

Some Multiplicative Temperature Indices of $HC_5C_7 [p, q]$ Nanotubes

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Abstract. In Chemical Science, connectivity indices are applied to measure the chemical characteristics of chemical compounds. In this paper, we compute the multiplicative first and second temperature indices, multiplicative first and second hyper temperature indices, multiplicative sum connectivity temperature index, multiplicative product connectivity temperature index, reciprocal multiplicative product temperature index, general multiplicative first and second temperature indices, multiplicative atom bond connectivity temperature index, multiplicative geometric-arithmetic temperature index, multiplicative arithmetic-geometric temperature index, multiplicative F -temperature index, general multiplicative temperature index of $HC_5C_7 [p, q]$ nanotubes.

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1. Introduction

Chemical Graph Theory has an important effect on the development of Chemical Sciences. A molecular graph or a chemical graph is a graph whose vertices correspond to atoms and edges to the bonds. A topological index or a graph index is a numeric quality from the structure of a molecule. Several graph indices are used in QSPR/QSAR study [1, 2].

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of a vertices adjacent to u . We refer [3] for undefined definitions and notations.

In [4], Fajtlowicz defined the temperature of a vertex u of a connected graph G as

$$T(u) = \frac{d_G(u)}{n - d_G(u)}, \quad \text{where } |V(G)| = n.$$

Motivated by the work on degree based multiplicative topological indices, we introduce the multiplicative temperature indices of a graph as follows:

The multiplicative first and second temperature indices of a graph G are defined as

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$$TH_1(G) = \prod_{uv \in E(G)} [T(u) + T(v)], \quad TH_2(G) = \prod_{uv \in E(G)} T(u)T(v).$$

The multiplicative first and second hyper temperature indices of a graph G are defined as

$$HTH_1(G) = \prod_{uv \in E(G)} [T(u) + T(v)]^2, \quad HTH_2(G) = \prod_{uv \in E(G)} [T(u)T(v)]^2.$$

Motivated by the work on multiplicative connectivity indices [2], we define the following connectivity temperature indices:

The multiplicative sum and product connectivity temperature indices of a graph G are defined as

$$STH(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{T(u) + T(v)}}, \quad PTH(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{T(u)T(v)}}.$$

The reciprocal multiplicative product connectivity temperature index of a graph G is defined as

$$RPTH(G) = \prod_{uv \in E(G)} \sqrt{T(u)T(v)}.$$

The general multiplicative first and second temperature indices of a graph G are defined as

$$TH_1^a(G) = \prod_{uv \in E(G)} [T(u) + T(v)]^a, \quad TH_2^a(G) = \prod_{uv \in E(G)} [T(u)T(v)]^a,$$

where a is a real number.

We introduce the following multiplicative temperature indices of a graph as follows:

The multiplicative atom bond connectivity temperature index of a graph G is defined as

$$ABCTH(G) = \prod_{uv \in E(G)} \sqrt{\frac{T(u) + T(v) - 2}{T(u)T(v)}}.$$

The multiplicative geometric-geometric temperature index of a graph G is defined as

$$GATH(G) = \prod_{uv \in E(G)} \frac{2\sqrt{T(u)T(v)}}{T(u) + T(v)}.$$

The multiplicative arithmetic-geometric temperature index of a graph G is defined as

$$AGTH(G) = \prod_{uv \in E(G)} \frac{T(u) + T(v)}{2\sqrt{T(u)T(v)}}.$$

The multiplicative F -temperature index of a graph G is defined as

$$FTH(G) = \prod_{uv \in E(G)} [T(u)^2 + T(v)^2].$$

The general multiplicative temperature index of a graph G is defined as

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$$TH_a(G) = \prod_{uv \in E(G)} [T(u)^a + T(v)^a].$$

Recently, some temperature indices were studied in [5]. Recently, some new multiplicative graph indices were studied, for example, in [6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24]. In this paper, some multiplicative temperature indices of $HC_5C_7 [p, q]$ nanotubes are computed.

2. Results for $HC_5C_7 [p, q]$ nanotube

In this section, we consider $HC_5C_7 [p, q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. The 2-D lattice of $HC_5C_7 [8, 4]$ nanotube is presented in Figure 1.

