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Characterization of Recent Concepts in Soft Bitopological Space

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Abstract. This paper aims at introducing a soft topology via soft preopen sets. We define soft preopen sets, soft preclosed sets, and prove some of its properties.

Keywords: soft preopen sets, soft preclosed sets, soft preclosure, soft preinterior, soft pre continuous, soft preirresolute.

AMS Mathematics Subject Classification (2010): 54A40

1. Introduction

Soft set was introduced by Molodtsov [9] in the year 1999. Soft topology was introduced by Shabir and Naz [13] in 2011. Cagman et al. [3] introduced soft limit points, soft Hausdroff space etc. Sabir and Naz [13] also defined and discussed properties of soft interior, soft exterior and soft boundary. Subhashini and Sekar[14] defined soft pre-open sets in a soft topological space. Alkazaleh [1] explained possibilities of fuzzy soft sets and Ahmad and Kharal [2] analyzed fuzzy soft sets. The parametrization of soft sets and its applications were explained by Chen [4] and Maji [8]. The topological structure of fuzzy soft sets defined by Tanay [15]. Feng [5] and Jun [6] introduced some basic properties on soft sets. Kelly [7] defined Bitopological spaces.

Recently, Rajesh and Srinivasan [11] introduced Fuzzy sets of second type and Prabhu et al. [10] explained about e* - continuous mapping and Fuzzy e*-compactness in topological spaces. Also Ranu et al. [12] showed some Fuzzy α_q opensets. In this paper we define some properties on soft pre open sets, soft pre closed sets, soft pre interior, soft pre closure, soft pre irresolute and soft pre continuous.

Definition 1.1. Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P (U).

Definition 1.2. Let (U,A,τ) be a soft topological space and let (G,A) be a soft set. Then
(i) The soft closure of (G,A) is the soft set
$\tilde{s}cl(G,A)= \cap \{(S,A):(S,A)$ is soft closed and $(G,A)\subseteq(S,A)\}$
(ii) The soft interior of (G,A) is the soft set
$\tilde{s}int(G,A)= \cup\{(S,A): (S,A)$ is soft open and $(S,A)\subseteq(G,A)\}$

Theorem 1.3. Let (U,A,τ) be a soft topological space and let (F,A) and (G,A) be soft sets over U. Then,
(i) (F,A) is soft closed iff (F,A) = $\tilde{s}$cl(F,A)
(ii)(G,A) is soft open iff (G,A) = $\tilde{s}$int(G,A)

Theorem 1.4. Arbitrary union of soft open sets is soft open and finite intersection of soft closed set is soft closed.

Remark 1.5. If $\{(G,A)_\alpha \mid \alpha \in I\}$ is a collection of soft sets, then
(i) $U\tilde{s}\mathrm{int}(G,A)_\alpha \subseteq \tilde{s}\mathrm{int}(U(G,A)_\alpha)$
(ii) $U\tilde{s}\mathrm{cl}(G,A)_\alpha \subseteq \tilde{s}\mathrm{cl}(U(G,A)_\alpha)$

Remark 1.6. $\tilde{s}$cl(G,A) is the smallest soft closed set containing (G,A) and $\tilde{s}$int(G,A) is the largest soft open set contained in (G,A).

Remark 1.7. If (F,A) and (K,A) are any two soft sets in (U,A,τ) then
$U_A - ((F,A)\cap(K,A)) = (U_A - (F,A))U(U_A - (K,A))$.

Theorem 1.8. (i) For every soft open set (G,A) in a soft topological space (U,A,τ) and every soft set (K,A) we have $\tilde{s}\mathrm{cl}(K,A)\cap(G,A) \subseteq \tilde{s}\mathrm{cl}((K,A) \cap (G,A))$
(ii) For every soft closed set (F,A) in a soft topological space (U,A,τ) and every soft set (K,A) we have $\tilde{s}\mathrm{int}(K,A)\, \tilde{U}(F,A) \subseteq \tilde{s}\mathrm{int}((K,A)\tilde{U}(F,A))$

Example 1.9. Suppose that there are five trees in the Universe. Let $U= \{t_1,t_2,t_3,t_4,t_5\}$ under consideration, and that $E = \{p_1,p_2,p_3,p_4,p_5,p_6,p_7,p_8\}$ is a set of decision parameters then the p_i(i =1,2,3,4,5,6,7,8} stands for the parameters "strong wood", "medicinal use", "beautiful flower", "tasty fruit", "speed growth", "slow growth", "small tree", "big tree", respectively. In this case, to define a soft set means to point out strong wood, medicinal use and so on.

