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Fuzzy γ-Hyper Connectedness in Fuzzy Topological Spaces

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Abstract. In this paper we introduce the concept of fuzzy γ-hyper connected space with the help of fuzzy γ-connected sets .Some basic theorems and results of these spaces are also discussed.

Keywords: Fuzzy γ-open set; Fuzzy γ-connected; fuzzy P-space; fuzzy Baire space; fuzzy hyper connected spaces.

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

The concept of fuzzy sets which formed the backbone of fuzzy mathematics was first introduced by Zadah [20] in his classical paper in the year of 1965. The theory of fuzzy topological spaces was introduced and developed by Chang [4] in 1968. H Since then much attention has been imply to develop and generalize the fundamental concepts of general topology in fuzzy setting by many authors. Thus a modern theory of fuzzy topology has been developed. In recent fuzzy topology has been found to be very useful in solving many industrial problems. In 2000, [9] Naschie showed that notion of fuzzy topology be relevant to Quatum particle physics.

The concept of fuzzy γ-open set was introduced by Hanfy et al. [8] in 1999 as a union of notation of fuzzy semi-open sets and fuzzy preopen sets. Hanafy introduced the fuzzy γ-open sets which are weaker than each of them. Using this notation, he studied fuzzy γ-continuous mapping on fuzzy topological space. In 2015, Thangaraj et al. [12] discussed fuzzy hyper- connected in fuzzy topological space and studied its properties.

We introduce and study the concepts of fuzzy γ-hyper connectedness by using fuzzy γ-connected sets in fuzzy topological space. Several characterizations of fuzzy γ-hyper connectedness in fts in terms of fuzzy G_σ sets, fuzzy γ-first category sets and fuzzy γ-no where dense sets are also established in this paper. Also we discuss some theorems and results on this spaces.

2. Preliminaries

Some basic definitions and properties which will be needed are recalled in this section. In this paper we use the notion of a fuzzy topology in the original sense of Chang (1968). Throughout this paper by (X,τ) we mean a fuzzy topological space (fts, shortly) .

For a fuzzy set λ in a fts (X,τ), clλ, intλ, intclint λ will respectively stand for the fuzzy closure, fuzzy interior and interior closure interior of λ in (X,τ).

Definition 2.1. [8] A fuzzy subset λ of fuzzy topological space (X,τ) is said to be fuzzy-γ open set (res. fuzzy γ closed set) if

$$\lambda \leq \text{clint}\,\lambda \vee \text{intcl}\lambda \quad (\text{res. } \lambda \geq \text{clint}\,\lambda \wedge \text{intcl}\,\lambda\,)$$

Also, it may be write that the complement of fuzzy γ-open set is fuzzy γ-closed set. Any union (resp. intersection)of Fuzzy of γ-open (resp. fuzzy γ-closed) set is fuzzy γ-open(resp. fuzzy γ-closed.). But intersection of two fuzzy γ open sets need not be fuzzy γ open. The intersection of a fuzzy open set which is a crisp subset and a fuzzy γ open set is fuzzy γ open. The union of fuzzy closed set which is a crisp subset and fuzzy γ closed set is fuzzy γ closed.

Remark 2.2. It is obvious that every fuzzy open (resp. fuzzy closed) set is a fuzzy γ-open set (resp. fuzzy γ-closed set).
But the converse need not be true in general.

It is also clear that a fuzzy γ open set is weaker than the concepts of fuzzy semi open or fuzzy pre open set and stronger than concepts of fuzzy semi open set.

Proposition 2.3. [1] If μ and λ are two fuzzy subsets of a fts (X,τ) then
(i) $\text{cl}\,\mu \vee \text{cl}\,\lambda = \text{cl}(\mu \vee \lambda)$ and $\text{cl}\,\mu \wedge \text{cl}\,\lambda \supseteq \text{cl}(\mu \wedge \lambda)$
(ii) $\text{int}\,\mu \wedge \text{int}\,\lambda = \text{int}(\mu \wedge \lambda)$ and $\text{int}\,\mu \vee \text{int}\,\lambda \subseteq \text{int}(\mu \vee \lambda)$
(iii) $1\text{-int}\,\lambda = \text{cl}(1-\lambda)$
(iv) $1\text{-cl}\,\lambda = \text{int}(1-\lambda)$.

