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Fuzzy Sadik Transform

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Abstract. In this work we show how to convert from Sadik transform to fuzzy Sadik transform (FST) and prove some properties of (FST). Also we find fuzzy Sadik transform of fuzzy derivatives about first, second and third order, and we generalized these formulas to the fuzzy nth- order derivative by using strongly generalized H-differentiability concepts. use this results to solve fuzzy initial value problem FIVP of first order.

Keywords: Sadik transform, generalization of fuzzy Sadik transform

AMS Mathematics Subject Classification (2010): 34A25

1. Introduction

One of the most important applications of integral transforms methods is solving the differential equations. Many researchers use fuzzy transform to solve fuzzy differential equations see [5, 6]. A new integral transform, which is denoted by Sadik transform, was suggested by Sadik Latif [9], he is find relationship between his transform and many transforms like Laplace transform, Sumudu transform, Elzaki transform, Kamal transform, Tarig transform, Laplace- Carson transform, Aboodh transform such that all these transform are particular cases from Sadik transform.

In this article, we find fuzzy Sadik transform for first, second and third order of fuzzy derivative, and generalize these formulas to the fuzzy nth-order derivative and prove some properties of fuzzy Sadik transform and apply that in fuzzy differential equation.

2. Basic concepts

We call the set of all real numbers by $\mathbb{R}$ and the set of all fuzzy numbers on R is indicated by E.

Definition 1. [11] A fuzzy number is a mapping $\zeta : \mathbb{R} \to [0,1]$ with the following properties:

(a) ζ is upper semi-continuous,

(b) ζ is fuzzy convex, i.e., $\zeta\left(\lambda\aleph + (1-\lambda)\Upsilon\right) \geq \min\left\{\zeta(\aleph), \zeta(\Upsilon)\right\}$,

$\forall\ \aleph, \Upsilon \in \mathbb{R}, \lambda \in [0,1]$.

(c) ζ is normal, i.e. $\exists \aleph_0 \in \mathbb{R}$ for which $\zeta(\aleph_0) = 1$.

(d) supp $\zeta = \{\aleph \in \mathbb{R} / \zeta(\aleph) > 0\}$ is the support of the ζ, and its closure cl (supp ζ) is compact.

Definition 2. [4] A fuzzy number ζ in parametric form is a pair $(\underline{\zeta}, \overline{\zeta})$ of functions $\underline{\zeta}(\gamma), \overline{\zeta}(\gamma)$, $0 \le \lambda \le 1$, which satisfy the following requirements:

1. $\underline{\zeta}(\gamma)$ is a bounded non-decreasing left continuous function in $(0,1]$, and right continuous at 0.
2. $\overline{\zeta}(\gamma)$ is a bounded non-increasing left continuous function in $(0,1]$, and right continuous at 0.
3. $\underline{\zeta}(\gamma) \le \overline{\zeta}(\gamma)$, $0 \le \gamma \le 1$

According to Zadeh's extension principle, the operation of addition on E is defined by

$$(\zeta + \upsilon) = \sup_{\Upsilon \in \Re} \min\{\zeta(\Upsilon), \upsilon(\aleph - \Upsilon)\}$$

and scalar multiplication of fuzzy numbers given by,

$$(k \odot \zeta)(\aleph) = \begin{cases} \zeta(\aleph / k), & k > 0 \\ 0, & k = 0 \end{cases} \quad \text{where} \quad 0 \in \mathbb{R}$$

The Hausdorff distance between fuzzy numbers given by $d : \mathrm{E} \times \mathrm{E} \to [0, +\infty]$

$$d(\zeta, \upsilon) = \sup_{\gamma \in [0,1]} \max\left\{\left|\underline{\zeta}(\gamma) - \underline{\upsilon}(\gamma)\right|, \left|\overline{\zeta}(\gamma) - \overline{\upsilon}(\gamma)\right|\right\} \text{where}$$

$\zeta = (\underline{\zeta}(\gamma), \overline{\zeta}(\gamma)), \upsilon = (\underline{\upsilon}(\gamma), \overline{\upsilon}(\gamma)) \subset \mathbb{R}$ is utilized in [1].