Consider the mapping f_E given by "trees(.)", where (.) is to be filled in by one of the parameters $p_i \in E$. For instance, $f_E(p_1)$ means "tree have(strong wood)", and its functional value is the set $\{t \in U$: t have strong wood} and so, let $A \subseteq E$, the soft set F_A that describes the "Best of trees" in the opinion of the forest officer say Kumar, may be defined like, $A = \{p_2,p_3,p_4\ ,p_5,p_7\}$, $F_A(p_2)=\{ t_2, t_3, t_5,\}$, $F_A(p_3)=\{ t_2, t_4\}$, $F_A(p_4)=\{ t_1\}$, $F_A(p_5) =\{U\}$ and $F_A(p_7)=\{t_3,t_5\}$.
The soft set F_A as consisting of the following collection of approximations:
$F_A = \{(p_2,\{t_2, t_3, t_5,\}),(p_3,\{t_2, t_4\}),(p_4,\{t_1\}),(p_5,\{U\}),(p_7,\{t_3,t_5\})\}$.

2. Soft pre open sets

Definition 2.1. Let (U,A,τ) be a soft topological space. If (G,A) is said to be soft pre open set then $(G,A) \subseteq \tilde{s}\mathrm{int}(\tilde{s}\mathrm{cl}(G,A))$

Definition 2.2. If (F,A) is said to be soft pre closed set then $(F,A) \supseteq \tilde{s}\mathrm{cl}(\tilde{s}\mathrm{int}(F,A))$

Remark 2.3. If$\{(G,A)_\alpha \mid \alpha \in I\}$ is a collection of soft sets, then
(i) $U\ \tilde{s}\mathrm{int}(G,A)_\alpha \subseteq \tilde{s}\mathrm{int}(U(G,A)_\alpha)$

(ii) U $\tilde{s}$cl$(G,A)_\alpha \subseteq \tilde{s}cl(U(G,A)_\alpha)$

Theorem 2.4. Arbitrary union of soft pre open sets is a soft pre open set.
Proof: Let (U,A,τ) be a soft topological space.
Let $\{(G,A)_\alpha \mid \alpha \in I\}$ be a collection of soft pre open set in the soft topological space (U,A,τ) then $(G,A)_\alpha \subseteq \tilde{s}int(\tilde{s}cl(G,A)_\alpha)$ $\forall$ $\alpha \in I$
To prove that $U(G,A)_\alpha \subseteq \tilde{s}int(\tilde{s}cl(U(G,A)_\alpha))$ $\forall$ $\alpha \in I$
By definition 2.1, We can write the collection of soft pre open set
$U(G,A)_\alpha \subseteq U\tilde{s}int(\tilde{s}cl(G,A)_\alpha)$ $\forall$ $\alpha \in I$
$\subseteq \tilde{s}$int$(U\tilde{s}$cl$(G,A)_\alpha)$ $\forall$ $\alpha \in I$ (by 1.5 Remark (i))
$\subseteq \tilde{s}$int$(\tilde{s}$cl$(U(G,A)_\alpha))$ $\forall$ $\alpha \in I$ (by 1.5 Remark (ii))
Therefore $U(G,A)_\alpha \subseteq \tilde{s}$int $(\tilde{s}$cl$(U(G,A)_\alpha))$ $\forall$ $\alpha \in I$

