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Multiplicative Kulli-Basava Indices

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Abstract. A topological index is a numeric quantity from the graph structure of a molecule. In this paper, we introduce the multiplicative total Kulli-Basava index, multiplicative modified first Kulli-Basava index, multiplicative *F*-Kulli-Basava index, general multiplicative Kulli-Basava index of a graph. We compute these indices for regular graphs, wheel, gear and helm graphs.

Keywords: Multiplicative *F*-Kulli-Basava index, general multiplicative Kulli-Basava index, regular, wheel, gear helm graphs

AMS Mathematics Subject Classification (2010): 05C05, 05C12, 05C35

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and the edge set. Let $d_G(v)$ denote the degree of a vertex v. The degree of an edge $e = uv$ in a graph G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. Let $S_e(v)$ denote the sum of the degrees of all edges incident to a vertex v. For undefined term and notation, we refer [1]. Topological indices have their applications in various disciplines of Science and Technology.

The modified first Kulli-Basava index was introduced in [2], defined as

$$KB_1^*(G) = \sum_{u \in V(G)} S_e(u)^2.$$

In [3], Kulli introduced the total Kulli-Basava index and *F*-Kulli-Basava index of a graph, defined as

$$TKB(G) = \sum_{u \in V(G)} S_e(u),$$

$$FKB(G) = \sum_{u \in V(G)} S_e(u)^3.$$

Recently, the hyper Kulli-Basava indices [4], connectivity Kulli-Basava indices [5], square Kulli-Basava index [6], multiplicative Kulli-Basava and multiplicative hyper Kulli-Basava indices [7] were introduced and studied.

We introduce the following multiplicative Kulli-Basava indices:

The multiplicative total Kulli-Basava index of a graph G is defined as

$$TKBII(G)=\prod_{u\in V(G)} S_e(u).$$

The multiplicative modified first Kulli-Basava index of a graph G is defined as

$$KB_1^*II(G)=\prod_{u\in V(G)} S_e(u)^2.$$

The multiplicative F-Kulli-Basava index of a graph G is defined as

$$FKBII(G)=\prod_{u\in V(G)} S_e(u)^3.$$

The multiplicative Kulli-Basava zeroth order index of a graph G is defined as

$$KBZII(G)=\prod_{u\in V(G)} \frac{1}{\sqrt{S_e(u)}}.$$

The multiplicative Kulli-Basava inverse degree of a graph G is defined as

$$KBIDII(G)=\prod_{u\in V(G)} \frac{1}{S_e(u)}.$$

The multiplicative modified inverse first Kulli-Basava index of a graph G is defined as

$$^{m}KB_1^*II(G)=\prod_{u\in V(G)} \frac{1}{S_e(u)^2}.$$

The general multiplicative Kulli-Basava index of a graph G is defined as

$$KB^aII(G)=\prod_{u\in V(G)} S_e(u)^a \tag{1}$$

where a is a real number.

In recent years, some multiplicative topological indices were studied, for example, in [8, 9, 10, 11, 12].

In this study, we compute explicit formulas for computing the multiplicative total Kulli-Basava index, multiplicative modified first Kulli-Basava index, multiplicative F-Kulli-Basava index, multiplicative zeroth order index, multiplicative Kulli-Basava inverse degree, multiplicative modified inverse first Kulli-Basava index and general multiplicative Kulli-Basava index of regular, complete, cycle, wheel, gear and helm graphs.

2. Results for regular graphs

A graph G is an r-regular graph if the degree of each vertex of G is r.

Theorem 1. Let G be an r-regular graph with n vertices. Then the general multiplicative Kulli-Basava index of G is

$$KB^aII(G)=[2r(r-1)]^{an}. \tag{2}$$

Proof: Let G be an r-regular graph with n vertices. Then $S_e(u) = 2r\ (r - 1)$ for every vertex u of G. Thus

$$KB^aII(G)=\prod_{u\in V(G)} S_e(u)^a=[2r(r-1)]^{an}.$$

Corollary 1.1. If G is an r-regular graph with n vertices, then

(i) $TKBII(G)=[2r(r-1)]^n$. (ii) $KB_1^*II(G)=[2r(r-1)]^{2n}$.

(iii) $FKBII(G)=[2r(r-1)]^{3n}$. (iv) $KBZII(G)=\left[\frac{1}{2r(r-1)}\right]^{\frac{n}{2}}$.

(v) $KBIDII(G)=\left[\frac{1}{2r(r-1)}\right]^{n}$. (vi) ${}^{m}KB_1^*II(G)=\left[\frac{1}{2r(r-1)}\right]^{2n}$.

