and shortest path length between source node and destination node in the network with type-2 fuzzy number.

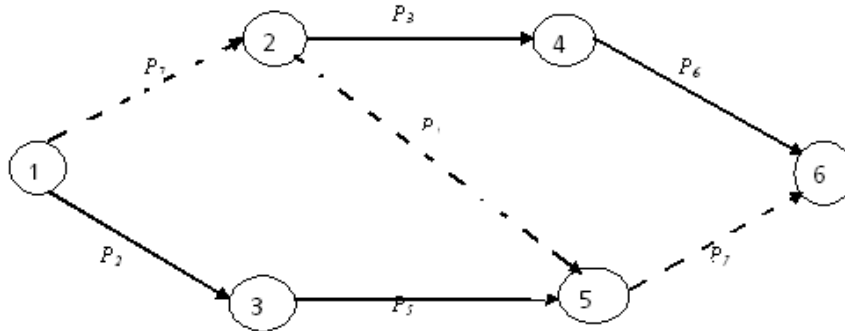


Figure 5.1:

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In this network each edge have been assigned to type-2 fuzzy number as follows:

$$P_1 = ((1.5, 1.8, 2.0, 2.5; 0.3), (1.2, 1.5, 2.0, 2.2), (1.1, 1.3, 1.5, 2.0), (1.0, 1.3, 1.5, 2.0))$$

$$P_2 = ((1.6, 1.7, 1.8, 2.0; 0.4), (1.5, 1.6, 2.0, 2.5), (1.3, 1.5, 2.0, 2.3), (1.2, 1.4, 1.5, 1.8))$$

$$P_3 = ((1.8, 1.9, 2.2, 2.6; 0.7), (1.6, 1.9, 2.2, 2.6), (1.5, 1.7, 2.1, 2.3), (1.2, 1.8, 2.3, 2.6))$$

$$P_4 = ((1.3, 1.5, 1.9, 2.0; 0.9), (1.3, 1.4, 2.3, 2.4), (1.2, 1.6, 2.3, 2.8), (1.0, 1.5, 1.9, 2.3))$$

$$P_5 = ((1.8, 1.9, 2.3, 2.5; 0.5), (1.6, 1.8, 2.3, 2.4), (1.5, 1.6, 1.9, 2.3), (1.3, 1.6, 1.9, 2.5))$$

$$P_6 = ((1.5, 1.7, 2.0, 2.5; 0.7), (1.3, 1.6, 2.2, 2.5), (1.2, 1.6, 2.3, 2.6), (1.1, 1.5, 1.8, 2.0))$$

$$P_7 = ((1.9, 2.0, 2.3, 2.4; 0.6), (1.8, 2.1, 2.3, 2.5), (1.5, 1.7, 2.0, 2.3), (1.4, 1.7, 2.0, 2.5))$$

Solution:

Step 1: Form the possible paths from starting node to destination node and compute the corresponding path lengths \tilde{L}_i , $i = 1, 2, \dots, n$ for possible n paths.

Possible paths are $1 - 2 - 4 - 6$; $1 - 2 - 5 - 6$; $1 - 3 - 5 - 6$.

Path lengths are $L_1 = 1 - 2 - 4 - 6 = P_1 + P_3 + P_6$

$$L_2 = 1 - 2 - 5 - 6 = P_1 + P_4 + P_7$$

$$L_3 = 1 - 3 - 5 - 6 = P_2 + P_5 + P_7$$

$$\tilde{L}_1 = ((1.5 + 1.8 + 1.5, 1.8 + 1.9 + 1.7, 2.0 + 2.2 + 2.0, 2.5 + 2.6 + 2.5; \min\{0.3, 0.7, 0.7\}), (1.2 + 1.6 + 1.3, 1.5 + 1.9 + 1.6, 2.0 + 2.2 + 2.2, 2.2 + 2.6 + 2.5), (1.1 + 1.5 + 1.2, 1.3 + 1.7 + 1.6, 1.5 + 2.1 + 2.3, 2.0 + 2.3 + 2.6), (1.0 + 1.2 + 1.1, 1.3 + 1.8 + 1.5, 1.5 + 2.3 + 1.8, 2.0 + 2.6 + 2.0))$$

$$\tilde{L}_1 = ((4.8, 5.4, 6.2, 7.6; 0.3), (4.1, 5.0, 6.4, 7.3), (3.8, 4.6, 5.9, 6.9), (3.3, 4.6, 5.6, 6.6))$$

$$\tilde{L}_2 = ((1.5 + 1.3 + 1.9, 1.8 + 1.5 + 2.0, 2.0 + 1.9 + 2.3, 2.5 + 2.0 + 2.4; \min\{0.3, 0.9, 0.6\}), (1.2 + 1.3 + 1.8, 1.5 + 1.4 + 2.1, 2.0 + 2.3 + 2.3, 2.2 + 2.4 + 2.5), (1.1 + 1.2 + 1.5, 1.3 + 1.6 + 1.7, 1.5 + 2.3 + 2.0, 2.0 + 2.8 + 2.3), (1.0 + 1.0 + 1.4, 1.3 + 1.5 + 1.7, 1.5 + 1.9 + 2.0, 2.0 + 2.3 + 2.5))$$

