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A New Ranking Method for Solving Hexadecagonal Fuzzy Assignment Problem

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Abstract. Assignment problem is a special case of linear programming problem. The objective of the optimal assignment is to minimize the total cost or maximize the profit. Fuzzy set theory has been applied in many fields of Science, Engineering and Management. In this paper a new ranking method is proposed for solving the Hexadecagonal fuzzy assignment problem. Fuzzy assignment problem transformed into crisp assignment problem and solved by Hungarian method. Numerical example is presented and the optimal solution is derived by using proposed method.

Keywords: Hexadecagonal Fuzzy number, Ranking Method, Fuzzy Assignment Problem

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

Assignment problem is the special case of linear programming problem. Assignment problem can be applied in all fields like Science, Engineering, and Management etc. Assignment problem plays an important role in industry and other applications. In assignment problem 'n' jobs are assigned 'n' persons depending on their efficiency to do the job. The objective of the optimal assignment is to minimize the total cost or maximize the profit. Fuzzy assignment problems have received great attention in the recent years. Here we investigate a more realistic problem, namely the assignment problem with fuzzy costs or times c_{ij}. The objective is to minimize the cost or to maximize the total profit, subject to some crisp constraints, the objective function is considered also as a fuzzy number.

Zadeh [25] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. Chen [2] proved some theorems and proved a fuzzy assignment model that considers all individuals to have same skills. Kumar and Gupta [1] were the first one who solved fuzzy assignment problem and travelling salesman problem with different membership function. Lin and Wen [6] solved fuzzy assignment problem using labeling algorithm. Rajarajeswari and Sahayasudha [14] were ranked the fuzzy numbers using centre of centroid method. Wang and Lee [23] proposed the revised method of ranking fuzzy numbers with an area between the centroid and original points.

Sujatha and Elizabeth [19] solved the fuzzy transportation and fuzzy unbalanced assignment problem using one point method. Jahirhussain and Jayaraman [4] solved fuzzy assignment problem using robust ranking method. Pavithra and Jenita [13] proposed a new method for solving a dodecagonal fuzzy assignment problem. S. Manimaran and Ananthanarayanan [7] were discussed a comparative study of two fuzzy numbers using average method. Thorani and Shankar [21] were studied the applications of fuzzy assignment problems

Dinagar and Kamalanathan [18] solved a fuzzy assignment problem with two ranking methods. Pandian and Kavitha [12] solved the assignment problems using parallel moving method. Nagoor Gani and Mohamed [10] proposed a new method for solving assignment problem for trapezoidal fuzzy numbers. Namarta, Thakur and Gupta [20] ranked the fuzzy numbers using centre of the centroids. Mehta, Thakur and Kaur [8] have approached a numerical method to find the solution of assignment problem. Selvi, Mary and Velammal [15] were finding the solution of assignment problem using magnitude method. Pal Singh, Mehta and Thakur [16] ranked the dodecagonal fuzzy number using value and ambiguity index. Srinivasan and Geetharamani [17] were solved fuzzy assignment method using ones assignment method. Thorani and Shankar [22] solved fuzzy assignment problem with generalized fuzzy number. Kar at al. [5] were finding solution of generalized fuzzy assignment problem with restriction on the cost of both job and person under fuzzy environment.

Choudhary et al. [3] were applied a branch and bound technique for solving fuzzy assignment problem. Nagoor Gani and Pareeeth [11] were finding the dual and primal solution for solving linear sum feasible fuzzy assignment problem. Muruganandhan and Hema [9] solved fuzzy assignment method using one suffix method. Mohamed and Divya [24] were solved fuzzy travelling salesman problem with α- cut and ranking technique.

In this paper a new ranking method is proposed in solving a hexadecogonal fuzzy assignment problem. Fuzzy assignment problem can be converted into crisp assignment problem using ranking method and an optimal solution is obtained by using Hungarian method.