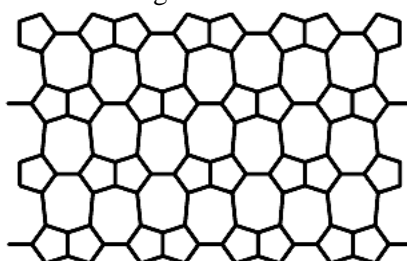


Figure 1: 2-D lattice of $HC_5C_7 [8, 4]$ nanotube

Let G be a graph of a nanotube $HC_5C_7 [p, q]$. By calculation, G has $4pq$ vertices and $6pq - p$ edges. Clearly, G has two types of edges based on the degree of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) \mid d_G(u)=2, d_G(v) = 3\}, \quad |E_1| = 4p$$

$$E_2 = \{uv \in E(G) \mid d_G(u)=d_G(v) = 3\}, \quad |E_2| = 6pq - 5p.$$

Therefore in G , there are two types of edges based on the temperature of the vertices of each edge as given in Table 1.

$T(u), T(v) \setminus uv \in E(G)$	$\left(\frac{2}{4pq-2}, \frac{3}{4pq-3} \right)$	$\left(\frac{3}{4pq-3}, \frac{3}{4pq-3} \right)$
Number of edges	$4p$	$6pq - 5p$

Table 1: Edge partition of G

Theorem 1. The general multiplicative first temperature index of a nanotube $HC_5C_7 [p, q]$ is

$$TH_1^a (HC_5C_7 [p, q]) = \left[\frac{20pq - 12}{(4pq - 2)(4pq - 3)} \right]^{4ap} \left(\frac{6}{4pq - 3} \right)^{a(6pq - 5p)}. \tag{1}$$

Proof: Let $G = HC_5C_7 [p, q]$. By definition, we have

$$TH_1^a (G) = \prod_{uv \in E(G)} [T(u) + T(v)]^a.$$

Thus by using Table 1, we obtain

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$$\begin{aligned} TII_1^a(HC_5C_7[p, q]) &= \left[\frac{2}{4pq-2} + \frac{3}{4pq-3} \right]^{4ap} \left(\frac{3}{4pq-3} + \frac{3}{4pq-3} \right)^{a(6pq-5p)} \\ &= \left[\frac{20pq-12}{(4pq-2)(4pq-3)} \right]^{4ap} \left(\frac{6}{4pq-3} \right)^{a(6pq-5p)}. \end{aligned}$$

From Theorem 1, we establish the following results.

Corollary 1.1. The multiplicative first temperature index of a nanotube $HC_5C_7[p, q]$ is

$$TII_1(HC_5C_7[p, q]) = \left[\frac{20pq-12}{(4pq-2)(4pq-3)} \right]^{4p} \left(\frac{6}{4pq-3} \right)^{6pq-5p}.$$

Corollary 1.2. The multiplicative first hyper temperature index of a number $HC_5C_7[p, q]$ is

$$HTII_1(HC_5C_7[p, q]) = \left[\frac{20pq-12}{(4pq-2)(4pq-3)} \right]^{8p} \left(\frac{6}{4pq-3} \right)^{12pq-10p}.$$

Corollary 1.3. The multiplicative sum connectivity temperature index of a nanotube $HC_5C_7[p, q]$ is

$$STII(HC_5C_7[p, q]) = \left[\frac{(4pq-2)(4pq-3)}{20pq-12} \right]^{2p} \times \left(\frac{4pq-3}{6} \right)^{3pq-\frac{5}{2}p}.$$

Proof: Put $a = 1, 2, -1/2$ in equation (1), we obtain the desired results

Theorem 2. The general multiplicative second temperature index of a nanotube $HC_5C_7[p, q]$ is given by

$$TII_2^a(HC_5C_7[p, q]) = \left[\frac{6}{(4pq-2)(4pq-3)} \right]^{4ap} \times \left(\frac{3}{4pq-3} \right)^{2a(6pq-5p)}. \quad (2)$$