$$\tilde{L}_2 = ((4.7, 5.3, 6.2, 6.9; 0.3), (4.3, 5.0, 6.6, 7.1), (3.8, 4.6, 5.8, 7.1), (3.4, 4.5, 5.4, 6.8))$$

$$\tilde{L}_3 = ((1.6 + 1.8 + 1.9, 1.7 + 1.9 + 2.0, 1.8 + 2.3 + 2.3, 2.0 + 2.5 + 2.4; \min\{0.4, 0.5, 0.6\}), (1.5 + 1.6 + 1.8, 1.6 + 1.8 + 2.0, 2.0 + 2.3 + 2.3, 2.5 + 2.4 + 2.5), (1.3 + 1.5 + 1.5, 1.5 + 1.6 + 1.7, 2.0 + 1.9 + 2.0, 2.3 + 2.3 + 2.3), (1.2 + 1.3 + 1.4, 1.4 + 1.6 + 1.7, 1.5 + 1.9 + 2.0, 1.8 + 2.5 + 2.5))$$

$$\tilde{L}_3 = ((5.4, 5.6, 6.4, 6.9; 0.4), (4.8, 5.4, 6.6, 7.4), (4.3, 4.8, 5.9, 6.9), (3.9, 4.7, 5.4, 6.8))$$

Step 2: Compute the ranking function for all possible lengths

$$R(\tilde{L}_i) = (a^L_1 + 2a^L_2 + 2a^L_3 + a^L_4 + 2a^M_1 + 4a^M_2 + 4a^M_3 + 2a^M_4 + 2a^N_1 + 4a^N_2 + 4a^N_3 + 2a^N_4 + a^U_1 + 2a^U_2 + 2a^U_3 + a^U_4) / 36.$$

$$\begin{aligned} R(\tilde{L}_1) &= (4.8 + 2(5.4) + 2(6.2) + 7.6 + 2(4.1) + 4(5.0) + 4(6.4) + 2(7.3) + 2(3.8) + 4(4.6) \\ &\quad + 4(5.9) + 2(6.9) + 3.3 + 2(4.6) + 2(5.6) + 6.6) / 36 \\ &= (4.8 + 10.8 + 12.4 + 7.6 + 8.2 + 20 + 25.6 + 14.6 + 7.6 + 18.4 + 23.6 + 13.8 \\ &\quad + 3.3 + 9.2 + 11.2 + 6.6) / 36 \end{aligned}$$

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$$= 197.7 / 36$$

$$R(\tilde{L}_1) = 5.492$$

$$\begin{aligned} R(\tilde{L}_2) &= (4.7 + 2(5.3) + 2(6.2) + 6.9 + 2(4.3) + 4(5.0) + 4(6.6) + 2(7.1) + 2(3.8) + 4(4.6) \\ &\quad + 4(5.8) + 2(7.1) + 3.4 + 2(4.5) + 2(5.4) + 6.8) / 36 \\ &= (4.7 + 10.6 + 12.4 + 6.9 + 8.6 + 20 + 26.4 + 14.2 + 7.6 + 18.4 + 23.2 + 14.2 \\ &\quad + 3.4 + 9.0 + 10.8 + 6.8) / 36 \\ &= 197.2 / 36 \end{aligned}$$

$$R(\tilde{L}_2) = 5.48$$

$$\begin{aligned} R(\tilde{L}_3) &= (5.4 + 2(5.6) + 2(6.4) + 6.9 + 2(4.8) + 4(5.4) + 4(6.6) + 2(7.4) + 2(4.3) + 4(4.8) \\ &\quad + 4(5.9) + 2(6.9) + 3.9 + 2(4.7) + 2(5.4) + 6.8) / 36 \\ &= (5.4 + 11.2 + 12.8 + 6.9 + 9.6 + 21.6 + 26.4 + 14.8 + 8.6 + 19.2 + 23.6 + 13.8 \\ &\quad + 3.9 + 9.4 + 10.8 + 6.8) / 36 \\ &= 204.8 / 36 \end{aligned}$$

$$R(\tilde{L}_3) = 5.69$$

Step 3: Decide the Shortest path length and corresponding path with minimum rank.

$$R(\tilde{L}_1) = 5.492$$

$$R(\tilde{L}_2) = 5.48$$

$$R(\tilde{L}_3) = 5.69$$

Path length L_2 is having the minimum rank ($R(\tilde{L}_2) = 5.48$)

The shortest path length is

((4.7, 5.3, 6.2, 6.9; 0.3), (4.3, 5.0, 6.6, 7.1), (3.8, 4.6, 5.8, 7.1), (3.4, 4.5, 5.4, 6.8))

And the corresponding shortest path is

$$1 - 2 - 5 - 6$$

6. Conclusion

In this paper, we proposed an algorithm for finding shortest path and shortest path length of type-2 fuzzy number by using linear ranking function for type-2 fuzzy numbers from source node to destination node on a network. This proves that it is more flexible and intelligent way in solving problems which deals with *uncertainty and ambiguity* for choosing the best decision since it uses type-2 fuzzy number rather than type-1 fuzzy number.

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