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2-Dimensional Subgroups of the Quasidihedral Group of Order 2^n with a Cyclic Group of Order 2

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Abstract. In this paper, the classification of finite p-groups is extended to the cartesian product of the quasidihedral group of order 2^n with a cyclic group of order 2.

Keywords: Finite p-Groups, Nilpotent Group, Fuzzy subgroups, Quasidihedral Group, Inclusion-Exclusion Principle,Maximal subgroups.Explicit formulae, non-cyclic subgroup

AMS Mathematics Subject Classification (2010): 20D15, 20E28, 20F18, 20N25, 20K27

1. Introduction

Denote by $h(G)$ the number of chains of subgroups of G which ends in G.This actually represents the distinct number of the fuzzy subgroups of G. Suppose that $M_1, M_2, \ldots, M_t$ are the maximal subgroups of G The method of computing $h(G)$ is based on the application of the Inclusion-Exclusion Principle. This had been extensively discussed in our article[1] Following our paper[1] the following equation(*) based on the usual Inclusive-Exclusive technique is appliied :

$$h(G) = 2\left(\sum_{r=1}^{t} h(M_r) - \sum_{1\le r_1<r_2\le t} h(M_{r_1}\cap M_{r_2}) + \cdots + (-1)^{t-1} h\left(\bigcap_{r=1}^{t} M_r\right)\right) \tag{1}$$

For more algebraic structures see [7-10].

2. The Cartesian product of the quasidihedral group of order 2^n and a cyclic group of order 2

The Quasidihedral (otherwise known as the semidihedral) group of order 2^n has the general structure of the form given by:

$$S_{2^n} = \langle x, y \mid x^{2^{n-1}} = y^2 = 1,\ \ yxy = x^{-1+2^{n-2}} \rangle, n > 3$$

There also exist seven distinct maximal subgroups for this structure.

Since $S_{2^3} \cong D_{2^3}$, we begin from the case, $n = 4$.

Set $G = S_{2^4} \times C_2$.

$$\frac{1}{2}h(G) = h(Z_2 \times Z_{2^3}) + 4h(S_{2^4}) + h(D_{2^3} \times C_2) + h(Q_{2^3} \times C_2) - 4h(Z_{2^3})$$
$$- 2h(Z_2 \times Z_{2^2}) - 4h(Q_{2^3}) - 4h(D_{2^3}) + 8h(Z_{2^2})$$
$$= 2^4 + 4(3)(2^5) + 432 + 176 - 4(16) - 4(32)$$

$\therefore \quad h(S_{2^4} \times C_2) = 2 \times 816 = 1632.$

3. Number of fuzzy subgroup for $S_2^n \times C_2$ in general

$$h(S_{2^n} \times C_2) = 2[2^n + 4h(S_{2^n}) + h(D_{2^{n-1}} \times C_2) + h(Q_{2^{n-1}} \times C_2) - (3)2^{2(n-1)}]$$
$$+ 2^{2n-3}(4n-5) - 2^n + 2^{-1}\sum_{k=1}^{n-3} 2^{n+k}$$
$$= 2^{2n}\left(\frac{3}{2} + \frac{6n-5}{4}\right) + 2^n \sum_{k=1}^{n-3} 2^k \quad = 2^{2n}\left(\frac{6+6n-5}{4}\right) + 2^n \sum_{k=1}^{n-3} 2^k$$
$$= 2^{2n-2}(6n+1+1) - 2^{n+1} = 2^{2n-2}(6n+2) - 2^{n+1}$$
$$= 2^{2n-1}(3n+1) - 2^{n+1}$$

Theorem 1. Let G be a nilpotent group of the quasidihedral group of order 2^n and a cyclic group of order 2, that is , $G = SD_{2^n} \times C_2$. Then, the number of distinct fuzzy subgroups of G is given by

$$h(G) = 2^{2n-1}(3n+1) - 2^{n+1}, \text{ for } n > 3$$

Proof: While 2^2 of the maximal subgroups are isomorphic to S_{2^n}, each of the remaining three is isomorphic to $Z_2 \times Z_{2^{n-1}}$, $D_{2^{n-1}} \times C_2$, and $Q_{2^{n-1}} \times C_2$, respectively.Using this on equation(3.2.1), we have the following:

$$h(SD_{2^n} \times C_2) = 2[2^n + 4h(S_{2^n}) + h(D_{2^{n-1}} \times C_2) + h(Q_{2^{n-1}} \times C_2) - (3)2^{2(n-1)}]$$
$$= 2^{2n-1}(3n+1) - 2^{n+1},$$

Alternatively, set F(n) = $h(SD_{2^n} \times C_2)$, and by assuming the truth of

F(k) = $h(SD_{2^k} \times C_2) = 2^{k+1} + 8h(SD_{2^k} + 2h(SD_{2^{k-1}} \times C_2)$
$+2h(Q_{2^{k-1}} \times C_2) - 3.\,2^{2(k-1)} + 1,$

we show the truth of F(k+1).

Thus,

F(k+1) = $h(SD_{2^{k+1}} \times C_2) = 2^{k+2} + 8h(SD_{2^{k+1}} + 2h(SD_{2^k} \times C_2)$
$+2h\big(Q_{2^k} \times C_2\big) - 3.\,2^{2k+1}$
$= 2^{2k+1}(3k+4) - 2^{k+2}$, which is true.

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