Definition 2.4. ([8] The γ-interior and γ-closure of a fuzzy set A in (X,τ) are denoted by γ-int(A) and γ-cl(A) respectively and are defined as
$\gamma\text{- int}(A) = \vee\{B: B \leq A, B$ is fuzzy γ-open set in X$\}$
$\gamma\text{- cl}(A) = \wedge\{C: C \geq A, C$ is fuzzy γ-closed set in X$\}$.
Here, let $\mu_A(x)$ and $\mu_B(x)$ be the membership function of every x in A and B respectively.Then a member of A is contained in a member of B which is denoted by A ≤ B iff $\mu_A(x) \leq \mu_B(x)$.

Proposition 2.5. [8] If λ is a subset of (X,τ) and λ' is its complement then
(i) γ-clλ'= $(\gamma\_int\lambda)'$
(ii) γ-intλ'= $(\gamma\_cl\lambda)'$.

Theorem 2.6. [8] i) Each fuzzy semi pre open set which is fuzzy closed is fuzzy γ-open.
ii) Each fuzzy γ-open set which is fuzzy closed is fuzzy semi open.

Definition 2.7. [11] i) A fuzzy set λ in a fuzzy topological space (X,τ) is called fuzzy dense if there exists no fuzzy closed set μ in (X,τ) such that $\lambda<\mu<1$. That is, cl(λ)=1.
(ii) A fuzzy set λ in a fts (X,τ) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,τ) such that $\mu < \text{cl}(\lambda)$, that is intcl(λ)=0.

Definition 2.8. [3] A fuzzy set λ in a fts (X,τ) is called a fuzzy G_δ-set in (X,τ) if $\lambda=\wedge^{\infty}_{i=1}(\lambda_i)$ where $\lambda_i\in\tau$ for $i\in\Lambda$.

Proposition 2.9. [1] For a family A of $\{\lambda_\alpha\}$ of fuzzy sets of a fts (X,τ), $\vee cl(\lambda_\alpha)\leq cl(\vee\lambda_\alpha)$. In case A is finite set, $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$.
Also $\vee int(\lambda_\alpha)\leq int(\vee\lambda_\alpha)$ in (X,τ).

Definition 2.10. [113] A fts (X,τ) is called a fuzzy Baire space if $int(\vee^{\infty}_{i=1}(\lambda_i))=0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,τ).

Definition 2.11. [13] A fuzzy set λ in fts(X,τ) is called a fuzzy first category set if $\lambda = \vee^{\infty}_{i=1}(\lambda_i)$, where (λ_i)'s are nowhere dense sets in (X,τ). Any other fuzzy set in (X,τ) is said to be of fuzzy second category.

Definition 2.12. [11] A fts (X,τ) is called a fuzzy first category set if $\vee^{\infty}_{i=1}(\lambda_i)=1_X$, where (λ_i)'s are nowhere dense sets in (X,τ).
Any other fts (X,τ) is said to be of fuzzy second category.

Theorem 2.13. [13] If λ is a fuzzy dense set and fuzzy G_δ set in a fuzzy topological space (X,τ) , then $1-\lambda$ is a fuzzy first category set in (X,τ).

Definition 2.14. [10] A fts (X,τ) is called fuzzy P-space if every non zero fuzzy G_δ-set in (X,τ) is fuzzy open in (X,τ). That is, if $\lambda = \wedge^{\infty}_{i=1}(\lambda_i)$ where $\lambda_i\in\tau$ for $i\in\Lambda$.

Definition 2.15. [16] A fts (X,τ) is called a fuzzy Volterra space if $cl(\wedge^{N}_{i=1}(\lambda_i) =1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,τ) .