2. Preliminaries

Definition 2.1. A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval {0,1}

i.e. $A = \{x, \mu_A(x)\ ; x \in X\}$.Here $\mu_A(x) = 1$

Definition 2.2. A fuzzy set A of the universe of discourse X is called normal fuzzy set implying that there exist at least one $x \in X$ Such that $\mu_A(x) = 1$

Definition 2.3. The support of fuzzy set in the Universal set X is the set that contains all the elements of X that have anon- zero membership grade in $\tilde{A}$. i.e.

$$Supp(\tilde{A}) = \{x \in X / \mu_{\tilde{A}}(x) > 0\}$$

Definition 2.4. Given a fuzzy set A defined on X and any number α∈[0,1] the α-cut, α_A is the crisp set $\alpha_A = \{x \in X / A(x) \geq \alpha, \alpha \in [0,1]$

Definition 2.5. A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be fuzzy number if its membership function $\mu_A(x): R \to [0,1]$ has the following properties

(i) A must be a normal and convex fuzzy set

(ii) α_A must be a closed interval for every $\alpha \in (0,1]$

(iii) The support of $\tilde{A}$ must be bounded

Definition 2.6. A fuzzy number $\tilde{A}$ is called triangular function is denoted by $\tilde{A} = (a_1, a_2, a_3)$ whose membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & a_2 \leq x \leq a_3 \\ 0 & x > a_3 \end{cases}$$

Definition 2.6. A fuzzy number $\tilde{A}$ is called trapezoidal function is denoted by $\tilde{A} = (a_1, a_2, a_3, a_4)$ whose membership function is defined as follows

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & a_3 \leq x \leq a_4 \\ 0 & x > a_3 \end{cases}$$

3. Hexadecagonal Fuzzy Number

In this section, Hexadecagonal Fuzzy number is defined.

A fuzzy number $\tilde{A}$ is a Hexadecagonal fuzzy number defined by

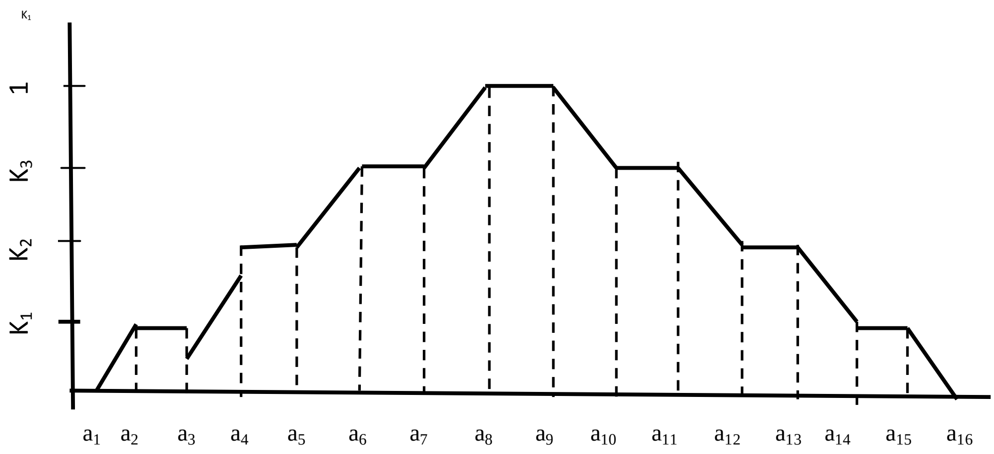
$\tilde{A}=(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16})$

where $a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16}$ are real numbers and its membership function is given by

$$\mu_A(x)=\begin{cases} 0 & x<a_1 \\ k_1\left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1\le x\le a_2 \\ k_1 & a_2\le x\le a_3 \\ k_1+\left(k_2-k_1\right)\left(\dfrac{x-a_3}{a_4-a_3}\right) & a_3\le x\le a_4 \\ k_2 & a_4\le x\le a_5 \\ k_2+\left(k_3-k_2\right)\left(\dfrac{x-a_5}{a_6-a_5}\right) & a_5\le x\le a_6 \\ k_3 & a_6\le x\le a_7 \\ k_3+\left(1-k_3\right)\left(\dfrac{x-a_7}{a_8-a_7}\right) & a_7\le x\le a_8 \\ 1 & a_8\le x\le a_9 \\ k_3+\left(1-k_3\right)\left(\dfrac{a_{10}-x}{a_{10}-a_9}\right) & a_9\le x\le a_{10} \\ k_3 & a_{10}\le x\le a_{11} \\ k_2+\left(k_3-k_2\right)\left(\dfrac{a_{12}-x}{a_{12}-a_{11}}\right) & a_{11}\le x\le a_{12} \\ k_2 & a_{12}\le x\le a_{13} \\ k_1+\left(k_2-k_1\right)\left(\dfrac{a_{14}-x}{a_{14}-a_{13}}\right) & a_{13}\le x\le a_{14} \\ k_1 & a_{14}\le x\le a_{15} \\ k_1\left(\dfrac{a_{16}-x}{a_{16}-a_{15}}\right) & a_{15}\le x\le a_{16} \\ 0 & a_{16}<x \end{cases}$$

where $0<k_1<k_2<k_3<1$

Figure 1: Graphical Representation of Hexadecagonal Fuzzy Numbers



3.1. Arithmetic operations on Hexadecagonal fuzzy number
In this section, Arithmetic operations on Hexadecagonal Fuzzy numbers are presented. Let

