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Somewhat *b-continuous and Somewhat *b-open Functions in Topological spaces

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Abstract. In this paper, we introduced and studied new classes of functions by making use of pre open sets,*b-open sets, D(c,*b)-sets. We established relationship between the new classes and other classes of functions and also given examples, counterexamples, properties and characterizations.

***Keywords*:** *b-open, somewhat pre continuous, somewhat *b-continuous, somewhat D(c,*b)-continuous, somewhat *b-open

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1. Introduction

Andrijevic [1] introduced and studied the concept of b-open sets in topological spaces. Mashhour [5] introduced pre open sets in topological spaces. Rekha and Indira [4, 6] introduced D(c,*b)-set, *b-open set in topological spaces. Gentry and Hoyle [3] introduced and studied the concept of somewhat continuous and somewhat open functions in topological spaces. Bechalli and Bansali [2] somewhat b-continuous functions and somewhat b-open functions in topological spaces. Santhileela and Balasubramanian [7] have made a similar study on semi continuous functions in Topological spaces. In this paper we introduced somewhat pre (*b, D(c,*b)) continuous, somewhat pre (*b, D(c,*b)) open functions,*b-dense set, *b-separable space. We established characterizations of somewhat *b-continuous functions. All throughout this paper, All spaces X, Y and Z are always topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively.

2. Preliminaries

Definition 2.1. A subset A of a space X is said to be
1. pre open[5] if $A \subseteq Int(Cl(A))$

2. *b-open [4]if $A \subseteq Cl(Int(A)) \cap Int(Cl(A))$
3. D(c,*b)-set[6] if $Int(A) = *bInt(A)$.

Definition 2.2. [3] A function $f : X \to Y$ is said to be somewhat continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

Definition 2.3. A function $f : X \to Y$ is said to be somewhat pre continuous if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists a pre open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.

Example 2.4. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a\}\}$, $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{2,3\}\}$. Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = 2; f(b) = 1; f(c) = 3$ .Then f is somewhat pre continuous.

Theorem 2.5. Every somewhat continuous function is somewhat pre continuous function.

Proof: Let $f : X \to Y$ be somewhat continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$ .Since f is somewhat continuous, there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$ .Since every open set is pre open, there exists a pre open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$, which implies that f is somewhat pre continuous function.

Remark 2.6. Converse of the above theorem is not true.

Example 2.7. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a,b\}\}$, $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{2,3\}\}$. Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = 1; f(b) = 2; f(c) = 3$ .Then f is a somewhat pre continuous function but not a somewhat continuous function.

Definition 2.8. [3] A function $f : X \to Y$ is said to be somewhat open function provided that for $U \in \tau$ and $U \neq \phi$, there exists an open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Definition 2.9. A function $f : X \to Y$ is said to be somewhat pre open function provided that for $U \in \tau$ and $U \neq \phi$, there exists a pre open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Example 2.10: Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a,b\}\}$, $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{1,3\}\}$. Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = 2; f(b) = 3; f(c) = 1$ .Then f is somewhat pre open function.

Theorem 2.11. Every somewhat open function is somewhat pre open function.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a somewhat open function. Let $U\in\tau$ and $U\neq\phi$ .Since f is somewhat open, there exists an open set V in Y such that $V\neq\phi$ and $V\subseteq f(U)$ .But every open set is pre open. Therefore there exists a pre open set V in Y such that $V\neq\phi$ and $V\subseteq f(U)$, which implies that f is somewhat pre open function.

Remark 2.12. Converse of the above theorem is not true.

Example 2.13. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a,b\}\}$, $Y=\{1,2,3\}$, $\sigma=\{Y,\phi,\{2,3\}\}$. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=1; f(b)=2; f(c)=3$ .Then f is somewhat pre open function but not a somewhat open function.

3. Somewhat *b - Continuous Functions

Definition 3.1. A function $f:X\to Y$ is said to be somewhat *b - continuous if for $U\in\sigma$ and $f^{-1}(U)\neq\phi$ there exists a *b - open set V in X such that $V\neq\phi$ and $V\subseteq f^{-1}(U)$.

Example 3.2. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,c\}\}$, $Y=\{1,2,3\}$, $\sigma=\{Y,\phi,\{1,3\}\}$. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=3; f(b)=1; f(c)=2$ .Then f is somewhat *b-continuous function.

Definition 3.3. A function $f:X\to Y$ is said to be somewhat D(c,*b) - continuous if for $U\in\sigma$ and $f^{-1}(U)\neq\phi$ there exists a D(c,*b) - open set V in X such that $V\neq\phi$ and $V\subseteq f^{-1}(U)$.