Proof: Put a = 1, 2, 3, $-\frac{1}{2}$, –1, –2 in equation (2), we obtain the desired results.

Corollary 1.2. If K_n is a complete graph with n vertices, then

(i) $TKBII(K_n)=[2(n-1)(n-2)]^n$.

(ii) $KB_1^*II(K_n)=4^n[(n-1)(n-2)]^{2n}$.

(iii) $FKBII(K_n)=8^n[(n-1)(n-2)]^{3n}$.

(iv) $KBZII(K_n)=\left[\frac{1}{2(n-1)(n-2)}\right]^{\frac{n}{2}}$.

(v) $KBIDII(K_n)=\left[\frac{1}{2(n-1)(n-2)}\right]^{n}$.

(vi) ${}^{m}KB_1^*II(K_n)=\left[\frac{1}{2(n-1)(n-2)}\right]^{2n}$.

Proof: Put r = n – 1 and a =1, 2, 3, $-\frac{1}{2}$, –1, –2, in equation (2), we get the desired results.

Corollary 1.3. If C_n is a cycle with n vertices, then

(i) $TKBII(C_n)=4^n$. (ii) $KB_1^*II(C_n)=16^n$.

(iii) $FKBII(C_n)=64^n$. (iv) $KBZII(C_n)=\left[\frac{1}{2}\right]^{n}$.

(v) $KBIDII(C_n)=\left[\frac{1}{4}\right]^{n}$. (vi) ${}^{m}KB_1^*II(C_n)=\left[\frac{1}{16}\right]^{n}$.

Proof: Put r = 2, and a = 1, 2, 3, $-\frac{1}{2}$, –1, –2 in equation (2), we obtain the required results.

3. **Results for wheel graphs**

A wheel W_n is the join of C_n and K_1. Clearly W_n has n+1 vertices and $2n$ edges. The vertices of C_n are called rim vertices and the vertex of K_1 is called apex. A graph W_n is presented in Figure 1.

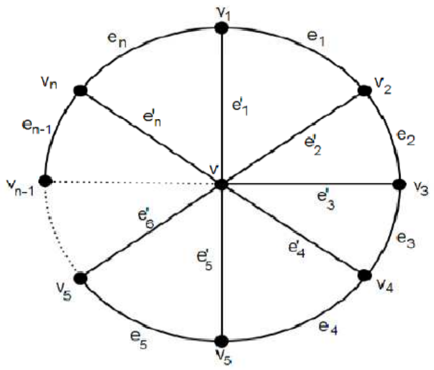

Figure 1: Wheel W_n

Let W_n be a wheel graph with n+1 vertices and 2n edges Then W_n has two types of vertices as given in Table 1.

$S_e(u) \setminus u \in V(W_n)$	$n(n+1)$	n+9
Number of vertices	1	n

Table 1: Vertex partition of W_n

Theorem 2. Let W_n be a wheel graph with n+1 vertices. Then the general multiplicative Kulli-Basava index of W_n is

$$KB^a II(W_n) = n^a (n+1)^a (n+9)^{an}. \tag{3}$$

Proof: From equation (1) and by using Table 1, we have

$$KB^a II(W_n) = \prod_{u \in V(W_n)} S_e(u)^a = n^a (n+1)^a (n+9)^{an}.$$

Corollary 2.1. If W_n is a wheel graph with n+1 vertices, then

(i) $TKBII(W_n) = n(n+1)(n+9)^n$.

(ii) $KB_1^* II(W_n) = n^2 (n+1)^2 (n+9)^{2n}$.

(iii) $FKBII(W_n) = n^3 (n+1)^3 (n+9)^{3n}$.

(iv) $KBZII(W_n) = \dfrac{1}{\sqrt{n(n+1)(n+9)^n}}$.

(v) $KBIDII(W_n) = \dfrac{1}{n(n+1)(n+9)^n}$.

(vi) $\quad {}^{m}KB_1^{*}II(W_n) = \dfrac{1}{n^2(n+1)^2(n+9)^{2n}}$.

Proof: Put a = 1, 2, 3, $-\frac{1}{2}$, –1, –2 in equation (3), we get the required results.

4. Results for gear graphs

A gear graph G_n is a graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices. Clearly G_n has $2n$+1 vertices and $3n$ edges. A gear graph G_n is shown in Figure 2.

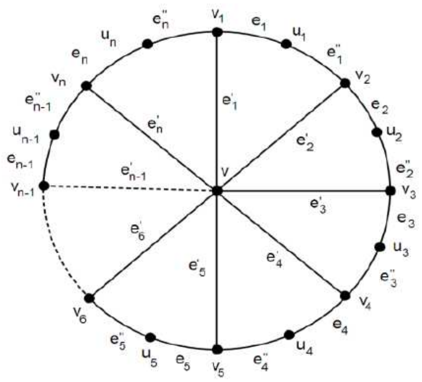

Figure 2: Gear graph G_n

A gear graph G_n has three types of vertices as given in Table 2.