$\tilde{A}_{HXDFN} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ &

$\tilde{B}_{HXDFN} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16})$

be two the hexadecogonal fuzzy numbers. Then the addition, subtraction and the scalar multiplication of these fuzzy numbers can be performed as follows

$$\tilde{A}_{HXDFN} + \tilde{B}_{HXFDN} = [a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8, a_9 + b_9, a_{10} + b_{10}, a_{11} + b_{11}, a_{12} + b_{12}, a_{13} + b_{13}, a_{14} + b_{14}, a_{15} + b_{15}, a_{16} + b_{16}]$$

$$\tilde{A}_{HXFDN} - \tilde{B}_{HXDFN} = [a_1 - b_{16}, a_2 - b_{15}, a_3 - b_{14}, a_4 - b_{13}, a_5 - b_{12}, a_6 - b_{11}, a_7 - b_{10}, a_8 - b_9, a_9 - b_8, a_{10} - b_7, a_{11} - b_6, a_{12} - b_5, a_{13} - b_4, a_{14} - b_3, a_{15} - b_2, a_{16} - b_1]$$

$$\lambda\tilde{A}_{HXDFN} = [\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7, \lambda a_8, \lambda a_9, \lambda a_{10}, \lambda a_{11}, \lambda a_{12}, \lambda a_{13}, \lambda a_{14}, \lambda a_{15}, \lambda a_{16}]$$

$$\lambda\tilde{B}_{HXDFN} = [\lambda b_1, \lambda b_2, \lambda b_3, \lambda b_4, \lambda b_5, \lambda b_6, \lambda b_7, \lambda b_8, \lambda b_9, \lambda b_{10}, \lambda b_{11}, \lambda b_{12}, \lambda b_{13}, \lambda b_{14}, \lambda b_{15}, \lambda b_{16}]$$

3.2. Measure of fuzzy number
In this section, measure of fuzzy number is defined.

The measure of $\tilde{A}_\omega$ is a measure is a function $M_o : R_\omega(I) \to R^+$ which assign a non-negative real numbers $M_o^{HXDFN}\left(\tilde{A}_\omega\right)$ that expresses the measure of

$$M_o^{HXDFN}\left(\tilde{A}_\omega\right) = \frac{1}{2}\int_\alpha^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{k_3}\left(h_1(t) + \bar{h}_1(t)\right)dt + \frac{1}{2}\int_{k_3}^{\omega}\left(l_1(w) + \bar{l}_1(w)\right)dw$$

where $0 \le \alpha < 1$

4. Ranking method
In this section, a new ranking method is defined to convert the Hexadecagonal fuzzy number to a crisp number.

Let $\tilde{A}$ be a normal Hexadecagonal fuzzy number. The measure of $\tilde{A}$ is calculated as follow

$$M_o^{HXDFN}\left(\tilde{A}_\omega\right) = \frac{1}{2}\int_0^{k_1}\left(f_1(r) + \bar{f}_1(r)\right)dr + \frac{1}{2}\int_{k_1}^{k_2}\left(g_1(s) + \bar{g}_1(s)\right)ds + \frac{1}{2}\int_{k_2}^{k_3}\left(h_1(t) + \bar{h}_1(t)\right)dt + \frac{1}{2}\int_{k_3}^{1}\left(l_1(w) + \bar{l}_1(w)\right)dw$$

$$M_o^{HXDFN}(\tilde{A}) = \frac{1}{4}\begin{Bmatrix} (a_1+a_2+a_{15}+a_{16})k_1 + (a_3+a_4+a_{13}+a_{14})(k_2-k_1) \\ +(a_5+a_6+a_{11}+a_{12})(k_3-k_2) + (a_7+a_8+a_9+a_{10})(1-k_3) \end{Bmatrix} \quad (4.1)$$

$$where\ 0 < k_1 < k_2 < k_3 < 1$$

5. Mathematical formulation assignment problems

In this section, the mathematical formulation of the assignment problem, fuzzy assignment problem and the conversion of assignment problem to crisp assignment problem are presented.