Theorem 2.5. Arbitrary intersection of soft pre closed set is soft pre closed.
Proof: Let (U,A,τ) be a soft topological space.
To prove that $\bigcap(F,A)_\alpha \supseteq \tilde{s}cl(\tilde{s}int(\bigcap(F,A)_\alpha))$ $\forall$ $\alpha \in I$
By theorem 2.4 ,Taking complement on both sides we get,
$(U(F,A)_\alpha)^c \subseteq (\tilde{s}int(\tilde{s}cl(U(F,A)_\alpha)))^c$ $\forall$ $\alpha \in I$
Therefore $\bigcap(F,A)_\alpha \supseteq \tilde{s}cl(\tilde{s}int(\bigcap(F,A)_\alpha))$ $\forall$ $\alpha \in I$

Example 2.6. Let $U = \{h_1,h_2\}$, $A=\{e_1,e_2\}$. We define
$(F,A)_1 = \{(e_1,\phi),(e_2,\phi)\}$
$(F,A)_2 = \{(e_1,\phi),(e_2,\{h_1\})\}$
$(F,A)_3 = \{(e_1,\phi),(e_2,\{h_2\})\}$
$(F,A)_4 = \{(e_1,\phi),(e_2,\{h_1,h_2\})\}$
$(F,A)_5 = \{(e_1,\{h_1\}),(e_2,\phi)\}$
$(F,A)_6 = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}$
$(F,A)_7 = \{(e_1,\{h_1\}),(e_2,\{h_2\})\}$
$(F,A)_8 = \{(e_1,\{h_1\}),(e_2,\{h_1,h_2\})\}$
$(F,A)_9 = \{(e_1,\{h_2\}),(e_2,\phi)\}$
$(F,A)_{10} = \{(e_1,\{h_2\}),(e_2,\{h_1\})\}$
$(F,A)_{11} = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$
$(F,A)_{12} = \{(e_1,\{h_2\}),(e_2,\{h_1,h_2\})\}$
$(F,A)_{13} = \{(e_1,\{h_1,h_2\}),(e_2,\phi)\}$
$(F,A)_{14} = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1\})\}$
$(F,A)_{15} = \{(e_1,\{h_1,h_2\}),(e_2,\{h_2\})\}$
are all soft set on universe set U under the parameter A.
$\tau=\{(F,A)_1, (F,A)_2,...., (F,A)_{16}\}$ is a soft topology over U.
Soft open sets are $\{(F,A)_1,(F,A)_5\ (F,A)_7, (F,A)_8, (F,A)_{16},\}$
Soft closed sets are $\{(F,A)_1, (F,A)_9, (F,A)_{10}, (F,A)_{12}, (F,A)_{16},\}$
Soft preopen sets are $\{(F,A)_1,(F,A)_5,(F,A)_6,(F,A)_7,(F,A)_8,(F,A)_{13},(F,A)_{14},(F,A)_{15},(F,A)_{16}\}$
Soft preclosed sets are
$\{(F,A)_1,(F,A)_2,(F,A)_3,(F,A)_4,(F,A)_9,(F,A)_{10},(F,A)_{11},(F,A)_{12},(F,A)_{16},\}$

Example 2.7. Let $U = \{ h_1,h_2 ,h_3,\}$, $A = \{e_1,e_2\}$
Let B,C,D,E,F,G,H be the mappings from A to P(U) defined by,

$(B,A) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$
$(C,A) =\{(e_1,\{h_2\}),(e_2,\{h_1,h_3\})\}$
$(D,A) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_1\})\}$
$(E,A)= \{(e_1,\{h_2\}),(e_2,\{h_1\})\}$
$(F,A)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2,h_3\})\}$
$(G,A)=\{(e_1,\{h_1,h_2,h_3\}),(e_2,\{h_1,h_2\})\}$
$(H,A)= \{(e_1,\{h_2,h_3\}),(e_2,\{h_1,h_3\})\}$ are soft sets over U
$\tau = \{\phi,U,(B,A),(C,A),(D,A),(E,A),(F,A),(G,A),(H,A)$ a soft topology over U.
Then $(S,A) = \{(e_1,\{h_1,h_2,h_3),(e_2,\{h_1,h_3\})\}$ is soft pre open set but not soft open.