Example 3.4. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a\}\}$, $\sigma=\{X,\phi,\{b\}\}$.
Then the identity function $f:(X,\tau)\to(X,\sigma)$ is somewhat D(c,*b)-continuous.

Theorem 3.5. Every somewhat *b-continuous function is somewhat pre continuous function

Proof: Let $f:X\to Y$ be somewhat *b-continuous function. Let U be any open set in Y such that $f^{-1}(U)\neq\phi$. Since f is somewhat *b-continuous, there exists a *b- open set V in X such that $V\neq\phi$ and $V\subseteq f^{-1}(U)$ .Since every *b-open set is pre open, there exists a pre open set V in X such that $V\neq\phi$ and $V\subseteq f^{-1}(U)$, which implies that f is somewhat pre continuous function.

Remark 3.6. Converse of the above theorem is not true.

Example 3.7. Let $X=\{a,b,c\}$, $\tau=\{X,\phi,\{a,b\}\}$, $Y=\{1,2,3\}$, $\sigma=\{Y,\phi,\{2,3\}\}$. Define a function $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=1; f(b)=2; f(c)=3$ .Then f is somewhat pre continuous function but not a somewhat *b-continuous function.

Theorem 3.8. For a function $f:X\to Y$, the following are equivalent

1)f is somewhat continuous

2)f is some what *b-continuous and somewhat
D(c,*b)-continuous

Proof: To prove: (i)$\Rightarrow$(ii)Let $f : X \to Y$ be somewhat continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$.Since f is somewhat continuous, there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.Since every open set is *b-open and D(c,*b)-set, there exists a *b-open and D(c,*b)-set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$, which implies that f is somewhat *b-continuous and D(c,*b)- continuous.
To prove: (ii)$\Rightarrow$(i)Let $f : X \to Y$ be somewhat *b-continuous and D(c,*b)-continuous function. Let U be any open set in Y such that $f^{-1}(U) \neq \phi$.Since f is somewhat *b-continuous and D(c,*b)-continuous, there exists a *b- open and D(c,*b)-set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.Since every *b-open and D(c,*b)-set is open, there exists an open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$, which implies that f is somewhat continuous function.

Theorem 3.9. Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be any two functions. If f is somewhat *b-continuous function and g is continuous function, then $g \circ f$ is somewhat *b-continuous function.

Proof: Let $U \in \eta$.Suppose that $g^{-1}(U) \neq \phi$. Since $U \in \eta$ and g is continuous function. Therefore $g^{-1}(U) \in \sigma$. Suppose that $f^{-1}(g^{-1}(U)) \neq \phi$ .Since by hypothesis f is somewhat *b-continuous function, there exists a *b-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(g^{-1}(U))$ .But $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$, which implies that $V \subseteq (g \circ f)^{-1}(U)$. Therefore $g \circ f$ is somewhat *b-continuous function.

Definition 3.10. Let M be a subset of a topological space (X,τ).Then M is said to be *b-dense in X if there is no proper *b-closed set C in X such that $M \subset C \subset X$.

Theorem 3.11. Let $f : (X,\tau) \to (Y,\sigma)$ be a function .Then the following are equivalent:
(i) f is somewhat *b-continuous function.
(ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper *b-closed subset D of X such that $D \supseteq f^{-1}(C)$.
(iii) If M is a *b-dense subset of X then $f(M)$ is a dense subset of Y .

Proof: To prove: $(i) \Rightarrow (ii)$ Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then $Y - C$ is an open set in Y such that $f^{-1}(Y - C) = X - f^{-1}(C) \neq \phi$. By hypothesis there exists a *b-open set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(Y - C) = X - f^{-1}(C)$ .This implies that $f^{-1}(C) \subset X - V$ and $X - V = D$ is a *b-closed set in X . Hence there is a proper *b-closed subset D of X such that $D \supseteq f^{-1}(C)$.

To prove: $(ii) \Rightarrow (iii)$ Let M be a *b-dense set in X. We have to prove that $f(M)$ is dense in Y. Suppose that $f(M)$ is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C \subset Y$. Since $f^{-1}(C) \neq X$. By (ii) there exists a proper *b-closed set D such that $M \subset f^{-1}(C) \subseteq D \subset X$, which is a contradiction to our assumption that M is *b-dense in X. Hence $f(M)$ is dense in Y.