5.1. Mathematical statement

The assignment problem can be stated in the form of $n \times n$ cost matrix c_{ij} of real numbers as follows:

Persons	Jobs			
	1	2	----	n
1	c_{11}	c_{12}		c_{1n}
2	c_{21}	c_{22}		c_{2n}
---	---	---	--	---
n	c_{n1}	c_{n2}		c_{mn}

The mathematical form of assignment problem is given by

$$minimize \quad Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1,\ i = 1,2,...n \qquad and$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ j = 1,2,...n$$

where x_{ij} is the decision variable

$$x_{ij} = \begin{cases} 1\, if\ the\, i^{th}\ person\, is\, assigned\, to j^{th}\ job \\ 0, otherwise \end{cases} \quad (5.1)$$

c_{ij} is the assignment cost of j^{th} job to i^{th} person. The objective is to minimize the total cost (or time) of assigning all the jobs to the available persons (one job to one person).

5.2. Mathematical formulation of fuzzy assignment problem

When the costs or time $\tilde{c}_{ij}$ are fuzzy numbers, then the total cost becomes a fuzzy number.

The assignment problem can be stated in the form of $n \times n$ cost matrix $\tilde{c}_{ij}$ of fuzzy numbers as follows:

Persons	**Jobs**			
	1	**2**	**----**	**n**
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$		$\tilde{c}_{1n}$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$		$\tilde{c}_{2n}$
---	**---**	**---**	**--**	**---**
n	$\tilde{c}_{1n}$	$\tilde{c}_{2n}$		$\tilde{c}_{mn}$

The mathematical formulation of the fuzzy assignment problem is given by

$$\min imize\ \tilde{Z}^* = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}{}^*$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1,\ i = 1,2,...n \qquad and$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ j = 1,2,...n$$

where x_{ij} is the decision variable and

$$x_{ij} = \begin{cases} 1 if\ the\ i^{th}\ person\ is\ assigned\ to j^{th}\ job \\ 0, otherwise \end{cases} \tag{5.2}$$

$\tilde{c}_{ij}$ is the fuzzy assignment cost of j^{th} job to i^{th} person. Hence it cannot be solved directly. For solving the problem, we first defuzzify the fuzzy cost coefficients into crisp ones by the above fuzzy number ranking method (4.1).

5.3. Proposed ranking method to convert fuzzy assignment problem to crisp assignment problem

Using our ranking method first we convert the model (5.2) to get the minimum value $\tilde{Z}^*$ as follows:

$$\Re\left(\tilde{Z}^*\right) = \min imize\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n} \Re\left(\tilde{c}_{ij}\right) x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1,\ i = 1,2,...n \qquad and$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ j = 1,2,...n$$

where x_{ij} is the decision variable and

$$x_{ij} = \begin{cases} 1 if\ the\ i^{th}\ person\ is\ assigned\ to j^{th}\ job \\ 0, otherwise \end{cases} \tag{5.3}$$

$\widetilde{c}_{ij}$ is the fuzzy assignment cost of j^{th} job to i^{th} person.The objective is to minimize the total cost (or time) of one job to one person.

5.4. Working rule for finding optimal solution of Hexadecagonal fuzzy assignment problem

The following steps to be followed to find the optimal solution of given Hexadecagonal fuzzy assignment problem

5.4.1. Working rule for solving Hexadecagonal fuzzy assignment problem

Step 1: First check whether the given fuzzy assignment problem (i.e the cost matrix is in fuzzy form) is balanced or not. If it is balanced (i.e number of jobs is equal to number of machines) then go to the step 4. If it is unbalanced (i.e number of jobs is not equal to number of machines) then go to step 2

Step 2: To convert the unbalanced assignment problem into balanced assignment problem, we introduce a dummy row (or column) and cost of that row (or column) is fuzzy zero cost

Step 3: Apply the ranking method in the each fuzzy cost matrix for converting the fuzzy cost into crisp cost using the proposed ranking method (4.1).

Step 4: After converting it into the form (5.1) we apply the following Hungarian method to find the optimum solution.

5.4.2. Hungarian algorithm

Step 1: First check whether the given assignment problem is balanced or not. If it is balanced (i.e. number of jobs is equal to number of machines) then go to the step 4. If it is unbalanced (i.e. number of jobs is not equal to number of machines) then go to step 2.

Step 2: To convert the unbalanced assignment problem into balanced assignment problem, we introduce a dummy row (or column) and cost of that row (or column) is zero cost.

Step 3: Identify the minimum cost of each row in the cost matrix and subtract that cost into from the other cost in that corresponding rows. The result should be atleast one zero in each row.

