Intern. J. Fuzzy Mathematical Archive
Vol. 18, No. 1, 2020, 7-11
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 19 March 2020
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v18n1a02207

International Journal of
Fuzzy Mathematical
Archive

An Alternate Formula for Addition of Discrete Fuzzy Numbers

Hemanta K. Baruah

Department of Mathematics, The Assam Royal Global University
Guwahati – 781035, Assam, India.
E-mail: hkbaruah@rgu.ac, hemanta@gauhati.ac.in

Received 2 February 2020; accepted 16 March 2020

Abstract. The formula available in the literature to find the sum of two discrete fuzzy numbers needs a major change. We have shown in this article with the help of numerical examples as to why the existing formula does not quite look logical. We are going to put forward a different formula to find the sum using the weighted average of the levels of presence of the discrete fuzzy numbers concerned. With numerical examples we have shown that our proposed formula does return expected results.

Keywords: Theory of fuzzy sets, discrete fuzziness.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

In the theory of fuzzy sets, the most basic idea is that an element in a set can be partially present, the level of presence being between 0 and 1. A fuzzy number $A = [a, b, c]$ is defined with reference to a membership function $\mu(x)$ which is a continuous non-decreasing function of x for $x \in [a, b]$ and is a continuous non-increasing function of x for $x \in [b, c]$. Here $\mu(x)$ for a given x in the intervals concerned gives the level of presence of x. There are standard procedures to add such fuzzy numbers. Such procedures are however not applicable to the case when the fuzzy numbers are discrete.

Let $x^{(\alpha)}$ be a discrete fuzzy number where x is a real number and α is the level of presence of x, $\alpha \in [0, 1]$. To add two discrete fuzzy numbers $x^{(\alpha)}$ and $y^{(\beta)}$, in the available literature the following formula is in use:

$$x^{(\alpha)} + y^{(\beta)} = (x + y)^{(\gamma)}$$

where γ = min (α, β)(Wang *et al.* [1], Casasnovas and Riera, [2]). This definition of addition of two discrete fuzzy numbers does not give any importance to the values of x and y whereas the definition in the continuous case considers all values of x in the two intervals $[a, b]$ and $[b, c]$.

It may be noted that very little work is available in the literature regarding operation of discrete fuzzy numbers. Casasnovas and Riera [3] have studied addition of discrete fuzzy numbers when Zadeh's Extension Principle [4] does not obtain a convex function. Wang and Wen [5] have studied discrete fuzzy number operations because the usual addition does not keep closeness of the operation. Fan [6] studied Zadeh's Extension Principle for operations on discrete fuzzy numbers. It may be noted that all of these works in [2, 3, 5, 6] are based on the work of Wang *et al.* [1], and we are going to propose in this article an alternative formula for discrete fuzzy number addition because the formula still being used dos not return logically acceptable results as would be seen in the discussions in what follows.

We would like to start our discussions with a few numerical examples. According to this definition, we see that

$$80^{(0.9)} + 20^{(0.1)} = 100^{(0.1)}$$

and

$$20^{(0.9)} + 80^{(0.1)} = 100^{(0.1)}.$$

In other words, according to this definition $80^{(0.9)} + 20^{(0.1)}$ and $20^{(0.9)} + 80^{(0.1)}$ are the same, which is not quite illogical because in $80^{(0.9)} + 20^{(0.1)}$ the level of presence of 80 is 0.9 while the level of presence of 20 is 0.1, and in $20^{(0.9)} + 80^{(0.1)}$ the level of presence of 80 is 0.1 while the level of presence of 20 is 0.9. This definition is therefore insensitive to the values of *x* as well as *y*; and it gives importance only to the levels of presence of *x* and *y*.

Indeed,

$$x^{(\alpha)} + y^{(\beta)} = (x + y)^{(\gamma)}$$

where γ = min (α, β) leads to gives us another result which too does not look quite logical:

$$80^{(0.9)} + 20^{(0.1)} = 80^{(0.5)} + 20^{(0.1)} = 100^{(0.1)}$$

which means that in what has been shown as an example, according to this definition it does not matter whether the level of presence of 80 is 0.9 or 0.5, the minimum value of the levels of presence of 20 only is important.