To prove: $(iii) \Rightarrow (ii)$ Suppose we assume that (ii) is not true . That is there exists a closed set C in Y such that $f^{-1}(C) \neq X$. But there is no proper *b-closed set D in X such that $f^{-1}(C) \subseteq D$. This means that $f^{-1}(C)$ is *b-dense in X. By (iii) $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to our assumption C is closed in Y. Hence (ii) is true.

To prove: $(ii) \Rightarrow (i)$ We have to prove that f is somewhat *b-continuous. Let $U \in \sigma$ and $f^{-1}(U) \neq \phi$. Then $Y - U$ is closed and $f^{-1}(Y-U) = X - f^{-1}(U) \neq \phi$. By our assumption there exists a proper *b-closed set D such that $D \supseteq f^{-1}(Y-U)$. This implies that there exists a *b-open set $X - D$ and $X - D \neq \phi$ such that $X - D \subseteq f^{-1}(U)$. Hence f is somewhat *b-continuous.

Theorem 3.12. Let (X,τ) and (Y,σ) be any two topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f:(X,\tau) \to (Y,\sigma)$ be a function such that f/A and f/B are somewhat *b-continuous functions. Then f is somewhat *b-continuous function.

Proof: We have to prove that f is somewhat *b-continuous. Let $U \in \sigma$ and $f^{-1}(U) \neq \phi$.
Then $(f/A)^{-1}(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$

Case (i): Suppose $(f/A)^{-1}(U) \neq \phi$

Since f/A is somewhat *b-continuous, there exists a *b-open set V in A such that $V \neq \phi$ and $V \subseteq (f/A)^{-1}(U) \subseteq f^{-1}(U)$. Since V is *b-open in A and A is open in X, which implies that V is *b-open in X. Thus f is somewhat *b-continuous function.

Case (ii): Suppose $(f/B)^{-1}(U) \neq \phi$

Since f/B is somewhat *b-continuous, there exists a *b-open set V in B such that $V \neq \phi$ and $V \subseteq (f/B)^{-1}(U) \subseteq f^{-1}(U)$. Since V is *b-open in B and B is open in X, which implies that V is *b-open in X. Thus f is somewhat *b-continuous function.

Case (iii): Suppose $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$

From case(i) and (ii), f is somewhat *b-continuous function.

Definition 3.13. A topological space X is said to be *b-separable if there exists a countable subset B of X which is *b-dense in X.

Example 3.14. Let $X=\{a,b,c,d\}$, $\tau=\{X,\phi,\{c\},\{d\},\{c,d\},\{a,c,d\},\{b,c,d\}\}$. The space X is *b-separable.

Theorem 3.15. If f is somewhat *b-continuous function from X onto Y and if X is *b-separable, then Y is separable.

Proof: Let $f:X\to Y$ be somewhat *b-continuous function such that X is *b-separable. Then by definition there exists a countable subset B of X which is *b-dense in X. Then by theorem 3.11, $f(B)$ is dense in Y. Since B is countable. Then $f(B)$ is also countable which is dense in Y. Hence Y is separable.

4. *b-Weakly Equivalent Topologies

Definition 4.1.[3] If X is a set and τ and σ are topologies for X, then τ is said to be weakly equivalent to σ provided if $U\in\tau$ and $U\neq\phi$,then there is an open set V in (X,σ) such that $V\neq\phi$ and $V\subset U$ and if $U\in\sigma$ and $U\neq\phi$,then there is an open set V in (X,τ) such that $V\neq\phi$ and $V\subset U$.

Definition 4.2. If X is a set and τ and σ are topologies for X, then τ is said to be *b-weakly equivalent to σ provided if $U\in\tau$ and $U\neq\phi$,then there is a *b- open set V in (X,σ) such that $V\neq\phi$ and $V\subset U$ and if $U\in\sigma$ and $U\neq\phi$,then there is a *b- open set V in (X,τ) such that $V\neq\phi$ and $V\subset U$.

Theorem 4.3. Let $f:(X,\tau)\to(Y,\sigma)$ be somewhat continuous function and let τ^* be a topology for X, which is *b-weakly equivalent to τ then the function $f:(X,\tau^*)\to(Y,\sigma)$ is somewhat *b-continuous function.

Proof: Let $U\in\sigma$ and $f^{-1}(U)\neq\phi$. Since by hypothesis $f:(X,\tau)\to(Y,\sigma)$ is somewhat continuous. By definition there exists an open set O in (X,τ) such that $O\neq\phi$ and $O\subseteq f^{-1}(U)$. Since O is an open set in (X,τ) such that $O\neq\phi$ and since by hypothesis τ is *b-weakly equivalent to τ^* by definition there exists a *b-open set V in (X,τ^*) such that $V\neq\phi$ and $V\subset O\subseteq f^{-1}(U)$.This implies $O\subseteq f^{-1}(U)$.Thus for any open set $U\in\sigma$ such that $f^{-1}(U)\neq\phi$ there exists a *b-open set V in (X,τ^*) such that $V\subseteq f^{-1}(U)$. Hence $f:(X,\tau^*)\to(Y,\sigma)$ is somewhat *b-continuous function.