Step 4: Identify the minimum cost in each column of cost matrix which obtained from step 4 and subtract that cost into the other costs in that corresponding columns. The result should be atleast one zero in each column. Then go to step 5

Step 5: After the completion step 5, nowwe start the procedure for optimal assignment.

(a) Identify the rows successively with single zero. Encircle ($\bigcirc$) that zero and cross off ($\times$) all zeros in its column. Continue this procedure in each rows

(b) Repeat the same procedure in each column in that reduced cost matrix

Step 6:

(a) If the row and each column contain exactly one encircled zero, then the current assignment is optimal.

(b) If atleast one row (or column) is without an assignment, then the current assignment is not an optimal. Then go to step 8

Step 7: Cover all the zeros by drawing a minimum number of lines as follows

(a) Mark ($\surd$) the rows that do not have assignment

(b) Mark (√) the columns that have zeros in marked rows.
(c) Mark (√) the rows that have assignments in marked columns.
(d) Repeat (b) and (c) until no more marking is required.
(e) Draw lines through all unmarked rows and marked columns. This gives us the desired minimum number of lines

Step 8: Identify smallest cost of the reduced cost matrix which is not covered by any lines. Subtract this smallest cost element from all other uncovered elements and add this to all those elements which are lying at the intersection of lines and do not change the remaining elements which lie on the straight lines.

Step 9: Go to step 6 and repeat the same procedure until an optimum solution attained.

5.4.3. Novelties of the proposed ranking method (4.1)

Major part of fuzzy logic is ranking of the fuzzy numbers. Ranking of fuzzy number plays an important role in decision making problems and some other fuzzy application system. Fuzzy numbers must be ranked before an action is taken by a decision maker. Ranking methods which convert a fuzzy number to a crisp number by applying a mapping function. Many methods have been proposed to deal with ranking fuzzy numbers. The ranking method presented here is a new flexible one of the Hexadecagonal fuzzy numbers. Proposed ranking method (4.1) should not just compare two fuzzy alternatives, nor pick the best choice from the list. This proposed ranking method (4.1) is simple and easy to calculate rank of fuzzy numbers which also gives perfect solution to the given problem. This ranking method is used to rank the all the Hexadecagonal fuzzy numbers. The advantage of the proposed model is illustrated by examples.

5.4.4. Working procedure for the Hexadecagonal fuzzy assignment problems

Hexadecagonal fuzzy assignment problems is first converted into balanced crisp assignment problem using the proposedfuzzy ranking method(4.1).Hungarian algorithm is used to find the optimal allocation of the crisp model.

Once the optimum value x^* of the assignment problem (5.2) are found, the fuzzy objective value of $\tilde{Z}^*$ the Hexadecagonal fuzzy assignment problem can be calculated using $\tilde{Z}^* = \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}{}^*$.

Using the above algorithm (5.4), one can find the optimum solution of all the Hexadecagonal fuzzy assignment problems.

6. Numerical example

In this section, validity of the ranking method is explained by numerical examples.

Example 6.1. Consider the following Hexadecagonal fuzzy assignment problem which consists of four jobs and four machines. The cost matrix $\tilde{c}_{ij}$ whose elements are Hexadecagonal fuzzy numbers. Here our objective is to find the optimum assignment so as to minimize the cost (or time).

Table 1: Hexadecagonal fuzzy assignment problem

	M_1	M_2	M_3	M_4
J_1	(1,2,3,4,5,6,7,8, 9,10,11,12,13,14, 15,16)	(0,1,2,3,4,5,6,7, 8,9,10,11,12,13, 14,15)	(1,3,5,7,9,11,13,15,17,19,21,23,25, 27, 29,31)	(1,3,4,5,7,9,10,12,14,15,16,18,20, 21, 22 ,24)
J_2	(2,4,6,8,10,12,14,16,18,20,22,24,26,28, 30,32)	(2,3,5,7,9,11,13, 17,19,23,29,31,35,37, 41,43)	(1,4,7,10,13,16,19,22,25,28,31,34, 37, 40,43, 46)	(0,4,6,9,11,12,13, 14,15,16,19,21, 23, 25, 27,29)
J_3	(1,2,3,4,7,10,13,15,16,17,22,26,30,34, 35,36)	(2,4,6,8,9,13,15, 16,18,20,21,25,27,28, 30,31)	(1,2,3,4,5,7,9,11, 13,17,21,25,27,31,32, 34)	(2,4,8,9,11,13,16, 19,20,22,24,25, 27, 29, 30,31)
J_4	(1,2,3,4,8,9,10, 12,13,15,17,18, 20,22,23, 24)	(0,2,3,5,6,7,9,10,13,15,16,18,21, 24,29, 34)	(3,4,7,10,11,13,15,16,18,21,24,28, 30,34,36,44)	(1,2,4,5,7,8,10,11,13,14,16,17,19, 20, 22,23)

