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Brief note
The General Classification of the Modular Group of the Structure $M_pn \times C_p$

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Abstract. In this paper, the classification of finite p-groups is extended to the group of the modular structure $M_pn\times C_p$, and the number of distinct subgroups were computed, making an entire classification of the given structure possible for any given prime p

Keywords: Finite p-Groups, Nilpotent Group, Fuzzy subgroups, Dihedral Group, Inclusion-Exclusion Principle, Maximal subgroups. Explicit formulae, non-cyclic subgroup, prime.

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1. Introduction

Let $h(G)$ be the number of chains of subgroups of G which ends in G. This actually represents the distinct number of the fuzzy subgroups of G (see [5]). Suppose that M_1, M_2, ..., M_t are the maximal subgroups of G The method of computing $h(G)$ is based on the application of the Inclusion-Exclusion Principle. This had been extensively discussed in our article [1]. Following our paper [1] (Also see [3] and [4]) the following equation (x) based on the usual Inclusive-Exclusive technique is applied :

$$h(G) = 2\left(\sum_{r=1}^{t} h(M_r) - \sum_{1\le r_1<r_2\le t} h(M_{r_1}\cap M_{r_2}) + \cdots + (-1)^{t-1} h\left(\bigcap_{r=1}^{t} M_r\right)\right) \quad (x)$$

The fuzzy subgroup for the nilpotent group of the form: $M_pn \times C_p$**.**

We approach this particular concept from two distinct perspectives namely; when $p = 2$ and in the subsequent case, $p > 2$. And, of course, p is a prime.

The nilpotent 2-group of the form $M_2n \times C_2$**.**

This case was already settled in one of our papers.

The derivation of $h(M_pn \times C_p)$ for $p > 2$

We begin with the case $p = 3$ and $n = 3$ By theorem (γ), there exists 13 distinct maximal subgroups for $M_33 \times C_3$.

By (x), we have:

$$\tfrac{1}{2}h(M_{3^3} \times C_3) = 3h(\mathbb{Z}_3 \times \mathbb{Z}_{3^2}) + 9h(M_{3^3}) + h(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3)$$

$$- 27h(\mathbb{Z}_{3^2}) - 12h(\mathbb{Z}_3 \times \mathbb{Z}_3) + 27h(\mathbb{Z}_3)$$

$$\therefore \quad h(M_33 \times C_3) = 420 + 2h(\mathbb{Z}_3 \times \mathbb{Z}_3 \times \mathbb{Z}_3),) = 736.$$

2. Determination of $h(M_pn \times C_2)$

Following a careful analysis and subsequent operations on the maximal subgroups, we have in general, an estimate given by:

$$\tfrac{1}{2}h(M_{p^n} \times C_p) = ph(\mathbb{Z}_p \times \mathbb{Z}_{p^{n-1}}) + p^2h(M_{p^n}) + h(\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}})$$
$$- p(p+1)h(\mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}}) - p^3h(\mathbb{Z}_{p^{n-1}}) + p^3h(\mathbb{Z}_{p^{n-2}})$$
$$\therefore \quad h(M_{p^n} \times C_p) = 2^{n-1}[p(p+1)(np+2) - p^3] + 2h(\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}})$$
$$= 2^{n-1}[p(p+1)(np+2) - p^3] + 2^{n-1}[(3n-5)p + (n^2-5)p^2 + (n^2-5n+8)p^3 + 4] - 4p^3$$
$$= 2^{n-1}[p^2(1+p)n^2 + p(3+p-4p^2)n + (7p^3 - 3p^2 - 3p + 4)] - 4p^3.$$

Theorem 1. Let $G = M_pn \times C_2$, the modular nilpotent group formed by taking the cartesian product of the modular p-group of order p^n and a cyclic group of order p, where p is a prime. Then, the number of distinct fuzzy subgroups of G for $n > 4$ is given by:

$$h(g) = \begin{cases} 2^n\left[3\left(n + \frac{5}{6}\right) - \frac{73}{12}\right], & p = 2 \\ 2^{n-1}[p^2(1+p)n^2 + p(3+p-4p^2)n + (7p^2 - 3p^2 - 3p + 4)] - 4p^3, & \text{for } p > 2 \end{cases}$$

Proof: For all values of p, there exist only one maximal subgroup which is isomorphic to the abelian type: $\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}}$, p of the maximal subgroups are isomorphic to : $\mathbb{Z}_p \times \mathbb{Z}_{p^{n-1}}$, while p^2 of them are isomorphic to M_pn. We have :

$$\frac{1}{2}h(M_{p^n} \times C_2) = 2^{n-2}[p(p+1)(np+2) - p^3] + h(\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}})$$
$$\therefore \ h(M_{p^n} \times C_p) = 2^{n-1}[p(p+1)(np+2) - p^3] + 2h(\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_{p^{n-2}})$$

We now have $h(M_pn \times C_2) = 2^{n-1}[p(p+1)(np+2) - p^3] + 2^{n-1}[(3n-5)p + (n^2-5)p^2 + (n^2 - 5n + 8)p^3 + 4] - 4p^3 = 2^{n-1}[p^2(1 + p)n^2 + p(3 + p - 4p^2)n + (7p^3 - 3p^2 - 3p + 4)] - 4p^3$. And $h(M_2n \times C_2) = 2^n(3n^2 + 5n - 4)$, for $n > 4, p = 2$

3. Conclusion

Finally, the general classification for the nilpotent p-groups of the specified modular structure given by: $M_pn \times C_p$ is thus hereby clearly made with the number of distinct fuzzy subgroups explicitly computed for all prime p and every non zero integer $n \geq 3$.

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