Proof: Let $G = HC_5C_7[p, q]$. By definition, we have

$$TII_2^a(G) = \prod_{uv \in E(G)} [T(u)T(v)]^a.$$

By using Table 1, we deduce

$$\begin{aligned} TII_2^a(HC_5C_7[p, q]) &= \left[\frac{2}{4pq-2} \times \frac{3}{4pq-3} \right]^{4ap} \times \left(\frac{3}{4pq-3} \times \frac{3}{4pq-3} \right)^{a(6pq-5p)} \\ &= \left[\frac{6}{(4pq-2)(4pq-3)} \right]^{4ap} \times \left(\frac{3}{4pq-3} \right)^{2a(6pq-5p)}. \end{aligned}$$

We obtain the following results by using Theorem 2.

Corollary 2.1. The multiplicative second temperature index of a nanotube $HC_5C_7[p, q]$ is

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$$TH_2 = (HC_5C_7[p, q]) = \left[\frac{6}{(4pq-2)(4pq-3)} \right]^{4p} \times \left(\frac{3}{4pq-3} \right)^{12pq-10p}.$$

Corollary 2.2. The multiplicative second hyper temperature index of a nanotube $HC_5C_7[p, q]$ is

$$HTH_2(HC_5C_7[p, q]) = \left[\frac{6}{(4pq-2)(4pq-3)} \right]^{8p} \times \left(\frac{3}{4pq-3} \right)^{24pq-20p}.$$

Corollary 2.3. The multiplicative product connectivity temperature index of a nanotube $HC_5C_7[p, q]$ is

$$PTH(HC_5C_7[p, q]) = \left[\frac{1}{6}(4pq-2)(4pq-3) \right]^{2p} \times \left[\frac{1}{3}(4pq-3) \right]^{6pq-5p}.$$

Corollary 2.4. The reciprocal multiplicative product connectivity temperature index of a nanotube $HC_5C_7[p, q]$ is

$$RPTH(HC_5C_7[p, q]) = \left[\frac{6}{(4pq-2)(4pq-3)} \right]^{2p} \times \left(\frac{3}{4pq-3} \right)^{6pq-5p}.$$

Proof: Put $a = 1, 2, -1/2, 1/2$ is equation (2), we get the desired results.

Theorem 3. The multiplicative atom bond connectivity temperature index of a nanotube $HC_5C_7[p, q]$ is given by

$$ABCTH(HC_5C_7[p, q]) = \left[\frac{10}{3}pq - 2 - \frac{1}{3}(4pq-2)(4pq-3) \right]^{2p} \\ \times \left[\frac{1}{9}\{6 - 2(4pq-3)\}(4pq-3) \right]^{3pq - \frac{5}{2}p}.$$

Proof: Let $G = HC_5C_7[p, q]$. By definition, we have

$$ABCTH(G) = \prod_{u,v \in E(G)} \sqrt{\frac{T(u)+T(v)-2}{T(u)T(v)}}.$$

By using Table 1, we deduce

$$ABCTH(HC_5C_7[p, q]) = \left[\frac{\frac{2}{4pq-2} + \frac{3}{4pq-3} - 2}{\frac{2}{4pq-2} \times \frac{3}{4pq-3}} \right]^{\frac{4p}{2}} \times \left[\frac{\frac{3}{4pq-3} + \frac{3}{4pq-3} - 2}{\frac{3}{4pq-3} \times \frac{3}{4pq-3}} \right]^{\frac{6pq-5p}{2}} \\ = \left[\frac{10}{3}pq - 2 - \frac{1}{3}(4pq-2)(4pq-3) \right]^{2p}$$

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$$\times \left[\frac{1}{9} \{6 - 2(4pq - 3)\}(4pq - 3) \right]^{3pq - \frac{5}{2}p}.$$

Theorem 4. The multiplicative geometric-arithmetic temperature index of a nanotube $HC_5C_7[p, q]$ is

$$GATH(HC_5C_7[p, q]) = \left[\frac{\sqrt{6(4pq - 2)(4pq - 3)}}{10pq - 6} \right]^{4p}.$$