The mathematical formulation of the given fuzzy assignment problem is

Minimize $\tilde{Z}^*$ = $\Re$ (1,2,3,4,5,6,7,8,9,10,11,12,13,14, 15,16) x_{11} + $\Re$ (0,1,2,3,4,5,6,7, 8,9,10,11,12,13,14,15) x_{21} + $\Re$ (1,3,5,7,9,11,13,15,17,19,21,23,25,27, 29,31) x_{13} + $\Re$ (1,3,4,5,7, 9,10,12,14,15,16,18,20,21, 22,24) x_{14} + $\Re$ (2,4,6,8,10,12,14,16,18,20 22,24,26,28,30, 32) x_{21} + $\Re$ (2,3,5,7,9,11,13,17,19,23,29,31,35,37, 41,43) x_{22} + $\Re$ (1,4,7,10,13,16,19,22,25, 28,31,34,37,40,43,46) x_{23} + $\Re$ (0,4,6,9,11,12,13,14,15,16, 19,21,23,25,27,29) x_{24} + $\Re$ (1,2,3,4,7,10,13,15,16,17,22,26,30,34,35,36) x_{31} + $\Re$ (2,4,6,8,9,13,15,16,18,20,21,25,27,28, 30,31) x_{32} + $\Re$ (1,2,3,4,5, 7,9,11,13,17, 21,25,27,31,32, 34) x_{33} + $\Re$ (2,4,8,9,11,13, 16,19, 20,22,24,25,27,29, 30,31) x_{34} + $\Re$ (1,2,3,4,8,9,10,12,13,15,17,18,20,22,23, 24) x_{41} + $\Re$ (0,2,3,5,6,7,9,10,13,15,16 ,18,21, 24,29, 34) x_{42} + $\Re$ (3,4,7,10,11,13,15,16,18,21,24,28,30,34,36,44) x_{43} + $\Re$ (1,2,4,5,7,8,10,11, 13,14,16,17,19,20, 22,23) x_{44}

Subject to the constraints

$$\sum_{i=1}^{4} x_{ij} = 1 \text{ and}$$

$$\sum_{j=1}^{4} x_{ij} = 1$$

where x_{ij} is the decision variable and

$$x_{ij} = \begin{cases} 1 \, if \; the \, i^{th} \; person \, is \, assigned \, to j^{th} \; job \\ 0, otherwise \end{cases} \quad (5.4)$$

$\tilde{c}_{ij}$ is the fuzzy assignment cost of j^{th} job to i^{th} person. The objective is to minimize the total cost of one job to one person.

6.2. Ranking of Hexadecagonal fuzzy number

In order to find the optimum value of the given Hexadecagonal fuzzy cost given in table 1, first we convert the fuzzy cost into the crisp cost using the proposed ranking method (4.1) as shown in table 2.

Take the values of $k_1 = 0.3,\ k_2 = 0.5,\ k_3 = 0.8$. The ranking of fuzzy numbers is done by using proposed ranking method (4.1).