$S_e(u) \setminus u \in V(G_n)$	$n(n+1)$	$n+7$	6
Number of vertices	1	n	n

Table 2: Vertex partition of G_n

Theorem 3. Let G_n be a gear graph with $2n$+1 vertices. Then the general multiplicative Kulli-Basava index of G_n is

$$KB^a II(G_n) = 6^{an} n^a (n+1)^a (n+7)^{an}. \tag{4}$$

Proof: By using equation (1) and Table 2, we deduce

$$KB^a II(G_n) = \prod_{u \in V(G_n)} S_e(u)^a = [n(n+1)]^a (n+7)^{an} 6^{an}.$$

$$= 6^{an} n^a (n+1)^a (n+7)^{an}.$$

Corollary 3.1. If G_n is a gear graph with $2n$+1 vertices, then

(i) $\quad TKBII(G_n) = 6^n n(n+1)(n+7)^n$.

(ii) $\quad KB_1^{*}II(G_n) = 6^{2n} n^2 (n+1)^2 (n+7)^{2n}$.

(iii) $\quad FKBII(G_n) = 6^{3n} n^3 (n+1)^3 (n+7)^{3n}$.

(iv) $KBZII(G_n)=\dfrac{1}{\sqrt{6^n n(n+1)(n+7)^n}}.$

(v) $KBIDII(G_n)=\dfrac{1}{6^n n(n+1)(n+7)^n}.$

(vi) ${}^m KB_1^* II(G_n)=\dfrac{1}{6^{2n} n^2 (n+1)^2 (n+7)^{2n}}.$

Proof: Put a = 1, 2, 3, $-\dfrac{1}{2}$, –1, –2 in equation (4), we obtain the required results.

5. Results for helm graphs

A helm graph H_n is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly H_n has 2n+1 vertices and 3n edges. A helm graph H_n is shown in Figure 3.

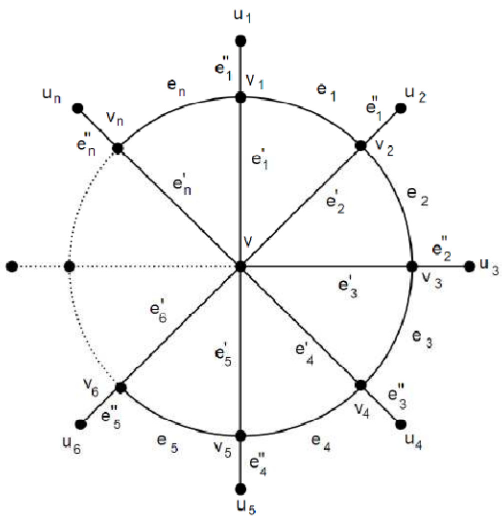

Figure 3: Helm graph H_n

A helm graph H_n has three types of vertices as given in Table 3.

$S_e(u) \setminus u \in V(H_n)$	$n(n+2)$	$n+17$	3
Number of vertices	1	n	n

Table 3: Vertex partition of H_n

Theorem 4. Let H_n be a helm graph with 2n+1 vertices and 3n edges. Then the general multiplicative Kulli-Basava index of H_n is

$$KB^a II(H_n)=3^{an} n^a (n+2)^a (n+17)^{an}. \tag{5}$$

Proof: By using equation (1) and Table 3, we derive

$$KB^a II(H_n)=\prod_{u\in V(H_n)} S_e(u)^a = n^a (n+2)^a (n+17)^{an} 3^{an}.$$

Corollary 4.1. Let H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then

(i) $TKBII(H_n) = 3^n n(n+2)(n+17)^n$.

(ii) $KB_1^* II(H_n) = 3^{2n} n^2 (n+2)^2 (n+17)^{2n}$.

(iii) $FKBII(H_n) = 3^{3n} n^3 (n+2)^3 (n+17)^{3n}$.

(iv) $KBZII(H_n) = \dfrac{1}{\sqrt{3^n n(n+2)(n+17)^n}}$.

(v) $KBIDII(G_n) = \dfrac{1}{3^n n(n+2)(n+17)^n}$.

(vi) ${}^m KB_1^* II(H_n) = \dfrac{1}{3^{2n} n^2 (n+2)^2 (n+17)^{2n}}$.

Proof: Put a = 1, 2, 3, $-\frac{1}{2}$, –1, –2 in equation (5), we get the desired results.

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