Definition 2.16. [5] A fuzzy topological space X is said to be fuzzy hyper connected if every non null fuzzy open subset of X is fuzzy dense in X. That is, a fuzzy topological space (X,τ) is fuzzy hyper connected if $cl(\mu_i) =1$, for all $\mu_i\in\tau$.

3. Fuzzy γ-hyper connected space

Definition 3.1. A fuzzy topological space X is called fuzzy γ-connected if there is no proper fuzzy set of X, which is both fuzzy γ-open and fuzzy γ-closed.

Definition 3.2. A fuzzy topological space (X,τ) is called fuzzy γ-hyper connected if every non null fuzzy γ-open subset of X is fuzzy dense set. That is, a fts (X,τ) is fuzzy γ-hyper connected if for every non null fuzzy γ-open subset μ of X, $cl(\mu) =1$.

Proposition 3.3. A fuzzy topological space (X,τ) is fuzzy γ-hyper connected for every non null fuzzy γ-open subset λ of X , if $clint\,\lambda \vee clintcl\,\lambda \leq 1$.

Proof. λ be a fuzzy subset of (X,τ). For every non null fuzzy γ-open subset λ

$$\Lambda \leq clint\,\lambda \vee intcl\,\lambda$$

Since λ is γ-hyper connected in (X,τ)

$\therefore cl\lambda=1$

$$cl\,\lambda \leq cl\,(clint\,\lambda \vee intcl\,\lambda)$$

$1 = cl\,(clint\,\lambda \vee intcl\,\lambda)$
$\Rightarrow clclint\,\lambda \vee clintcl\,\lambda = 1$ [λ is finite fuzzy open set, $cl(\vee\lambda) \geq \vee cl\,\lambda$]
$\Rightarrow clint\,\lambda \vee clintcl\,\lambda \leq 1$.

Proposition 3.4. A fts (X,τ) is fuzzy γ-hyper connected for non null finite fuzzy γ-closed subsets μ of X if $clint\,\mu \wedge clintcl\,\mu \leq 1$.
Proof: Let μ is fuzzy γ-closed set.
Then by definition we have $\mu \geq clint\,\mu \wedge intcl\,\mu$
$1-\mu \leq 1-[clint\,\mu \wedge intcl\,\mu]$
$\therefore int(1-\mu) \leq int[1-(clint\,\mu \wedge intcl\,\mu)]$
$1-cl\,\mu \leq 1-cl[clint\,\mu \wedge intcl\,\mu]$
since μ is hyper connected, $cl\,\mu=1$ and $cl(\wedge\mu_i) \subseteq \wedge cl\,\mu_i$
$\therefore 1-1 \leq 1-clclint\,\mu \wedge clintcl\,\mu$
$\Rightarrow 0 \leq 1-clint\,\mu \wedge clintcl\,\mu$
$\Rightarrow clint\,\mu \wedge clintcl\,\mu \leq 1$

Theorem 3.5. Arbitrary union of fuzzy γ-hyper connected subset of X is fuzzy γ-hyper connected set.
Proof: Let $\{\lambda_i : i \in \Lambda\}$ be collection of fuzzy γ-hyper connected sets of X.
Then, for each $i \in \Lambda$, we have λ_i
$\lambda_i \leq clint\,\lambda_i \vee intcl\,\lambda_i$
$\Rightarrow \vee\lambda_i \leq \vee(clint\,\lambda_i \vee intcl\,\lambda_i)$
$= (clint \vee \lambda_i) \vee (intcl \vee \lambda_i)$ [since $\vee cl\,\lambda_i \leq cl \vee \lambda_i$ and $\vee int\,\lambda_i \leq int \vee \lambda_i$]
Taking closure:
$cl(\vee\lambda_i) \leq cl((clint \vee \lambda_i) \vee (intcl \vee \lambda_i))$
$= [clclint(\vee\lambda_i) \vee (clintcl(\vee\lambda_i)]$ [inequality holds in respect of closure property]
$= clint(\vee\lambda_i) \vee clintcl(\vee\lambda_i)$
$\therefore clint(\vee\lambda_i) \vee clintcl(\vee\lambda_i) = 1$.
Imply that $(\vee\lambda_i)$ is a fuzzy γ-hyper connected in fts(X,τ).