Table 2: Crisp cost for the corresponding Hexadecagonal fuzzy cost

Hexadecagonal Number	Ranking values
$\tilde{c}_{11} = (1,2,3,4,5,6,7,8,9,10, 11,12,13,14,15,16)$	$M_o^{HXDFN} = 8.5$
$\tilde{c}_{12} = (0,1,2,3,4,5,6,7,8,9,10, 11,12,13,14,15)$	$M_o^{HXDFN} = 7.5$
$\tilde{c}_{13} = (1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31)$	$M_o^{HXDFN} = 16$
$\tilde{c}_{14} = (1,3,4,5,7,9, 10,12,14,15, 16,18,20,21, 22,24)$	$M_o^{HXDFN} = 12.6$
$\tilde{c}_{21} = (2,4,6,8,10,12,14,16, 18,20,22,24,26,28,30,32)$	$M_o^{HXDFN} = 17$
$\tilde{c}_{22} = (2,3,5,7,9,11,13,17,19, 23,29,31,35,37,41,43)$	$M_o^{HXDFN} = 20.5$
$\tilde{c}_{23} = (1,4,7,10,13,16,19,22, 25,28,31,34,37,40,43, 46)$	$M_o^{HXDFN} = 23.5$
$\tilde{c}_{24} = (0,4,6,9,11,12,13,14, 15,16,19,21,23,25,28,30)$	$M_o^{HXDFN} = 15.3$
$\tilde{c}_{31} = (1,2,3,4,7,10,13,15,16,17,22,26,30,34,35,36)$	$M_o^{HXDFN} = 17$
$\tilde{c}_{32} = (2,4,6,8,9,13,15,16,18, 20,21,25,27,28,30,31)$	$M_o^{HXDFN} = 15.3$
$\tilde{c}_{33} = (1,2,3,4,5,7,9,11,13,17,21,25,27,31,32,34)$	$M_o^{HXDFN} = 20.6$
$\tilde{c}_{34} = (2,4,8,9,11,13,16,19, 20, 22,24,25,27,29,30,31)$	$M_o^{HXDFN} = 18$
$\tilde{c}_{41} = (1,2,3,4,8,9,10,12,13,15,17,18,20,22,24,26)$	$M_o^{HXDFN} = 12.6$
$\tilde{c}_{42} = (0,2,3,5,6,7,9,10,13,15,16,18,21,24,29,34)$	$M_o^{HXDFN} = 13.4$
$\tilde{c}_{43} = (3,4,7,10,11,13,15,16,18,21,24,28,30,34,36,44)$	$M_o^{HXDFN} = 19.8$
$\tilde{c}_{44} = (1,2,4,5,7,8,10,11,13,14,16,17,19,20,22,23)$	$M_o^{HXDFN} = 14.4$

6.3. Crisp assignment problem of the corresponding Hexadecagonal fuzzy assignment problem

The crisp assignment problem of the corresponding Hexadecagonal fuzzy assignment problem is given in table 3.

Table 3: Crisp assignment problem of the corresponding Hexadecagonal fuzzy assignment problem

	M_1	M_2	M_3	M_4
J_1	8.5	7.5	16	12.6
J_2	17	20.5	23.5	15.3
J_3	17	15.3	20.6	18
J_4	12.6	13.4	19.8	14.4

The given fuzzy assignment problem is a balanced assignment problem. By applying the Hungarian method, we find the optimal assignment schedule and the optimum assignment cost. The optimal assignment schedule is given by

$$J_1 \to M_2,\ J_2 \to M_4,\ J_3 \to M_3,\ J_4 \to M_1$$

Using (5.2) the assignment cost is given by

$\tilde{c}_{12} + \tilde{c}_{24} + \tilde{c}_{33} + \tilde{c}_{41} =$ $\Re$ (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15) x_{12} + $\Re$ (0,4,6,9,11,12, 13,14, 15,16,19,21,23,25,27,29) x_{24} + $\Re$ (1,2,3,4,5,7,9,11,13,17,21,25,27,31,32, 34) x_{33} + $\Re$ (1,2,3,4,8,9,10,12,13,15,17,18,20,22,23, 24) x_{41} = 7.5 + 15.3 + 20.6 + 12.6 = 56 units.

The optimal assignment cost =7.5+15.3+20.6+12.6=56 units.

Example 6.2. Consider the following Hexadecagonal fuzzy assignment problem which consists of three jobs and three machines. The cost matrix $\tilde{c}_{ij}$ whose elements are Hexadecagonal fuzzy numbers.Here our objective is to find the optimum assignment so as to minimize the cost (or time).

Table 4: Hexadecagonal fuzzy assignment problem

	M_1	M_2	M_3
J_1	(0,1,2,3,4,5,6,7,8,9, 10,11,12,13,14, 15)	(1,2,3,4,5,6,7,8,9,10,1 1,12,13,14,15,16)	(-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8,9,10,11)
J_2	(-3,-2,-1, 0, 1 ,2, 3, 4,5,6,7,8,9,10,11,12)	(-1, 0, 1 ,2, 3, 4, 5, 6, 7,8,9,10,11,12,13,14)	(-6,-5,-4,-3,-2,-1,0, 1,2,3,4, 5,6,7,8,9,10)
J_3	(-5,-4,-3,-2,-1,0,1,2, 3,.4, 5,6,7,8,9,10)	(-2.-1, 0, 1 ,2, 3, 4, 5, 6,7,8,9,10,11,12,13)	(-7,-6,-5,-4,-3,-2,-1,0, 1,2,3,4, 5,6,7,8)