Definition 3. [8] Let $\aleph, \Upsilon \in \mathrm{E}$. If there exists $\rho \in \mathrm{E}$ such that $\aleph + \Upsilon = \rho$ then ρ is called the Hukuhara – difference of $\aleph$ and Υ and it is denoted by $\aleph \ominus \Upsilon$. The sign $\ominus$ always stands for H–difference and.

Definition 4. [1] Let $f(\aleph) : (a,b) \to \mathrm{E}$ and $\aleph_0 \in (a,b)$, we say that f is strongly generalized differential at $\aleph_0$ if there exists an element $f'(\aleph_0) \in \mathrm{E}$ such that :

$\forall\ h > 0$ sufficiently small $\exists f(\aleph_0 + h) \ominus f(\aleph_0), \exists f(\aleph_0) \ominus f(\aleph_0 - h)$ and the limits (in metric D)

$$f'(\aleph_0) = \lim_{h \to 0^+} \frac{f(\aleph_0 + h) \ominus f(\aleph_0)}{h} = \lim_{h \to 0^+} \frac{f(\aleph_0) \ominus f(\aleph_0 - h)}{h}$$

or

$\forall\ h > 0$ sufficiently small $\exists f\ (\aleph_0) \ominus f\ (\aleph_0 + h), \exists f\ (\aleph_0 - h) \ominus f\ (\aleph_0)$ and the limits (in metric D)

$$f\,'(x_0) = \lim_{h \to 0^+} \frac{f\ (\aleph_0) \ominus f\ (\aleph_0 + h)}{-h} = \lim_{h \to 0^+} \frac{f\ (\aleph_0 - h) \ominus f\ (\aleph_0)}{-h}$$

Theorem 1. [3] Let $f\ : R \to \mathrm{E}$ be a function and denote $f\ (\aleph) = \left(\underline{f}\ (\aleph;\gamma), \underline{f}\ (\aleph;\gamma)\right)$ for each $\gamma \in [0,1]$ Then

1. If f is the first form, then $\underline{f}\ (\aleph;\gamma)$ and $\overline{f}\ (\aleph;\gamma)$ are differentiable functions and $f\,'(\aleph) = \underline{f}\ (\aleph;\gamma), \overline{f}\ (\aleph;\gamma)$
2. If f is the second form, then $\underline{f}\ (\aleph;\gamma)$ and $\overline{f}\ (\aleph;\gamma)$ are differentiable functions and $f\,'(\aleph) = \overline{f}\ (\aleph;\gamma), \underline{f}\ (\aleph;\gamma)$

Theorem 2. [9] Laplace – Sadik transform duality theorem

If (s) is Laplace transform of $f(t)$ and $G\left(v^{\alpha}, \beta\right)$ is a Sadik transform of $f(t)$ then

$$G\left(v^{\alpha}, \beta\right) = \frac{1}{v^{\beta}} F\left(v^{\alpha}\right) \tag{1}$$

3. Fuzzy Sadik transforms

Theorem 3. [10] Let $f : R \to F(R)$ and it is represented by $\left[\underline{f}_{\alpha}(x), \overline{f}_{\alpha}(x)\right]$. For any fixed $\alpha \in (0,1]$ assume $\underline{f}_{\alpha}(x)$ and $\overline{f}_{\alpha}(x)$ are Riemann-integrable on [a, b] for every $b \geq a$, and assume there are two positive $\underline{M}_{\alpha}$ and $\overline{M}_{\alpha}$ such that $\int_a^b \left|\underline{f}_{\alpha}(x)\right| dx \leq \underline{M}_{\alpha}$ and $\int_a^b \left|\overline{f}\ (x)\right| dx \leq \overline{M}_{\alpha}$ for every $b \geq a$ .Then, $F(R)$ is improper fuzzy Riemann-integrable on $[a, \infty)$ and the improper fuzzy Riemann-integrable is a fuzzy number. Furthermore, we have

$$\int_a^{\infty} f\ (x)\ dx = \left[\int_a^{\infty} \underline{f}\ (x)\ dx, \int_a^{\infty} \overline{f}\ (x)\ dx\right]$$