Example 3.6. Let μ_1, μ_2, μ_3 be fuzzy sets on $X=[0,1]$ defined as

$\mu_1(x) = 0$ if $0 \leq x \leq \frac{1}{2}$
$= 2x-1$ if $\frac{1}{2} \leq x \leq 1$

$\mu_2(x) = 1$, if $0 \leq x \leq \frac{1}{4}$
$= -4x+2$, $\frac{1}{4} \leq x \leq \frac{1}{2}$
$= 0$, $\frac{1}{2} \leq x \leq 1$

$\mu_3(x) = 0$, if $0 \leq x \leq \frac{1}{4}$
$= \frac{4x-1}{3}$, $\frac{1}{4} \leq x \leq 1$

Consider $\tau = \{0, 1, \mu_1, \mu_2, \mu_3, \mu_1 \vee \mu_2\}$ on X. In (X,τ) μ_2, μ_3 and $\mu_1 \wedge \mu_2$ are fuzzy γ-open sets but $\mu_2 \wedge \mu_3$ is not fuzzy γ-open set.
Indeed $\mu_2 \wedge \mu_3 \nleq clint\,(\mu_2 \wedge \mu_3) \vee intcl((\mu_2 \wedge \mu_3)$,
But $\mu_1 \wedge \mu_2 \leq clint\,(\mu_2 \wedge \mu_1) \vee intcl((\mu_2 \wedge \mu_1)$. $\therefore \mu_1 \wedge \mu_2$ is fuzzy γ-open set
and $cl(\mu_2 \wedge \mu_1) = 0$
so, $\mu_1 \wedge \mu_2$ is not fuzzy γ-hyper connected set in (X,τ).
But μ_2, μ_3 are fuzzy γ-open set

And also clμ_2= 1, clμ_3=1.
Therefore, μ_2,μ_3 are fuzzy γ-hyper connected sets.

Definition 3.7. A fuzzy γ-open set λ in fts (X,τ) is called fuzzy γ dense if there exists no fuzzy γ-closed set μ in(X,τ) such that $\lambda<\mu<1$; That is cl(λ)=1.

Definition 3.8. A fuzzy γ-open set λ in fts (x,τ) is said fuzzy nowhere γ-dense if there exists no non zero γ-open setμ in (X,τ) such that $\mu<$ cl(λ). That is, intcl(λ)= 0.

Definition 3.9. A fuzzy γ-open set λ in fts(X,τ) is called fuzzy γ- $F_\sigma$set in (X,τ) if $\lambda=\vee^{\infty}_{i=1}(\lambda_i)$, where $1-\lambda_i\in\tau$ for $i\in I$,

Definition 3.10. A fuzzy γ-open set λ in fts(X,τ) is called fuzzy γ- $G_\delta$set in (X,τ) if $\lambda=\wedge^{\infty}_{i=1}(\lambda_i)$, where $\lambda_i\in\tau$ for $i\in I$.

Definition 3.11. A fts(X,τ) is called a fuzzy γ-Baire space if $int(\vee^{\infty}_{i=1}(\lambda_i))=0$, where (λ_i)'s are fuzzy nowhere dense sets in (X,τ).

Definition 3.12. A fuzzy γ-open set λ in a fts (X,τ) is fuzzy first category γ set if $\lambda=\vee^{\infty}_{i=1}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere γ-dense sets in (X,τ). Any other fuzzy γ set in (X,τ) is said to be fuzzy second category.