Further, if one of *x* and *y* is 0 with a non-zero presence level, this definition gives an unacceptable result. For example, it gives us:

$$100^{(0.9)} + 0^{(0.1)} = 100^{(0.1)},$$

whereas addition of the discrete fuzzy number $100^{(0.9)}$ with the discrete fuzzy number $0^{(0.1)}$ should be $100^{(0.9)}$ itself.

It is clear that a definition that leads to such conclusions must have some defect. In fact, it has the following defects:

1. First, this definition does not give any weightage to the numerical values of *x* and *y*.
2. Secondly, this definition uses the value of min (α, β), while it is insensitive to changes in max (α, β).
3. Finally, any discrete fuzzy number $x^{(\alpha)}$ added to the discrete fuzzy number $0^{(\beta)}$ must give us $x^{(\alpha)}$ back again anyway.

Therefore, we need to define the operation of addition of discrete fuzzy numbers in such a way that it does not have the defects mentioned above. In what follows, we are going to show how the operation of addition of discrete fuzzy numbers should be performed.

2. Addition of two discrete fuzzy numbers

Our objective is to define the sum of two discrete fuzzy numbers $x^{(\alpha)}$ and $y^{(\beta)}$ where *x* and *y* are real and $\alpha,\ \beta \in [0,1]$. The sum must be so defined that proper weightage be given to *x* and *y*, and not just min (α, β) but both α and β must be given importance. The only logical way to do so is to take the weighted average of α and β. It may be noted that weighted average is a very classical concept used in finding sample mean in statistical matters, and we are going to use this classical concept to define addition of discrete fuzzy numbers. According to us, the correct way to get the sum of $x^{(\alpha)}$ and $y^{(\beta)}$ is

$$x^{(\alpha)} + y^{(\beta)} = (x+y)^{\left(\frac{\alpha x + \beta y}{(x+y)}\right)}, \quad x \neq 0, \quad y \neq 0.$$

Here ($\frac{\alpha x + \beta y}{x+y}$) is the weighted average of α and β.

We would now like to go back to the numerical examples discussed in Section 1 above.

$$80^{(0.9)} + 20^{(0.1)} = \ 100^{\left(\frac{72+2}{100}\right)} = 100^{(\,0.74\,)}$$

and

$$20^{(0.9)} + 80^{(0.1)} = \ 100^{\left(\frac{18+8}{100}\right)} = 100^{(0.26)}.$$

As expected, $80^{(0.9)} + 20^{(0.1)}$ and $20^{(0.9)} + \ 80^{(0.1)}$ are not same if we use our proposed formula. It should be obvious that 80 with 90% presence added to 20 with 10% presence cannot be same as 20 with 90% presence added to 80 with 10% presence. In the existing definition available in the literature cited above, $80^{(0.9)} + 20^{(0.1)}$ and $20^{(0.9)} + \ 80^{(0.1)}$ were the same.

We now come to the second numerical example mentioned in Section 1 above. It can be seen that

$$80^{(0.9)} + 20^{(0.1)} = \ 100^{(0.74)}$$

and

$$80^{(0.5)} + 20^{(0.1)} = 100^{(0.42)}.$$

As expected, $80^{(0.9)} + 20^{(0.1)}$ and $80^{(0.5)} + 20^{(0.1)}$ are not same if we use our proposed formula. In the definition available in the literature, $80^{(0.9)} + 20^{(0.1)}$ and $80^{(0.5)} + 20^{(0.1)}$ were numerically same. Obviously, 80 with 90% presence must not be same as 80 with 50% presence.

Finally, coming to the third numerical example mentioned above, using our proposed procedure we see that

$$100^{(0.9)} + 0^{(0.1)} = \ 100^{(0.9)}$$

as should be the case. If the definition available in the literature is followed, it is equal to $100^{(0.1)}$ which does not seem logical.

3. The rationale behind

We have shown numerically why using the classical concept of weighted average of the levels of presence of the fuzzy numbers is the logical way to add two discrete fuzzy numbers. We now proceed to explain the mathematical basis of using the weighted average of the presence levels.

