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An Introduction to Fuzzy Soft Tritopological Spaces

Asmhan Flieh Hassan

Department of Mathematics, Faculty of Education for Girls
University of Kufa, Iraq
Email: asmhanf.alzuhairy@uokufa.edu.iq

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Abstract. The aim of this paper is to introduce the concept of fuzzy soft tritopological Space, Where we define it as a space equipped with three fuzzy soft topologies, i.e. triple of three fuzzy soft topologies τ_1, τ_2 and τ_3 on the same initial universal set $\mathcal{U}$ and the set of parameters E. And investigate some of its fundamental properties. Also we defined some new kinds of fuzzy soft open sets in fuzzy soft tritopological spaces, which are called fuzzy soft $\tau_1\tau_2\tau_3$-open set, fuzzy soft $\tau_1\tau_2\tau_3$-pre-open set, fuzzy soft $\tau_1\tau_2\tau_3$-semi-open set, fuzzy soft $\tau_1\tau_2\tau_3$-α-open set (fuzzy soft tri-α-open set) and fuzzy soft δ^*-open set. We can consider this work as an introduction of this concept.

Keywords: Fuzzy soft set, fuzzy soft topological space, fuzzy soft bitopological space, fuzzy soft tritopological space

AMS Mathematics Subject Classification (2010): 54A05, 54A10, 54E55, 54E99, 54A40

1. Introduction

In 1965, Fuzzy set was introduced by Zadeh in [1] as a mathematical way to represent and deal with vagueness in everyday life. And the applications of fuzzy set theory can be found in many branches of sciences (see [2, 3])

In 1999, soft set theory was initiated by Molodtsov [4], he defined the concept of soft set theory as a new mathematical tool, and presented several fundamental results and successfully applied it to several mathematical directions such as smoothness of functions, theory of probability, Riemann-integration, operations research, Perron integration, etc. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parametrization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Some important applications of soft sets are in decision making problems and information systems [5][6].

In 2001, Maji et al. [7] presented the concept of the fuzzy soft sets by embedding the ideas of fuzzy sets. By using this definition of fuzzy soft sets many interesting applications of soft set theory have been expanded by researchers. Roy and Maji [8] gave some applications of fuzzy soft sets. Aktas and Cagman [9] compared soft sets with the related concepts of fuzzy sets and rough sets. Yang et al. [10] defined the operations on fuzzy soft sets which are based on three fuzzy logic operators: negation, triangular norm and triangular conorm. Xiao et al. [11] presented the combination of interval-valued fuzzy set and soft set.

In 1963, Kelly first initiated the concept of bitopological spaces [12], where defined a bitopological space to set with two topologies and initiated the systematic study of bitopological spaces. In later, many researchers studied bitopological spaces (see [13, 14]) where carrying out a wide scope for the generalization of topological results in bitopological environment.

In 2014, Ittanagi [15] introduced the concept of soft bitopological spaces, which is defined it over an initial universal set with fixed set of parameters, and he introduced some types of soft separation axioms in soft bitopological spaces. A study of fuzzy soft bitopological spaces is a generalization of the study of fuzzy soft topological spaces.

In 2015, Mukherjee and Park [16] were first introduced the notion of fuzzy soft bitopological space and studied some of their basic properties, and to more information (see [17,18]).

In 2000, Kovar.[19], initiated the concept of tritopological spaces by modify θ-regularity for spaces with three topologies, where they define it as a spaces equipped with three topologies, i.e. triple of topologies on the same set, Palaniammal [20] studied tritopological spaces and introduced semi-open and pre-open sets in tritopological spaces and he also introduced fuzzy tritopological space.

In 2004, Asmhan was introduced the definition of δ^*-open set in tritopological spaces[21]. And in [22] she defined the δ^*-connectedness in tritopological spaces, also Asmhan et al. [23] defined the δ^*-base in tritopological spaces. In [24, 25] the reader can find a relationships among separation axioms, and a relationships among some types of continuous and open functions in topological, bitopological and tritopological spaces, and in 2017, Asmhan introduced the new definitions of countability and separability in tritopological spaces namely δ^*-countability and δ^*-separability [26].

In 2017, Asmhan F.H. presented the concept of the soft tritopological spaces [27].

In the present paper, concept of fuzzy soft topological spaces have been generalized to initiate the study of fuzzy soft tritopological spaces. In addition, we introduce and characterize a new types of fuzzy soft open sets in a fuzzy soft tritopological spaces namely fuzzy soft $\tau_1\tau_2\tau_3$-open set, fuzzy soft $\tau_1\tau_2\tau_3$-pre-open set, fuzzy soft $\tau_1\tau_2\tau_3$-α-open set (or fuzzy soft tri-α-open set), fuzzy soft δ^*-open set. And investigate some basic properties.