Definition 5. Fuzzy Sadik transform is defined by

$$G\left(v^{\alpha}, \beta\right) = S\left[f\ (\aleph)\right] = \frac{1}{v^{\beta}} \int_0^{\infty} e^{-\aleph v^{\alpha}} f\ (\aleph)\, d\,\aleph$$

from theorem (3) we get:

$$\frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}} f(\aleph)d\aleph = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}} \underline{f}(\aleph)d\aleph, \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}} \overline{f}(\aleph)d\aleph$$

By using definition of Sadik transform we get:

$$S\left[\underline{f}(\aleph;\gamma)\right] = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}} \underline{f}(\aleph;\gamma)d\aleph,$$

$$S\left[\overline{f}(\aleph;\gamma)\right] = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}} \overline{f}(\aleph;\gamma)d\aleph$$

then,

$$S\left[f(\aleph;\gamma)\right] = S\left[\underline{f}(\aleph;\gamma)\right], S\left[\overline{f}(\aleph;\gamma)\right]$$

where, $f(\aleph)$ continuous fuzzy – valued function
v is complex variable,
α is any non zero real numbers, and
β is any real number.

Theorem 4. [7] Assume that $f(\aleph), f'(\aleph), \ldots, f^{n-1}(\aleph)$ be continuous fuzzy-valued functions on $[0,\infty)$ and of exponential order and that $f^{(n)}(\aleph)$ is piecewise continuous fuzzy-valued function on $[0,\infty)$. Let $f^{(i_1)}(\aleph), f^{(i_2)}(\aleph), \ldots, f^{(i_m)}(\aleph)$ be (ii)-differentiable functions for $0 \le i_1 \le i_2 \ldots \le i_m \le n-1$ and $f^{(p)}$ be (i)-differentiable function for $p \ne i_j$, $j = 1, 2, \ldots, m$, then:

(1) If m is an even number, we have

$$L\left(f^{(n)}(\aleph)\right) = p^n L\left(f(\aleph)\right) \ominus p^{n-1} f(0) \otimes \sum_{k=1}^{n-1} p^{n-(k+1)} f^{(k)}(0)$$

such that

$$\otimes = \begin{cases} \ominus, & \text{if the number of repetitions of the number 2 among } i_1,\ldots,i_k \\ & \text{is an even number} \\ -, & \text{if the number of repetitions of the number 2 among } i_1,\ldots,i_k \\ & \text{is an odd number} \end{cases}$$

2. If m is an odd number, we have

$$L\left(f^{(n)}(\aleph)\right) = -p^{n-1} f(0) \ominus \left(-p^n\right) L\left(f(\aleph)\right) \otimes \sum_{k=1}^{n-1} p^{n-(k+1)} f^{(k)}(0),$$

such that

$$\otimes = \begin{cases} \ominus, & \text{if the number of repetitions of the number 2 among } i_1,...,i_k \\ & \text{is an odd number} \\ -, & \text{if the number of repetitions of the number 2 among } i_1,...,i_k \\ & \text{is an even number} \end{cases}$$

Theorem 5. Let $f(\aleph), h(\aleph)$ be continuous fuzzy- valued function and assume that a_1, a_2 are constant, then

$$S\left[\left(a_1 \odot f(\aleph)\right) \oplus \left(a_2 \odot h(\aleph)\right)\right] = \left(a_1 \odot S\left[f(\aleph)\right]\right) \oplus \left(a_2 \odot S\left[h(\aleph)\right]\right)$$

Proof:

$$S\left[\left(a_1 \odot f(\aleph)\right) \oplus \left(a_2 \odot h(\aleph)\right)\right] = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}}\left[\left(a_1 \odot f(\aleph)\right) \oplus \left(a_2 \odot h(\aleph)\right)\right] d\aleph$$

$$= \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}}\left[\left(a_1 \odot f(\aleph)\right)\right] d\aleph \oplus \frac{1}{v^{\beta}}\int_0^{\infty} e^{-\aleph v^{\alpha}}\left(a_2 \odot h(\aleph)\right) d\aleph$$

$$= a_1 \frac{1}{v^{\beta}}\int_0^{\infty} e^{-tv^{\alpha}} f(\aleph) d\aleph \oplus a_2 \frac{1}{v^{\beta}}\int_0^{\infty} e^{-tv^{\alpha}} h(\aleph) d\aleph$$

$$= \left(a_1 \odot S\left[f(\aleph)\right]\right) \oplus \left(a_2 \odot S\left[h(\aleph)\right]\right)$$