Example 2.8. Let $U =\{ h_1, h_2,h_3\ h_4\}$, $A =\{e_1,e_2,e_3\}$ and $\tau =\{\phi, U,(F,A)_1, (F,A)_2,..., (F,A)_{15}\}$ is a soft topology over U. And $(F,A)_1, (F,A)_2,..., (F,A)_{15}\}$ is a soft set over U, defined as follows:
$(F,A)_1 = \{(e_1,\{h_1\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1,h_4\})\}$
$(F,A)_2 = \{(e_1,\{h_2,h_4\}),(e_2,\{h_1,h_3,h_4\}),(e_3,\{h_1,h_2,h_4\})\}$
$(F,A)_3 = \{(e_2,\{h_3\}),(e_3,\{h_1\})\}$
$(F,A)_4 = \{(e_1,\{h_1,h_2,h_4\}),(e_2,\tilde{A})(e_3,\tilde{A})\}$
$(F,A)_5 =\{(e_1,\{h_1,h_3\}),(e_2,\{h_2,h_4\}),(e_3,\{h_2\})\}$
$(F,A)_6 =\{(e_1,\{h_1\}),(e_2,\{h_2\})\}$
$(F,A)_7 = \{(e_1,\{h_1,h_3\}),(e_2,\{h_2,h_3,h_4\}),(e_3,\{h_1,h_2,h_4\})\}$
$(F,A)_8 = \{(e_2,\{h_4\}),(e_3,\{h_2\})\}$
$(F,A)_9 = \{(e_1,\tilde{A}),(e_2,\tilde{A}),(e_3,\{h_1,h_2,h_3\})\}$
$(F,A)_{10} =\{(e_1,\{h_1,h_3\}),(e_2,\{h_2,h_3,h_4\}),(e_3,\{h_1,h_2\})\}$
$(F,A)_{11}=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\tilde{A}), (e_3,\{h_1,h_2,h_3\})\}$
$(F,A)_{12}=\{(e_1,\{h_1\}),(e_2,\{h_2,h_3,h_4\}),(e_3,\{h_1,h_2,h_4\})\}$
$(F,A)_{13} = \{(e_1,\{h_1\}),(e_2,\{h_2,h_4\}),(e_3,\{h_2\})\}$
$(F,A)_{14}=\{(e_1,\{h_3,h_4\}),(e_2,\{h_1,h_2\})\}$
$(F,A)_{15}=\{(e_1,\{h_1\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1\})\}$
Now soft closed sets are $\{\phi,U,\{(F,A)_1,(F,A)_2,..., (F,A)_{15}\}^c\}$
Let $(Sp, A)=\{(e_3,\{h_2\})\}$ then $cl(Sp, A)= (F,A)_1{}^c$,
$Int(cl((Sp, E)))= (F,A)_8$, and so $(Sp, A)\tilde{\subseteq}\ int(cl(Sp, A)))$
Hence (Sp, A) is soft pre open set.

Theorem 2.9. If (G,A) is soft pre open set such that
$(G_1,A) \subseteq (G,A) \subseteq \tilde{s}cl(G_1,A)$then (G_1,A) is also a soft pre open set.
Proof: Given that $(G_1,A) \subseteq(G,A) \subseteq \tilde{s}cl(G_1,A)$ and (G_1,A) and(G,A) is soft pre open

By 2.1 Definition, $(G,A) \subseteq \tilde{s}int(\tilde{s}cl(G,A))$ (1)

$\tilde{s}cl(G,A) \subseteq \tilde{s}cl(G_1,A)$ (2)

$$\begin{aligned}
\Longrightarrow\ \tilde{s}int\ \tilde{s}cl(G,A) &\subseteq \tilde{s}int\ \tilde{s}cl(G_1,A) \quad \text{and} \\
&\subseteq \tilde{s}int\ \tilde{s}cl(G,A) \qquad \text{by (1)} \\
&\subseteq \tilde{s}int\ \tilde{s}cl(G_1,A) \qquad \text{by (2)}
\end{aligned}$$

Therefore $(G_1,A) \subseteq\tilde{s}int\ \tilde{s}cl(G_1,A)$

Therefore (G_1,A) is soft pre open set.