Definition 3.13. A fts (X,τ) is called γ-P space if countable intersection of fuzzy γ-open sets in (X,τ) is fuzzy γ-open . That is, every non zero fuzzy γ-G_δ-set in (X,τ) is fuzzy γ-open in (X,τ).

Theorem 3.14. If a fuzzy γ-P space (X,τ) is a fuzzy γ-hyper connected space, then (X,τ) is a fuzzy γ-Baire space
Proof: Here, let λ be a fuzzy γ- G_δ space in fuzzy γ-P space (X,τ).
Since, fts (X,τ) is fuzzy γ-hyper connected space, the fuzzy γ-open set λ is dense set in (X,τ)
i.e. Cl(λ) =1.
Also, we know, since λ is fuzzy γ-P space and γ-dense set,
Therefore, (1-λ) is first category set in (X,τ) .
$\therefore (1-\lambda)=\vee^{\infty}_{i=1}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,τ)
$int(\vee^{\infty}_{i=1}(\lambda_i))=int(1-\lambda)=1-cl(\lambda_i)=1-1=0$.
Since $int(\vee^{\infty}_{i=1}(\lambda_i)=0$. Where $\lambda_i$are fuzzy nowhere dense set,
⇒ (X,τ) is fuzzy γ-Baire space.

Theorem 3.15. If a fuzzy γ-P-space (X,τ) is a fuzzy γ-hyper connected space, then (X,τ) is a fuzzy γ-second category space.
Proof: Let the fuzzy γ P-space (X,τ) be a fuzzy γ-hyper connected space.By above theorem, it may write that (X,τ) is fuzzy γ-Baire space.
$\therefore Int(\vee^{\infty}_{i=1}(\lambda_i)=0$. Where (λ_i)'s are no where dense sets in (X,τ)
We have to show that $\vee^{\infty}_{i=1}(\lambda_i)\neq.1$. Suppose$\vee^{\infty}_{i=1}(\lambda_i)=1$
This imply that

$\text{Int}[\vee^{\infty}_{i=1}(\lambda_i)]=\text{int}[1]=1\neq 0.$, a contradiction.
Hence we must have $\vee^{\infty}_{i=1}(\lambda_i)\neq.\ 1$, where (λ_i) are fuzzy γ-nowhere dense set in (X,τ).
Therefore (X,τ) is fuzzy γ-second category space.

Definition 3.16. A fuzzy topological space(X,τ) is called a fuzzy γ-Volterra space if $cl(\wedge^{N}_{i=1}(\lambda_i) = 1$,where (λ_i)'s are fuzzy γ-dense and fuzzy γ-G_δ sets in (X,τ). Otherwise, fts (X,τ) is fuzzy γ-voltera space if $clint(\wedge^{N}_{i=1}(\lambda_i) = 1$, where (λ_i)'s are fuzzy dense and G_δ set in (X,τ).

Theorem 3.17. If there are k fuzzy γ- G_δ sets, (λ_i)'s (i= 1 to k) in fuzzy γ- hyper connected and fuzzy γ- P-space (X,τ), then (X,τ) is a fuzzy γ- Volterra space.
Proof: (λ_i)'s are G_δ– sets in a fuzzy γ- hyper connected and fuzzy γ- P space(X,τ).
Since (X,τ) is fuzzy P-space, then $\wedge^{k}_{i=1}(\lambda_i)=\lambda$, is also a fuzzy γ- open set.
Since the fuzzy space (X,τ) is a fuzzy hyper connected space, the fuzzy open set λ in (X,τ) is a fuzzy dense set. That is, $cl\lambda=1$
Hence λ is a fuzzy G_δ set and a fuzzy dense set in (X,τ).
Therefore, $(1-\lambda)$ is ist category n (X,τ).
$\therefore (1-\lambda)=\vee^{\infty}_{i=1}(\lambda_i)$, λ_i is nowhere dense set in X.
Then $\text{int}(\vee^{\infty}_{i=1}(\lambda_i))=\text{int}(1-\lambda)=1-cl\lambda=1-1=0$.
Therefore, (X,τ) is a fuzzy γ-Volterra space.