Theorem 6. Let f is continuous fuzzy-value function and $S\left[f(\aleph)\right] = G\left(v^{\alpha}, \beta\right)$ Then,

$$S\left[e^{a\aleph} \odot f(\aleph)\right] = G\left(v^{\alpha} - \mathrm{a}, \beta\right)$$

Proof:

$$S\left[e^{a\aleph} f(\aleph)\right] = \frac{1}{v}\int_0^{\infty} e^{a\aleph - \aleph v^{\alpha}} f(\aleph) d\aleph$$

$$= \frac{1}{v}\int_0^{\infty} e^{a\aleph - \aleph v^{\alpha}} \underline{f}(\aleph) d\aleph, \frac{1}{v}\int_0^{\infty} e^{a\aleph - \aleph v^{\alpha}} \overline{f}(\aleph) d\aleph$$

$$= \frac{1}{v}\int_0^{\infty} e^{-\left(a - v^{\alpha}\right)\aleph} \underline{f}(\aleph) d\aleph, \frac{1}{v}\int_0^{\infty} e^{-\left(a - v^{\alpha}\right)\aleph} \overline{f}(\aleph) d\aleph$$

$$= G\left(v^{\alpha} - a, \beta\right)$$

Remark 1. Let $f(\aleph)$ be continuous fuzzy-value function on $[0, \infty)$ and $\mu \geq 0$, then, $S\left[\mu \odot f(\aleph)\right] = \mu \odot S\left[f(\aleph)\right]$

Proof:

$$S\left[\mu \odot f(\aleph)\right]=\frac{1}{v}\int_{0}^{\infty} e^{-\aleph v^{\alpha}}(\mu \odot f(\aleph))d\aleph=\mu\frac{1}{v}\int_{0}^{\infty} e^{-\aleph v^{\alpha}} f(\aleph)d\aleph$$

$$=\mu S\left[f(\aleph)\right].$$

Theorem 7. assume that $f'(\aleph)$ be continuous fuzzy-valued function and $f(\aleph)$ the primitive of $f'(\aleph)$ on $[0,\infty)$, we have:

1. $S\left[f'(\aleph)\right]=v^{\alpha}S\left[f(\aleph)\right]\ominus v^{-\beta}f(0)$, where f is the first form differentiable
2. $S\left[f'(\aleph)\right]=-v^{-\beta}f(0)\ominus\left(-v^{\alpha}\right)S\left[f(\aleph)\right]$, where f is the second form differentiable

Proof: for any arbitrary $\gamma\in[0,1]$

$$v^{\alpha}S\left[f(\aleph)\right]\ominus v^{-\beta}f(0)=v^{\alpha}S\left[\underline{f}(\aleph,\gamma)\right]-v^{-\beta}\underline{f}(0,\gamma),v^{\alpha}S\left[\overline{f}(\aleph,\gamma)\right]-v^{-\beta}\overline{f}(0,\gamma)$$

since

$$s\left[\underline{f}'(\aleph,\gamma)\right]=v^{\alpha}S\left[\underline{f}(\aleph,\gamma)\right]-v^{-\beta}\underline{f}(0,\gamma),$$

$$s\left[\overline{f}'(\aleph,\gamma)\right]=v^{\alpha}S\left[\overline{f}(\aleph,\gamma)\right]-v^{-\beta}\overline{f}(0,\gamma)$$

$$v^{\alpha}S\left[f(\aleph)\right]\ominus v^{-\beta}f(0)=S\left[\underline{f}'(\aleph,\gamma)\right],S\left[\overline{f}'(\aleph,\gamma)\right]$$

by linearity of Sadik transform we get:

$$v^{\alpha}S\left[f(\aleph)\right]\ominus v^{-\beta}f(0)=S\left(\left[\underline{f}'(\aleph,\gamma)\right],\left[\overline{f}'(\aleph,\gamma)\right]\right)$$

Since f is the first form, it follows that

$$v^{\alpha}S\left[f(\aleph)\right]\ominus v^{-\beta}f(0)=S\left[f'(\aleph)\right]$$