Notation : We denote the collection of all soft pre open set of the soft topological space (U,A,τ) by $(SPOS(U_A))$and the collection of all soft pre closed set of the soft topological space (U,A,τ) by $(SPCS(U_A))$.

3. Soft preclosure and soft pre interior

Definition 3.1. Let (U,A,τ) be the soft topological space and let (G,A) be soft set over U The soft pre closure of (G,A) is

$\tilde{s}p$cl(G,A)= $\cap$\{(S,A) : (G,A)$\subseteq$(S,A)/(S,A)$\in$ SPCS(U_A)

Definition 3.2. Let (U,A,τ) be the soft topological space and let (G,A) be soft set over U The soft pre interior of (G,A) is $\tilde{s}p$int(G,A)= $\cup$\{(S,A) : (S,A)$\subseteq$(G,A)/(S,A)$\in$ SPOS(U_A)

Remark 3.3. $\tilde{s}p$cl(G,A) is the smallest soft pre closed set containing (G,A).

Remark 3.4. $\tilde{s}pint$(G,A) is the largest soft pre open set containing (G,A).

Theorem 3.5. Let (U,A,τ) be the soft topological space and (G,A) be a soft set over U. Then

(i) (G,A) $\in$ SPCS(U_A) iff (G,A)= $\tilde{s}p$cl(G,A)

(ii) (G,A) $\in$ SPOS(U_A) iff (G,A)= $\tilde{s}pint$(G,A)

Proof: Let (U,A,τ) be the soft topological space and (G,A) be a soft set over U.

(i) Let (G,A) be a soft pre closed set

To prove that (G,A)= $\tilde{s}p$cl(G,A)

Using 3.3 Remark, the smallest soft pre closed set containing (G,A).

Therefore (G,A) = $\tilde{s}p$cl(G,A)

Conversely assume that (G,A) = $\tilde{s}pcl(G,A)$

Since finite intersection of soft closed set is soft closed

Therefore $\tilde{s}p$cl(G,A) also the intersection of soft pre closed set is soft pre closed which is in SPCS(U_A),

i.e $\tilde{s}p$cl(G,A) $\in$ SPCS(U_A)

$\Longrightarrow$ (G,A) $\in$ SPCS(U_A)

(i) Let (G,A) be a soft pre open set

To prove that (G,A)= $\tilde{s}pint$(G,A)

Using 3.4 Remark, the largest soft pre open set containing (G,A)

Therefore (G,A)= $\tilde{s}pint$(G,A)

Since finite intersection of soft open set is soft open

Therefore $\tilde{s}pint$(G,A) also the intersection of soft pre open set is soft pre open which is in SPOS(U_A)

i.e, $\tilde{s}pint$(G,A) $\in$ SPOS(U_A)

(G,A) $\in$ SPOS(U_A)

Theorem 3.6. Let (U,A,τ) be the soft topological space and (G,A) be a soft set over U. Then (i) ($\tilde{s}p$cl(G,A))^c =$\tilde{s}$ $pint$(G^c,A)

(ii)($\tilde{s}pint$(G,A))^c = $\tilde{s}p$cl (G^c,A)

Proof: We prove this theorem using 3.1Definition and 3.2 Definition

Now $\tilde{s}p$cl(G,A) = $\tilde{\cap}$\{(S,A):(G,A)$\tilde{\subseteq}$ (S,A)/(S,A) $\in$ SPCS(U_A)\}

Taking complement on both sides

($\tilde{s}p$cl(G,A))^c =($\tilde{\cap}$\{(S,A):(G,A)$\tilde{\subseteq}$(S,A)/(S,A) $\in$ SPCS(U_A)\})^c

=$\tilde{\cup}$\{(S,A)^c:(G,A) $\tilde{\supseteq}$ (S,A)/(S,A)^c $\in$ SPCS(U_A)\}