In section 2, some preliminary concepts about fuzzy soft topological spaces, fuzzy soft bitopological spaces and tritopological spaces are given. The main section of the manuscript is third which the definition of fuzzy soft tritopological spaces with examples and some theorems are given. Section 4 is devoted for the definitions of some types of fuzzy soft open sets in fuzzy soft tritopological spaces with some examples. Finally in section 5 the conclusions and some ideas of future work is suggested.

This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application of fuzzy soft tritopological spaces.

2. Preliminaries

In this section, we present the basic definitions of fuzzy soft set theory, soft set theory and fuzzy set theory that are useful for subsequent discussions and which will be a central role in our work.

Throughout this work, $\mathcal{U}$ refers to an initial universe, E is a set of parameters, $P(\mathcal{U})$ is the power set of $\mathcal{U}$, and $A \subseteq E$.

Definition 2.1. [1] Let $\mathcal{U}$ be a universe. A fuzzy set X over $\mathcal{U}$ is a set defined by a function μ_X representing a mapping $\mu_X: \mathcal{U} \to [0,1]$, μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in \mathcal{U}$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over $\mathcal{U}$ can be represented as follows: $X = \{ (\mu_X(u)/u) \ : u \in \mathcal{U}, \mu_X(u) \in [0,1]\}$
Note that the set of all the fuzzy sets over $\mathcal{U}$ will be denoted by $F(\mathcal{U})$.

Definition 2.2. [4] A soft set F_A over $\mathcal{U}$ is a set defined by a function f_A representing a mapping: $\mathrm{f}_A : E \to P(\mathcal{U})$ such that $\mathrm{f}_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called approximate function of the soft set F_A, and the value $\mathrm{f}_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth noting that the sets $\mathrm{f}_A(x)$ may be arbitrary, empty, or have nonempty intersection. Thus a soft set over $\mathcal{U}$ can be represented by the set of ordered pairs $F_A = \{(x, \mathrm{f}_A(x)) : x \in E, \mathrm{f}_A(x) \in P(U)\}$. Note that the set of all soft sets over $\mathcal{U}$ will be denoted by $S(\mathcal{U})$.

Definition 2.3. [7,28] A fuzzy soft set $\mathcal{S}_A$ over $\mathcal{U}$ is a set defined by a function ξ_A representing a mapping $\xi_A : E \longrightarrow F(\mathcal{U})$ such that $\xi_A(x) = \emptyset$ if $x \notin A$. Here, ξ_A is called fuzzy approximate function of the fuzzy soft set $\mathcal{S}_A$, and the value $\xi_A(x)$ is a set called x-element of the fuzzy soft set for all $x \in E$. Thus, an fuzzy soft set $\mathcal{S}_A$ over $\mathcal{U}$ can be represented by the set of ordered pair $\mathcal{S}_A = \{(x, \xi_A(x)): x \in E,\ \xi_A(x) \in F(\mathcal{U})\}$. Note that the set of all fuzzy soft sets over $\mathcal{U}$ will be denoted by $FS(\mathcal{U})$ or $FS(\mathcal{U}, E)$

Definition 2.4. [7] Let $\mathcal{S}_A \in FS(\mathcal{U})$. If $\xi_A(x) = \emptyset$ for all $x \in E$, then $\mathcal{S}_A$ is called an empty fuzzy soft set, denoted by $\mathcal{S}_{\Phi}$ or (0_E).

Definition 2.5. [7] Let $\mathcal{S}_A \in FS(\mathcal{U})$. If $\xi_A(x) = \mathcal{U}$ for all $x \in A$, then $\mathcal{S}_A$ is called A -universal fuzzy soft set, denoted by $\mathcal{S}_{\breve{A}}$. If $A = E$, then the A-universal fuzzy soft set is called universal fuzzy soft set, denoted by $\mathcal{S}_{\breve{E}}$ or (1_E) .

Definition 2.6. [7] Let $\mathcal{S}_A$, $\mathcal{S}_B \in FS(\mathcal{U})$. Then $\mathcal{S}_A$ is called a fuzzy soft subset of $\mathcal{S}_B$, denoted by $\mathcal{S}_A \sqsubseteq \mathcal{S}_B$ If $\xi_A(x) \subseteq \xi_B(x)$ for all $x \in E$.

Remark 2.7. [29] $\mathcal{S}_A \sqsubseteq \mathcal{S}_B$ does not mean as in the classical subset. (i.e. does not imply that every element of $\mathcal{S}_A$ is an element of $\mathcal{S}_B$).