4. Conclusion

In this paper, we introduce and study a new concept fuzzy γ -hyper-connectedness in fuzzy topological spaces. The notion of fuzzy open sets are fundamental structure of fuzzy topology. Fuzzy γ- open sets form a link between pre open sets and semi pre-open sets.

This work may be extended to fuzzy strongly γ-connectedness and totally fuzzy γ-disconnectedness with the concepts of fuzzy hyper γ-connectedness spaces. The fuzzy γ-hyper-connectedness may play an important role in computation of fuzzy topology and may have applications in quantum particle physics and quantum gravity, particularly in connection with string theory.

REFERENCES

1. K.K.Azad, On fuzzy semicontinuity,fuzzy almost continuity and fuzzy weakly continuity, *J. Math. Anal. Appl*., 82 (1981) 14-32.
2. G.Balasubramanian and V.Chandrasekar, Fuzzy α-connectedness and fuzzy α-disconnectedness in fuzzy topological spaces, *Math. Bechnk, UDK,* 56, 47-55.
3. G.Balasubramanian, Maximal fuzzy topologies, *Kybernetika*, 31 (1995) 459-464.
4. C.L.Chang, Fuzzy topological spaces, *J. Math. Anal. Appl*., 24 (1968) 182-190.
5. M.Caldas, G.Navalagi and R.Saraf, On fuzzy weakly semi-open functions, *Proyeccions -Revistade Mate*. Chile, 21 (2002) 51-63.
6. D.De, Fuzzy generalized γ-closed set in fuzzy topological spaces, *Annals of Pure and Applied Mathematics*, 7 (2014) 104-109.
7. D.De, On fuzzy α-hyper-connectedness in fuzzy topological spaces, IFSA-SCIS 2017, Otsu, Japan.
8. I.M.Hanafy, Fuzzy γ-open sets and fuzzy γ-continuity, *J. Fuzzy Mathematics*, l7 (2) (1999) 419-430.

9. M.S.El.Naschie, On the certifcation of heteroyic strings, M theory and $\in^{\infty}$theory, *Chaos, Solutions and Fractals*, (2000) 2397-2408.
10. G.Thangaraj and G.Balsubramunian, On fuzzy basically disconnected spaces, *J. Fuzzy Math*., 9 (2001) 103-110.
11. G.Thangaraj and G.Balsubramunian, On somewhat fuzzy continuous functions," *J. Fuzzy Math*. 11 (2003) 725-736.
12. G.Thangaraj and C.Anbazhagan, Some remarks on fuzzy P-spaces, *Gen. Math. Notes*, 26 (2015) 8-16.
13. G.Thangaraj and S.Anjalmose, On fuzzy Baire spaces, *J. Fuzzy Math*, 21 (2013) 667-676.
14. T.Senapati, C.Jana, M.Bhowmik and M.Pal, L-fuzzy G-subalgebras of G-algebras, *Journal of the Egyptian Mathematical Society*, 23 (2) (2015) 219-223.
15. G.Thangaraj and S.Soundararajan, On fuzzy regular volterra spaces, *J. Computational Mathematica*, 4(1) (2020) 17-25.
16. G.Thangaraj and S.Soundararajan, On fuzzy Volterra spaces, *J. Fuzzy Math*., 21 (2013) 895.
17. G.Thangaraj and C.Anbazhagan and P.Vivakanandan, On fuzzy P-spaces, weak fuzzy P-spaces and almost P-spaces, *Gen. Math. Notes*, 18(2) (2013) 128-139.
18. M.Bhowmik and M.Pal, Some results on intuitionistic fuzzy matrices and intuitionistic circulant fuzzy matrices, *International Journal of Mathematical Sciences*, 7 (1-2) (2008) 177-192.
19. A.K.Shyamal and M.Pal, Distances between intuitionistics fuzzy matrices, *VU. J. Physical Sciences*, 8 (2002) 81-91.
20. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.