Let $x^{(\alpha)}$ be a discrete fuzzy number where *x* is a real number and $\alpha \in [0,1]$ is the presence level of *x*. We would like to put forward a standpoint that the number 80 with level of presence 40% is numerically the same as the number 40 with level of presence 80%. In other words, from our standpoint $80^{(0.4)}$ = $40^{(0.8)}$, and both of them are equal to 32 with level of presence 100%. In other words, from this standpoint, $80^{(0.4)}$ = $40^{(0.8)}$ = $32^{(1.0)}$ and this means that $80^{(0.4)}$ is numerically equal to the product of 80 and its level of presence 0.4. The same can be said about $40^{(0.8)}$. So, we are proposing that $x^{(\alpha)}$ and αx are numerically the same. This would give us an alternative way to define fuzzy numbers.

Now, if it is accepted that $x^{(\alpha)}$ and αx are numerically the same where $x^{(\alpha)}$ is a discrete fuzzy number *x* being a real number and $\alpha \in [0, 1]$ is the presence level of *x*, and therefore if it is accepted that $x^{(\alpha)}$ can be expressed as

$$x^{(\alpha)}=\alpha x$$

we get

$$x^{(\alpha)}+ y^{(\beta)} = \alpha x + \beta y =(x + y).\frac{(\alpha x+ \beta y)}{(x + y)} = (x + y)^{(\frac{\alpha x+ \beta y}{(x+y)})}, x \neq 0, y \neq 0,$$

where

$$\frac{\alpha x + \beta y}{x + y}$$

is nothing but the weighted average of *α* and *β*. For $x = 0$, y = 0,

$$x^{(\alpha)}+ y^{(\beta)} = \alpha x + \beta y = 0$$

because both αx and βy are equal to 0 already. For example, $0^{(0.1)}$ + $0^{(0.2)}$ = 0.
Indeed, fuzziness is rooted at partial presence, and if it is accepted that partial presence can be defined using the relation $x^{(\alpha)} = \alpha x$, the formula for addition of discrete fuzzy numbers comes out automatically.

4. Conclusions and discussions

In the Theory of Fuzzy Sets, the essence is that a real number can be partially present, the level of presence being a number between 0 and 1. Incidentally, what was proposed as the formula to add two discrete fuzzy numbers gives importance to the minimum value of the levels of presence of the numbers concerned. We have in this article shown with three numerical examples why that definition looks not quite logical. We have shown that in adding discrete fuzzy numbers, we should use the classical weighted average of the levels of presence of the discrete fuzzy numbers concerned.

In this context, we would like to mention that we have only put forward a correction with reference to addition of discrete fuzzy numbers. In our eyes, there is an error in the theory concerned, and our objective was to point that out and to put forward an alternative formula. We have explained the rationale behind using the weighted mean of the levels of presence of the discrete fuzzy numbers when we proceed to find the sum of the fuzzy numbers. Even without this rationale, using the classical concept of weighted averages leads us to the expected results.

REFERENCES

1. G.Wang, C.Wu and C.Zhao, Representation and operations of discrete fuzzy numbers, *Southeast Asian Bulletin of Mathematics*, 28 (2005) 1003 – 1010.

2. J.Casasnovas and J.V.Riera, On the addition of discrete fuzzy numbers, *Proceedings of the 5th WSEAS International Conference on Telecommunications and Informatics*, Istanbul, (2006) 432 – 437.
3. J.Casasnovas and J.V.Riera, Discrete fuzzy numbers defined on a subset of natural numbers, In: O.Castilo, P.Melin, O.M.Ross, C.R.Sepulveda, W.Pedrycz, J.Kacprzyk (Eds.) *Theoretical Advances and Applications of Fuzzy Logic and Soft Computing*, 42 (2007) 573 – 582, Springer, Berlin, Heidelberg.
4. L.C.de Barros, R.C.Bassanezi, W.A.Lodwick, The extension principle of Zadeh and fuzzy numbers, In: A First Course in Fuzzy Logic, Fuzzy Dynamical Systems, and Biomathematics. *Studies in Fuzziness and Soft Computing*, 347 (2016) 23 – 41.
5. G.Wang and C.Wen, A new fuzzy arithmetic for discrete fuzzy numbers, *Fourth International Conference on Fuzzy Systems and Knowledge Discovery (FSKD 2007)*, Haikou, (2007) 52 – 56.
6. T.Fan, Notes on discrete fuzzy numbers, *Quantitative Logic and Soft Computing*, (2012) 643 – 648.