$=\tilde{\cup}\{(S^c,A):(G^c,A)\,\tilde{\supseteq}(S^c,A)/(S^c,A)\in SPCS(U_A)\}$

$(\tilde{s}pcl(G,A))^c = \tilde{s}\,pint(G^c,A)$

(ii) Now $\tilde{s}pint(G,A)=\tilde{\cup}\{(S,A):(S,A)\,\tilde{\subseteq}(G,A)/(S,A)\in SPOS(U_A)\}$

Taking complement on both sides

$\tilde{s}pint(G,A))^c=(\tilde{\cup}\{(S,A):(S,A)\,\tilde{\subseteq}(G,A)/(S,A)\in SPOS(U_A)\})^c$

$=\tilde{\cap}\{(S,A)^c:(S,A)^c\,\tilde{\supseteq}(G,A)^c/(S,A)^c\in SPOS(U_A)\}$

$=\tilde{\cap}\{(S^c,A):(S^c,A)\,\tilde{\supseteq}(G^c,A)/(S^c,A)\in SPOS(U_A)\}$

$\tilde{s}pint(G,A))^c = \tilde{s}pcl\,(G^c,A)$

Theorem 3.7. Let (U,A,τ) be the soft topological space and $(G_1,A)$and (G_2,A) be two soft sets over U. Then (i) $(G_1,A)\,\tilde{\subseteq}\,(G_2,A) \Rightarrow \tilde{s}pint(G_1,A)\,\tilde{\subseteq}\,\tilde{s}pint(G_2,A)$

(ii) $(G_1,A)\,\tilde{\subseteq}\,(G_2,A) \Rightarrow \tilde{s}pcl(G_1,A)\,\tilde{\subseteq}\,\tilde{s}pcl(G_2,A)$

Proof:

(i) Using Definition 3.2, we can write,

$\tilde{s}pint(G_1,A)=\tilde{\cup}\{(S_1,A):(S_1,A)\,\tilde{\subseteq}(G_1,A)/(S_1,A)\in SPOS(U_A)\}$ and

$\tilde{s}pint(G_2,A)=\tilde{\cup}\{(S_2,A):(S_2,A)\,\tilde{\subseteq}(G_2,A)/(S_2,A)\in SPOS(U_A)\}$

Now $\tilde{s}pint(G_1,A)\ \tilde{\subseteq}(G_1,A)$

$\tilde{\subseteq}\,(G_2,A)$ (1)

Since $\tilde{s}pint(G_2,A)$ is the largest soft pre openset contained in(G_2,A)

Therefore (1) $\Longrightarrow \tilde{s}pint(G_1,A)\,\tilde{\subseteq}\,\tilde{s}pint(G_2,A)$

Using Definition 3.1, we can write,

$\tilde{s}pcl(G_1,A) = \tilde{\cap}\{(S_1,A):(G_1,A)\tilde{\subseteq}(S_1,A)/(S_1,A)\in SPCS(U_A)\}$ and

$\tilde{s}pcl(G_2,A) = \tilde{\cap}\{(S_2,A):(G_2,A)\tilde{\subseteq}(S_2,A)/(S_2,A)\in SPCS(U_A)\}$

Now $\tilde{s}pcl(G_1,A)\ \tilde{\subseteq}(G_1,A)$

$\tilde{\subseteq}\,(G_2,A)$ (2)

Since $\tilde{s}pcl(G_2,A)$ is the smallest soft pre openset contained in(G_2,A)

Therefore (2) $\Longrightarrow \tilde{s}pcl(G_1,A)\,\tilde{\subseteq}\,\tilde{s}pcl(G_2,A)$

Theorem 3.8. Let (U,A,τ) be the soft topological space and $(G_1,A)$and (G_2,A) be two soft sets over U. Then

(i) $\tilde{s}pcl((G_1,A)\tilde{\cap}(G_2,A))\,\tilde{\subseteq}\,\tilde{s}pcl(G_1,A)\tilde{\cap}\,\tilde{s}pcl(G_2,A)$