Now we suppose that f is the second form, for any arbitrary $\gamma\in[0,1]$

$$-v^{-\beta}f(0)\ominus\left(-v^{\alpha}\right)S\left[f(\aleph)\right]=-v^{-\beta}\overline{f}(0,\gamma)+v^{\alpha}S\left[\overline{f}(\aleph,\gamma)\right],-v^{-\beta}\underline{f}(0,\gamma)+v^{\alpha}S\left[\underline{f}(\aleph,\gamma)\right]$$

since,

$$s\left[\underline{f}'(\aleph,\gamma)\right]=-v^{-\beta}\underline{f}(0,\gamma)+v^{\alpha}S\left[\underline{f}(\aleph,\gamma)\right],$$

$$s\left[\overline{f}'(\aleph,\gamma)\right]=-v^{-\beta}\overline{f}(0,\gamma)+v^{\alpha}S\left[\overline{f}(\aleph,\gamma)\right]$$

$$-v^{-\beta}f(0)\ominus\left(-v^{\alpha}\right)S\left[f(\aleph)\right]=S\left[\overline{f}'(\aleph,\gamma)\right],S\left[\underline{f}'(\aleph,\gamma)\right]$$

$$-v^{-\beta}f(0)\ominus\left(-v^{\alpha}\right)S\left[f(\aleph)\right]=S\left(\left[\overline{f}'(\aleph,\gamma)\right],\left[\underline{f}'(\aleph,\gamma)\right]\right)$$

Since f is the second form, it follows that

$$-v^{-\beta}f(0)\ominus\left(-v^{\alpha}\right)S\left[f(\aleph)\right]=S\left[f'(\aleph)\right]$$

by same way we can get fuzzy Sadik transform of fuzzy derivatives about, second and three orders respectively :

1. If f, f' are first form then:

$$S[f''(\aleph)] = v^{2\alpha}S[f(\aleph)] \ominus v^{\alpha-\beta}f(0) \ominus v^{-\beta}f'(0)$$

2. If f is first form and f' second form then:

$$S[f''(\aleph)] = -v^{\alpha-\beta}f(0) \ominus (-v^{2\alpha})S[f(\aleph)] \ominus v^{-\beta}f'(0)$$

3. If f is second form and f' first form then:

$$S[f''(\aleph)] = -v^{\alpha-\beta}f(0) \ominus (-v^{2\alpha})S[f(\aleph)] - v^{-\beta}f'(0)$$

4. If f, f' are second form then:

$$S[f''(\aleph)] = v^{2\alpha}S[f(\aleph)] \ominus v^{\alpha-\beta}f(0) - v^{-\beta}f'(0)$$

Now if fuzzy Sadik transform of fuzzy derivatives about, third order.

1.If f, f', f'' are first form then:

$$S[f'''(\aleph)] = v^{3\alpha}S[f(\aleph)] \ominus v^{2\alpha-\beta}f(0) \ominus v^{\alpha-\beta}f'(0) \ominus v^{-\beta}f''(0)$$

2. If f', f'' are first form and f second form then:

$$S[f'''(\aleph)] = -v^{2\alpha-\beta}f(0) \ominus (-v^{3\alpha})S[f(\aleph)] - v^{\alpha-\beta}f'(0) - v^{-\beta}f''(0)$$

3. If f, f'' are first form and f' second form then:

$$S[f'''(\aleph)] = -v^{2\alpha-\beta}f(0) \ominus (-v^{3\alpha})S[f(\aleph)] \ominus v^{\alpha-\beta}f'(0) \ominus v^{-\beta}f''(0)$$

4. If f, f' are first form and f'' second form then:

$$S[f'''(\aleph)] = -v^{2\alpha-\beta}f(0) \ominus (-v^{3\alpha})S[f(\aleph)] - v^{\alpha-\beta}f'(0) \ominus v^{-\beta}f''(0)$$

5. If f, f' are second form and f'' first form then:

$$S[f'''(\aleph)] = v^{3\alpha}S[f(\aleph)] \ominus v^{2\alpha-\beta}f(0) - v^{\alpha-\beta}f'(0) - v^{-\beta}f''(0)$$