Theorem 4.4. Let $f:(X,\tau)\to(Y,\sigma)$ be somewhat *b-continuous function and let σ^* be a topology for Y, which is weakly equivalent to σ. Then the function $f:(X,\tau)\to(Y,\sigma^*)$ is somewhat *b-continuous function.

Proof: Let $U\in\sigma^*$ and $f^{-1}(U)\neq\phi$ which implies $U\neq\phi$. Since σ and σ^* are weakly equivalent there exists an open set W in (Y,σ) such that $W\neq\phi$ and $W\subset U$. Since W is an open set in (Y,σ) such that $W\neq\phi$,which implies $f^{-1}(W)\neq\phi$. By hypothesis $f:(X,\tau)\to(Y,\sigma)$ be somewhat *b-continuous function. Therefore there exists a *b-open set V in X, such that $V \subseteq f^{-1}(W)$. Now $W \subset U$ implies $f^{-1}(W) \subset f^{-1}(U)$. We have $V \subseteq f^{-1}(U)$. Hence $f:(X,\tau) \to (Y,\sigma^*)$ is somewhat *b-continuous function.

5. Somewhat *b-Open Functions

Definition 5.1. A function $f: X \to Y$ is said to be somewhat *b-open function provided that for $U \in \tau$ and $U \neq \phi$, there exists a *b-open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Example 5.2. Let $X = \{1,2,3,4\}$, $\tau = \{X, \phi, \{2,3,4\}, \{2\}, \{1,2\}\}$, $Y = \{a,b,c,d\}$, $\sigma = \{Y, \phi, \{c\}, \{d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$.Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(1) = c; f(2) = d; f(3) = a; f(4) = b$.Then f is somewhat *b-open function.

Definition 5.3. A function $f: X \to Y$ is said to be somewhat D(c,*b)-open function provided that for $U \in \tau$ and $U \neq \phi$, there exists a D(c,*b)-open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.

Example 5.4. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{c\}\}$, $\sigma = \{X, \phi, \{a\}\}$.
Then the identity function $f:(X,\tau) \to (X,\sigma)$ is somewhat D(c,*b)-open function.

Theorem 5.5. Every somewhat *b-open function is somewhat pre open function
Proof: Let $f:(X,\tau) \to (Y,\sigma)$ be a somewhat *b-open function. Let $U \in \tau$ and $U \neq \phi$ .Since f is somewhat *b-open, there exists a *b-open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.But every *b-open set is pre open.Therefore there exists a pre open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$, which implies that f is somewhat pre open function.

Remark 5.6. Converse of the above theorem is not true.

Example 5.7. Let $X = \{a,b,c\}$, $\tau = \{X, \phi, \{a,b\}\}$, $Y = \{1,2,3\}$, $\sigma = \{Y, \phi, \{2,3\}\}$.
Define a function $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = 1; f(b) = 2; f(c) = 3$ .Then f is somewhat pre open function but not a somewhat *b-open function.

Theorem 5.8. For a function $f: X \to Y$, the following are equivalent
1) f is somewhat open
2) f is some what *b-open and somewhat D(c,*b)-open
Proof: To prove: (i)$\Rightarrow$(ii)Let $f: X \to Y$ be somewhat open function. Let $U \in \tau$ and $U \neq \phi$.Since f is somewhat open function, there exists an open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$. Since every open set is *b-open and D(c,*b)-set.Therefore there exists a*b-open and D(c,*b)-set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$, which implies that f is somewhat *b-open function and somewhat D(c,*b)-open function.

To prove: (ii)$\Rightarrow$(i)Let $f : X \to Y$ be somewhat *b-open and D(c,*b)-set. Let $U \in \tau$ and $U \neq \phi$.Since f is somewhat *b-open function and D(c,*b)-open function , there exists a *b-open and D(c,*b)-set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$. Since every *b-open and D(c,*b)-set is open .Therefore there exists an open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$, which implies that f is somewhat open function.

Theorem 5.9. Let $f : (X,\tau) \to (Y,\sigma)$ and $g : (Y,\sigma) \to (Z,\eta)$ be any two functions. If f is an open map and g is somewhat *b-open map, then $g \circ f : (X,\tau) \to (Z,\eta)$ is somewhat *b-open map.