(ii) $\tilde{s}pint((G_1,A)\tilde{\cup}(G_2,A))\,\tilde{\supseteq}\,\tilde{s}pint(G_1,A)\tilde{\cup}\tilde{s}pint(G_2,A)$

Proof:

(i) We have $(G_1,A)\tilde{\cap}(G_2,A)\tilde{\subseteq}\,(G_1,A)$ and $(G_1,A)\tilde{\cap}(G_2,A)\tilde{\subseteq}\,(G_2,A)$

$\Rightarrow \tilde{s}pcl\,((G_1,A)\tilde{\cap}(G_2,A))\tilde{\subseteq}\,\tilde{s}pcl\,(G_1,A)$ and

$\tilde{s}pcl\,((G_1,A)\tilde{\cap}(G_2,A))\tilde{\subseteq}\,\tilde{s}pcl\,(G_2,A)$

$\Rightarrow \tilde{s}pcl\,((G_1,A)\tilde{\cap}(G_2,A))\,\tilde{\subseteq}\,\tilde{s}pcl(G_1,A)\,\tilde{\cap}\,\tilde{s}pcl(G_2,A)$

(ii) Similiar to (i)

Theorem 3.9. If (O,A) is soft open and (G,A) is soft pre open, then $(G,A)\tilde{\cap}(O,A)$ is soft pre open.

Proof: Using Theorem 1.8,

$(G,A)\tilde{\cap}(O,A)\,\tilde{\subseteq}\,\tilde{s}int(\tilde{s}cl(G,A))\tilde{\cap}(O,A)\,\tilde{\subseteq}\,\tilde{s}int(\tilde{s}cl(G,A)\tilde{\cap}(O,A))$

$\tilde{\subseteq}\,\tilde{s}int(\tilde{s}cl((G,A)\tilde{\cap}(O,A)))$

4. Soft pre continuous

Definition 4.1. A soft mapping f : X→Y is said to be soft pre continuous (SP-continuous) if the inverse image of each soft open set of Y is a soft pre open set in X.

Theorem 4.2. Let f : X→Y be a mapping from a soft topological space X to soft topological space Y and let f is soft pre continuous iff the inverse image of each soft closed set in Y is soft pre closed in X.
Proof: Let (G,K) be a soft closed set in Y then $(G,K)^c$ is soft open set.
Therefore $f^{-1}((G,K)^c) \in SPOS(X)$
$\Rightarrow X - f^{-1}((G,K)) \in SPOS(X)$
Hence $f^{-1}((G,K))$ is soft pre closed set in X.
Conversely let (O,K) is soft open set in Y.
Then $(O,K)^c$ is soft closed set and the inverse image of each soft closed set in Y is soft pre closed in X.
We have $f^{-1}((O,K)^c) \in SPCS(X)$
$\Rightarrow X - f^{-1}((O,K)) \in SPCS(X)$
Hence $f^{-1}((O,K))$ is soft pre open set in X.
Therefore f is soft pre continuous function.

Theorem 4.3. Every soft continuous function is soft pre continuous function.
Proof: Let f : X→Y be a soft continuous function and let (F,K) be a soft open set in Y.
Since f is soft continuous, $f^{-1}((F,K))$is soft preopen set in X
Therefore f is soft pre continuous function.

Theorem 4.4. Let f: $(X,\tau,E) \rightarrow (Y,\tau_1,K)$, g: $(Y,\tau_2,K) \rightarrow (Z,\tau_3,T)$ be two functions then $g \circ f$: X→Z is a soft pre continuous, if f is soft pre continuous and g is soft continuous.
Proof: Let (H,T) be soft closed set of Z. Since g: Y→Z is a soft continuous, by definition $g^{-1}((H,T))$is soft closed set of Y.
Now f : X→Y be a soft pre continuous function and $g^{-1}((H,T))$is soft closed set of Y, by 4.1 definition, $f^{-1}(g^{-1}((H,T))) = (g \circ f)^{-1}(H,T)$ is soft pre closed in X.
Hence $g \circ f$: X→Z is soft pre continuous.