6. If f, f'' are second form and f' first form then:

$$S[f'''(\aleph)] = v^{3\alpha}S[f(\aleph)] \ominus v^{2\alpha-\beta}f(0) \ominus v^{\alpha-\beta}f'(0) - v^{-\beta}f''(0)$$

7.If f', f'' are second form and f first form then:

$$S[f'''(\aleph)] = v^{3\alpha}S[f(\aleph)] \ominus v^{2\alpha-\beta}f(0) - v^{\alpha-\beta}f'(0) \ominus v^{-\beta}f''(0)$$

8. If f, f', f'' are second form then:

$$S[f'''(\aleph)] = -v^{2\alpha-\beta}f(0) \ominus (-v^{3\alpha})S[f(\aleph)] \ominus v^{\alpha-\beta}f'(0) - v^{-\beta}f''(0)$$

4. Fuzzy Sadik transform for fuzzy nth-order derivative

Theorem 8. Assume that $f(\aleph), f'(\aleph), \ldots, f^{n-1}(\aleph), f^{(n)}(\aleph)$ be continuous fuzzy-valued functions on $[0, \infty)$. Let $f^{(i_1)}(\aleph), f^{(i_2)}(\aleph), \ldots, f^{(i_\varphi)}(\aleph)$ be second form differentiable functions for $0 \le i_1 \le i_2 ... \le i_\varphi \le n-1$ and $f^{(p)}$ be first form differentiable function for $p \ne i_j$, $j = 1, 2, ..., \varphi$, then:

(1) If φ is an even number, we have

$$S\left(f^{(n)}(\aleph)\right) = v^{n\alpha} S\left(f(\aleph)\right) \ominus v^{(n-1)\alpha-\beta} f(0) © \sum_{\kappa=1}^{n-1} v^{(n-(\kappa+1))\alpha-\beta} f^{(\kappa)}(0)$$

such that

$$© = \begin{cases} \ominus, & \text{if the number of repetitions of the second form differentiable between } i_1, ..., i_\kappa \\ & \text{is an even number} \\ -, & \text{if the number of repetitions of the second form differentiable between } i_1, ..., i_\kappa \\ & \text{is an odd number} \end{cases}$$

2. If φ is an odd number, we have

$$S\left(f^{(n)}(\aleph)\right) = -v^{(n-1)\alpha-\beta} f(0) \ominus \left(-v^{n\alpha}\right) S\left(f(\aleph)\right) © \sum_{\kappa=1}^{n-1} v^{(n-(\kappa+1))\alpha-\beta} f^{(\kappa)}(0),$$

such that

$$© = \begin{cases} \ominus, & \text{if the number of second form differentiable between } i_1, ..., i_\kappa \\ & \text{is an odd number} \\ -, & \text{if the number of second form differentiable between } i_1, ..., i_\kappa \\ & \text{is an even number} \end{cases}$$

Proof: we shall prove by using the duality relation between fuzzy Laplace and Sadik transforms as follows

$$G\left(v^\alpha, \beta\right) = S\left[f(\aleph)\right] \qquad F(p) = L\left[f(\aleph)\right]$$

$$G_n\left(v^\alpha, \beta\right) = S\left[f^{(n)}(\aleph)\right] \quad \text{and} \quad F_n(p) = L\left[f^{(n)}(\aleph)\right]$$

From duality relation (1), we have

$$G_n\left(v^\alpha, \beta\right) = S\left[f^{(n)}(\aleph)\right] = \frac{1}{v^\beta} F_n\left(v^\alpha\right) \tag{2}$$

Let *m* is an even number. Then from theorem 7 when φ is an even number, equation (2) becomes

$$G_n\left(v^\alpha, \beta\right) = \frac{1}{v^\beta}\left[v^{n\alpha} F\left(v^\alpha\right) \ominus v^{(n-1)\alpha} f(0) © \sum_{k=1}^{n-1} v^{(n-(k+1))\alpha} f^{(k)}(0)\right]$$

$$= v^{n\alpha}\left[\frac{1}{v^\beta} F\left(v^\alpha\right)\right] \ominus v^{(n-1)\alpha-\beta} f(0) © \sum_{k=1}^{n-1} v^{(n-(k+1))\alpha-\beta} f^{(k)}(0)$$

$$= v^{n\alpha} S\left[f(\aleph)\right] \ominus v^{(n-1)\alpha-\beta} f(0) © \sum_{\kappa=1}^{n-1} v^{(n-(\kappa+1))\alpha-\beta} f^{(\kappa)}(0)$$