Proof: Let $G = HC_5C_7[p, q]$. By definition, we have

$$GATH(G) = \prod_{uv \in E(G)} \frac{2\sqrt{T(u)T(v)}}{T(u) + T(v)}.$$

By using Table 1, we derive

$$\begin{aligned} GATH(HC_5C_7[p, q]) &= \left[\frac{2\sqrt{\frac{2}{4pq-2} \times \frac{3}{4pq-3}}}{\frac{2}{4pq-2} + \frac{3}{4pq-3}} \right]^{4p} \times \left[\frac{2\sqrt{\frac{3}{4pq-3} \times \frac{3}{4pq-3}}}{\frac{3}{4pq-3} + \frac{3}{4pq-3}} \right]^{6pq-5p} \\ &= \left[\frac{\sqrt{6(4pq-2)(4pq-3)}}{10pq-6} \right]^{4p}. \end{aligned}$$

Theorem 5. The multiplicative arithmetic-temperature index of a nanotube $HC_5C_7[p, q]$ is

$$AGTH(HC_5C_7[p, q]) = \left[\frac{10pq - 6}{\sqrt{6(4pq - 2)(4pq - 3)}} \right]^{4p}.$$

Proof: Let $G = HC_5C_7[p, q]$. By definition, we have

$$AGTH(G) = \prod_{uv \in E(G)} \frac{T(u) + T(v)}{2\sqrt{T(u)T(v)}}.$$

By using Table 1, we deduce

$$\begin{aligned} AGTH(HC_5C_7[p, q]) &= \left[\frac{\frac{2}{4pq-2} + \frac{3}{4pq-3}}{2\sqrt{\frac{2}{4pq-2} \times \frac{3}{4pq-3}}} \right]^{4p} \times \left[\frac{\frac{3}{4pq-3} + \frac{3}{4pq-3}}{2\sqrt{\frac{3}{4pq-3} \times \frac{3}{4pq-3}}} \right]^{6pq-5p} \\ &= \left[\frac{10pq - 6}{\sqrt{6(4pq - 2)(4pq - 3)}} \right]^{4p}. \end{aligned}$$

Theorem 6. The general multiplicative index of a nanotube $HC_5C_7[p, q]$ is

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$$TH_a(HC_5C_7[p, q]) = \left[\left(\frac{2}{4pq-2} \right)^a + \left(\frac{3}{4pq-3} \right)^a \right]^{4p} \times \left[2 \left(\frac{3}{4pq-3} \right)^a \right]^{6pq-5p} \quad (3)$$

Proof: Let $HC_5C_7[p, q]$. By definition, we have

$$TH_a(G) = \prod_{uv \in E(G)} [T(u)^a + T(v)^a].$$

Thus by using Table 1, we obtain

$$\begin{aligned} TH_a(HC_5C_7[p, q]) &= \left[\left(\frac{2}{4pq-2} \right)^a + \left(\frac{3}{4pq-3} \right)^a \right]^{4p} \times \left[\left(\frac{3}{4pq-3} \right)^a + \left(\frac{3}{4pq-3} \right)^a \right]^{6pq-5p} \\ &= \left[\left(\frac{2}{4pq-2} \right)^a + \left(\frac{3}{4pq-3} \right)^a \right]^{4p} \times \left[2 \left(\frac{3}{4pq-3} \right)^a \right]^{6pq-5p} \end{aligned}$$

From Theorem 6, we establish the following result.

Corollary 6.1. The multiplicative F -temperature index of a nanotube $HC_5C_7[p, q]$ is

$$FTII(HC_5C_7[p, q]) = \left[\frac{4}{(4pq-2)^2} + \frac{9}{(9pq-3)^a} \right]^{4p} \times \left[\frac{18}{(4pq-3)^2} \right]^{6pq-5p}.$$

Proof: Put $a = 2$ in equation (3), we get the desired result.

3. Conclusion

In this paper, we have established the expressions for the multiplicative first and second temperature indices, multiplicative first and second hyper temperature indices, multiplicative sum and product connectivity temperature indices, multiplicative atom bond connectivity temperature index, multiplicative geometric-arithmetic temperature index, multiplicative arithmetic-geometric temperature index, multiplicative F -temperature index of $HC_5C_7[p, q]$ nanotubes.

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