Definition 4.5. A mapping f : X→Y is said to be soft pre open map if the image of every soft open set in X is soft pre open set in Y.

Definition 4.6. A mapping f : X→Y is said to be soft pre closed map if the image of every soft closed set in X is soft pre closed set in Y.

Theorem 4.7. If f : X→Y is soft closed function and g: Y→Z is soft closed function, then $g \circ f$ is soft pre closed function.
Proof: For a soft closed set (C,A) in X, f((C,A)) is soft closed set in Y. Since g: Y→Z is a soft pre closed function, g(f(C,A)) is soft pre closed set in Z.
$g(f((C,A))) = (g \circ f)((C,A))$ is soft pre closed set in Z.
Therefore $g \circ f$ is soft pre closed function.

Theorem 4.8. Let f : X→Y, g : Y→Z be two maps such that $(g \circ f)$:X→Z is soft pre closed map. If f is soft continuous and surjective then g is soft pre closed map

Proof: Let (C,A) be a soft closed set of Y. Then f^{-1}((C,A))is soft closed set in X as f is soft continuous. Since g ₒ f is soft pre closed map, (g ₒ f)(f^{-1}((C,A)))=g((C,A)) is soft pre closed set in Z.
Therefore g : Y→Z is soft pre closed map.

5. Soft pre irresolute

Definition 5.1. A mapping f : X→Y is said to be soft pre irresolute if f^{-1}((C,A)) is soft pre closed set in X, for every soft pre closed set (C,A) in Y.

Theorem 5.2. A mapping f : X→Y is soft pre irresolute mapping iff the inverse image of every soft pre open set in Y is soft pre open set in X.

Theorem 5.3. Every soft pre irresolute mapping is soft pre continuous mapping
Proof: Let f : X→Y be soft pre irresolute mapping and let (C,A) be a soft closed set in Y, then (C,A) is soft pre closed set in Y.
Since f is soft pre irresolute mapping , f^{-1}((C,A))is soft closed set in X.
Therefore f is soft pre continuous mapping.

Theorem 5.4. Let f: $(X,\tau,E) \rightarrow (Y,\tau_1,K)$, g: $(Y,\tau_2,K) \rightarrow (Z,\tau_3,T)$ be two functions then
(i) g ₒf: X→Z is a soft pre irresolute, if f and g is soft pre irresolute functions.
(ii) g ₒf: X→Z is soft pre continuous if f is soft pre irresolute and g is soft pre continuous.
Proof: (i) Let g: Y→Z is soft pre irresolute, and let (C,A) be soft pre closed set of Z
Since g is soft pre irresolute , by 5.1definition, g^{-1}((C,A))is soft closed set of Y,
Also f : X→Y be soft pre irresolute, so $f^{-1}(g^{-1}$((C,A))) =(g ₒ f)$^{-1}$ ((C,A)) is soft pre closed.
Therefore g ₒ f: X→Z is soft pre irresolute.
(ii) Let (C,A) be soft pre closed set of Z. Since g:Y→Z is soft precontinuous,
g^{-1}((C,A))is soft pre closed set of Y and f : X→Y be soft pre irresolute, so every soft pre closed set of Y .
Therefore, $f^{-1}(g^{-1}$((C,A))) =(g ₒ f)$^{-1}$ ((C,A)) is soft pre closed set of X.
Therefore g ₒ f: X→Z is soft pre continuous.

Theorem 5.5. Let f : X→Y, g: Y→Z be two maps such that g ₒ f: X→Z is soft pre closed map. If g is soft pre irresolute and injective then f is soft pre closed map.
Proof: Let (C,A) be a soft closed set in X. Then (g ₒ f)((C,A)) is soft pre closed set in Z and g^{-1}(g ₒ f)((C,A))=f((C,A)) is soft pre closed set in Y. Since g is soft pre irresolute and injective.
Hence f is soft pre closed map.

6. Conclusion

The work in this paper step forward to strengthen the theoretical foundation of soft topology via soft preopen sets. We defined soft preopen sets, soft preclosed sets.

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