Let φ is an odd number. Then from theorem 7, equation (2) becomes:

$$G_n\left(v^{\alpha},\beta\right)=\frac{1}{v^{\beta}}\left[-v^{(n-1)\alpha}f\ (0)\ominus\left(-v^{n\alpha}\right)F\left(v^{\alpha}\right)©\sum_{\kappa=1}^{n-1}v^{(n-(\kappa+1))\alpha}f^{(\kappa)}(0)\right]$$

$$=-v^{(n-1)\alpha-\beta}f\ (0)\ominus\left(-v^{n\alpha}\right)\left[\frac{1}{v^{\beta}}F\left(v^{\alpha}\right)\right]©\sum_{\kappa=1}^{n-1}v^{(n-(\kappa+1))\alpha-\beta}f^{(\kappa)}(0)$$

$$=-v^{(n-1)\alpha-\beta}f\ (0)\ominus\left(-v^{n\alpha}\right)S\left[f\ (\aleph)\right]©\sum_{\kappa=1}^{n-1}v^{(n-(\kappa+1))\alpha-\beta}f^{(\kappa)}(0)$$

Example 1. Consider the following first order FIVP

$$y'(\aleph)=-y\ (\aleph) \tag{3}$$

$\underline{y}\ (0;\gamma)=\alpha-2,\overline{y}\ (0;\gamma)=2-\alpha$.

We note that:

$$y\ (\aleph)=\left(\underline{y}\ (\aleph;\gamma),\overline{y}\ (\aleph;\gamma)\right)$$

By using fuzzy Sadik transform for both sides of equation 3 we get:

$$S\left[y'(\aleph)\right]=S\left[-y\ (\aleph)\right]$$

Now, we shall solve this example for 2 cases as follows:

Case 1.

$$v^{\alpha}S\left[y\ (\aleph)\right]\ominus v^{-\beta}y\ (0)=S\left[-y\ (\aleph)\right] \tag{4}$$

Equation (4) becomes:

$$v^{\alpha}S\left[\underline{y}(\aleph;\gamma)\right]-v^{-\beta}\underline{y}(0;\gamma)=-S\left[\underline{y}(\aleph;\gamma)\right],$$

$$v^{\alpha}S\left[\overline{y}(\aleph;\gamma)\right]-v^{-\beta}\overline{y}(0;\gamma)=-S\left[\overline{y}(\aleph;\gamma)\right]$$

Then by solve above equation we get:

$$S\left[\underline{y}\ (\aleph;\gamma)\right]=\frac{v^{-\beta}}{\left(v^{\alpha}+1\right)}\alpha-1,S\left[\overline{y}\ (\aleph;\gamma)\right]=\frac{v^{-\beta}}{\left(v^{\alpha}+1\right)}1-\alpha$$

By using inverse fuzzy Sadik transform we get:

$$\underline{y}\ (\aleph;\gamma)=e^{-\aleph}\left(\alpha-1\right),\overline{y}\ (\aleph;\gamma)=e^{-\aleph}\left(1-\alpha\right)$$

Case 2.

$$-v^{-\beta}y\ (0)\ominus\left(-v^{\alpha}\right)S\left[y\ (\aleph)\right]=-S\left[y\ (\aleph)\right] \tag{5}$$

Equation 5 becomes:

$$-v^{-\beta}\overline{y}(0;\gamma)+v^{\alpha}S\left[\overline{y}(\aleph;\gamma)\right]=-S\left[\underline{y}(\aleph;\gamma)\right],$$

$$-v^{-\beta}\underline{y}(0;\gamma)+v^{\alpha}S\left[\underline{y}(\aleph;\gamma)\right]=-S\left[\overline{y}(\aleph;\gamma)\right]$$

Then by solve above equation and using inverse fuzzy Sadik transform we get:

$$\bar{y}(\aleph;\gamma)=\cosh(\aleph)(1-\alpha),\underline{y}(\aleph;\gamma)=\sinh(\aleph)(\alpha-1)$$

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