The mathematical formulation of the given fuzzy assignment problem

Minimize $\tilde{Z}^* = \Re$ (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15) x_{11} + $\Re$ (1,2,3,4,5,6,7,8,9, 10,11,12,13,14,15,16) x_{12} + $\Re$ (-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11) x_{13} + $\Re$ (-3,-2,-1,0, 1,2,3,4,5,6,7,8,9,10,11,12) x_{21} + $\Re$ (-,0,1,23,4,5,6,7,8,9,10,11,12,13,14) x_{22} + $\Re$ (-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10) x_{23} + $\Re$ (-5,-4,-3,-2,-1,0,1,2,3,.4,5,6,7,8,9,10) x_{31} + $\Re$ (-2.-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13) x_{32} + $\Re$ (-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8) x_{33}

Subject to the constraints

$$\sum_{i=1}^{3} x_{ij} = 1 \text{ and } \sum_{j=1}^{3} x_{ij} = 1$$

where x_{ij} is the decision variable and

$$x_{ij} = \begin{cases} 1 \; if \; the \; i^{th} \; person \; is \; assigned \; to j^{th} \; job \\ 0, otherwise \end{cases} \quad (5.5)$$

Table 5: Crisp cost for the corresponding Hexadecagonal fuzzy cost

Take the values of $k_1 = 0.3, \; k_2 = 0.5, \; k_3 = 0.8$. The ranking of fuzzy numbers is done by using (4.1)

Hexadecagonal Number	Ranking values
$\tilde{c}_{11} =$ (0,1,2,3,4,5,6,7,8,9, 10,11,12,13,14, 15)	$M_o^{HXDFN} = 7.5$
$\tilde{c}_{12} =$ (1,2,3,4,5,6,7,8,9,10, 11,12,13,14,15,16)	$M_o^{HXDFN} = 8.5$
$\tilde{c}_{13} =$ (-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8,9,10,11)	$M_o^{HXDFN} = 3.5$
$\tilde{c}_{21} =$ (-3,-2,-1, 0, 1 ,2, 3, 4,5,6,7,8,9,10,11,12)	$M_o^{HXDFN} = 4.5$
$\tilde{c}_{22} =$ (-1, 0, 1 ,2, 3, 4, 5, 6, 7,8,9,10,11,12,13,14)	$M_o^{HXDFN} = 6.5$
$\tilde{c}_{23} =$ (-6,-5,-4,-3,-2,-1,0, 1,2,3,4, 5,6,7,8,9,10)	$M_o^{HXDFN} = 1.5$
$\tilde{c}_{31} =$ (-5,-4,-3,-2,-1,0,1,2, 3,.4, 5,6,7,8,9,10)	$M_o^{HXDFN} = 3.25$
$\tilde{c}_{32} =$ (-2.-1, 0, 1 ,2, 3, 4, 5, 6,7,8,9,10,11,12,13)	$M_o^{HXDFN} = 5.5$
$\tilde{c}_{33} =$ (-7,-6,-5,-4,-3,-2,-1,0, 1,2,3,4, 5,6,7,8,9)	$M_o^{HXDFN} = 0.5$

Table 6: Crisp assignment problem of the corresponding Hexadecagonal fuzzy assignment problem

	M_1	M_2	M_3
J_1	7.5	8.5	3.5
J_2	4.5	6.5	1.5
J_3	3.25	5.5	0.5

The given fuzzy assignment problem is a balanced assignment problem. By applying the Hungarian method, we find the optimal assignment schedule and the optimum assignment cost. The optimal assignment schedule is given by

$$J_1 \to M_3, \; J_2 \to M_1, \; J_3 \to M_2$$

Using (5.2) the assignment cost is given by

$\tilde{c}_{13} + \tilde{c}_{21} + \tilde{c}_{32} = \Re$ (-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8,9,10,11) x_{13} + $\Re$ (-3,-2,-1, 0, 1 ,2, 3, 4,5,6,7,8,9,10,11,12) x_{21} + $\Re$ (-2.-1, 0, 1 ,2, 3, 4, 5, 6,7,8,9,10,11,12,13) x_{32}

=3.5+4.5+5.5=13.5 units

The optimal assignment cost =3.5+4.5+5.5=13.5 units.

7. Conclusion

In this paper, a new method is proposed for solving Hexadecagonal fuzzy assignment problem. Fuzzy assignment problem is transformed into crisp assignment problem and solved by Hungarian method. Numerical example is presented and the optimal solution is obtained. In future, this method is applied to assigning jobs to suitable persons in a real life problem.

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