Proof: Let $U \in \tau$. Suppose that $U \neq \phi$. Since f is an open map $f(U)$ is open and $f(U) \neq \phi$. Thus $f(U) \in \sigma$ and $f(U) \neq \phi$. Since g is somewhat *b-open map and $f(U) \in \sigma$ such that $f(U) \neq \phi$, there exists a *b-open set $V \in \eta$, $V \subseteq g(f(U))$, which implies that $g \circ f$ is somewhat *b-open function.

Theorem 5.10. Let $f : (X,\tau) \to (Y,\sigma)$ is a one-one and onto mapping , then the following are equivalent:

(i) f is somewhat *b-open map.

(ii) If C is a closed subset of X such that $f(C) \neq Y$, then there is a *b-closed subset D of Y such that $D \neq Y$ and $D \supseteq f(C)$.

Proof: To prove: (i)$\Rightarrow$(ii)

Let C be any closed subset of X such that $f(C) \neq Y$. Then $X - C$ is open in X and $X - C \neq \phi$. Since f is somewhat *b-open map, there exists a *b-open set $V \neq \phi$ in Y such that $V \subseteq f(X - C)$. Put $D = Y - V$. Then D is *b-closed in Y. We claim that $D \neq Y$. Suppose $D = Y$,then $V = \phi$ which is a contradiction. So that $D \neq Y$ .Since $V \subseteq f(X - C)$, $D = Y - V \supseteq Y - [f(X - C)] = f(C)$.

To prove: (ii)$\Rightarrow$(i)

Let U be any non-empty open set in X. Put $C = X - U$. Then C is a closed subset of X and $f(X - U) = f(C) = Y - f(U)$ implies $f(C) \neq \phi$ .Therefore, by (ii) there is a *b-closed subset D of Y such that $D \neq Y$ and $f(C) \subset D$. Put $V = Y - D$. Clearly V is a *b-open set and $V \neq \phi$ .Since $V = Y - D \subseteq Y - f(C) = Y - [Y - f(U)] = f(U)$ .Hence f is somewhat *b-open map.

Theorem 5.11. Let $f : (X,\tau) \to (Y,\sigma)$ be somewhat *b-open function and A be any open subset of X. Then $f/A : (A, \tau/A) \to (Y,\sigma)$ is also somewhat *b-open function.

Proof: Let $U \in \tau/A$ such that $U \neq \phi$. Since U is open in A and A is open in (X,τ), U is open in (X,τ) and since by hypothesis $f : (X,\tau) \to (Y,\sigma)$ is somewhat *b-open function, there exists a *b-open set V in Y, such that $V \subseteq f(U)$. Thus, for any open set

$U \in (A, \tau/A)$ with $U \neq \phi$, there exists a *b-open set V in Y such that $V \subseteq f(U)$. Hence f/A is somewhat *b-open function.

Theorem 5.12. Let (X, τ) and (Y, σ) be any two topological spaces, $X = A \cup B$ where A and B are open subsets of X and $f : (X, \tau) \to (Y, \sigma)$ be a function such that f/A and f/B are somewhat *b-open, then f is also somewhat *b-open function.

Proof: Let U be any open subset of (X, τ) such that $U \neq \phi$. Since $X = A \cup B$,either $A \cap U \neq \phi$ or $B \cap U \neq \phi$ or both $A \cap U \neq \phi$ and $B \cap U \neq \phi$. Since U is open in (X, τ) , U is open in $(A, \tau/A)$ and $(B, \tau/B)$.

Case: (i) Suppose that $U \cap A \neq \phi$ where $U \cap A$ is open in τ/A. Since by hypothesis f/A is somewhat *b-open function , there exists a *b-open set $V \in (Y, \sigma)$ such that $V \subseteq f(U \cap A) \subseteq f(U)$, which implies that f is somewhat *b-open function.

Case: (ii) Suppose that $U \cap B \neq \phi$ where $U \cap B$ is open in τ/B. Since by hypothesis f/B is somewhat *b-open function , there exists a *b-open set $V \in (Y, \sigma)$ such that $V \subseteq f(U \cap B) \subseteq f(U)$, which implies that f is somewhat *b-open function.

Case: (iii) Suppose that both $U \cap A \neq \phi$ and $U \cap B \neq \phi$ .From case (i) and (ii) , f is somewhat *b-open function.

Conclusion

As an extension of this paper, Somewhat **b-continuous functions and Somewhat **b-open functions can be defined and can obtain theorems based on the above defined concepts.

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