Definition 2.8. [7] Let $\mathcal{S}_A$, $\mathcal{S}_B \in FS(\mathcal{U})$. Then the two fuzzy soft sets $\mathcal{S}_A$ and $\mathcal{S}_B$ are equal, written as $\mathcal{S}_A = \mathcal{S}_B$ If and only if $\xi_A(x) = \xi_B(x)$ for all $x \in E$.

Definition 2.9. [7]Let $\mathcal{S}_A \in FS(\mathcal{U})$. Then the complement $\mathcal{S}_A^c$ of $\mathcal{S}_A$ is a fuzzy soft set such that $\xi_{A^c}(x) = \xi_A^c(x)$ for all $x \in E$, where $\xi_A^c(x)$ is complement of the set $\xi_A(x)$. Clear that $(\mathcal{S}_A^c)^c = \mathcal{S}_A$, $\mathcal{S}_\Phi^c = \mathcal{S}_{\check{E}}$ and $\mathcal{S}_{\check{E}}^c = \mathcal{S}_\Phi$

Definition 2.10. [7] Let $\mathcal{S}_A$, $\mathcal{S}_B \in FS(\mathcal{U})$. Then the union of $\mathcal{S}_A$ and $\mathcal{S}_B$, denoted by $\mathcal{S}_A \sqcup \mathcal{S}_B$, is defined by its fuzzy approximate function $\xi_{A\sqcup B}(x) = \xi_A(x) \cup \xi_B(x)$ for all $x \in E$.

Definition 2.11. [7] Let $\mathcal{S}_A$, $\mathcal{S}_B \in FS(\mathcal{U})$. Then the intersection of $\mathcal{S}_A$ and $\mathcal{S}_B$, denoted by $\mathcal{S}_A \sqcap \mathcal{S}_B$, is defined by its fuzzy approximate function $\xi_{A\sqcap B}(x) = \xi_A(x) \cap \xi_B(x)$ for all $x \in E$.

Definition 2.12. [30] Let τ be the collection or sub family of fuzzy soft sets over $\mathcal{U}$ (i.e. $\tau \sqsubseteq FS(\mathcal{U}, E)$). Then τ is said to be a fuzzy soft topology on the universal set $\mathcal{U}$ if satisfying the following properties:

(i) $\mathcal{S}_\Phi$, $\mathcal{S}_{\check{E}} \in \tau$

(ii) If $\mathcal{S}_A$, $\mathcal{S}_B \in \tau$, then $\mathcal{S}_A \sqcap \mathcal{S}_B \in \tau$

(iii) If $\mathcal{S}_{Aj} \in \tau$, $\forall j \in \Lambda$, where Λ is some index set, then $\sqcup_{j\in\Lambda} \mathcal{S}_{Aj} \in \tau$.

Then the triple $(\mathcal{U}, E, \tau)$ is called a fuzzy soft topological space over $\mathcal{U}$. And each member of τ is called fuzzy soft open set in $(\mathcal{U}, E, \tau)$. Also a fuzzy soft set is called fuzzy soft closed if and only if its complement is fuzzy soft open.

Definition 2.13. [30] Let $(\mathcal{U}, E, \tau)$ be a fuzzy soft topological space and $\mathcal{S}_A$ be a fuzzy soft set over $\mathcal{U}$, then

i. Fuzzy soft interior of $\mathcal{S}_A$ is defined as the union of all fuzzy soft open sets contained in $\mathcal{S}_A$ and is denoted by $FS - int(\mathcal{S}_A)$.
ii. Fuzzy soft closure of $\mathcal{S}_A$ over $\mathcal{U}$ is defined as the intersection of all fuzzy soft closed super sets of $\mathcal{S}_A$ and is denoted by $FS - cl(\mathcal{S}_A)$.

Definition 2.14. [16] Let $(\mathcal{U}, E, \tau_1)$ and $(\mathcal{U}, E, \tau_2)$ be the two fuzzy soft topological spaces over $\mathcal{U}$. Then $(\mathcal{U}, E, \tau_1, \tau_2)$ is called a fuzzy soft bitopological space.

Definition 2.15. [19] Let X be a nonempty set and $\mathcal{T}, \mathcal{P}$ and $\mathcal{Q}$ be a three topologies on X. The set X together with three topologies is called a tritopological space and is denoted by $(\mathrm{X}, \mathcal{T}, \mathcal{P}, \mathcal{Q})$.

Definition 2.16. [21] Let $(\mathrm{X}, \mathcal{T}, \mathcal{P}, \mathcal{Q})$be a tritopological space, a subset A of X is said to be δ^*-open set iff $A \subseteq \mathcal{T} - \mathrm{int}(\mathcal{P} - \mathrm{cl}(\mathcal{Q} - \mathrm{int}(A)))$, and the family of all δ^*-open sets over X is denoted by $\delta^*.\mathrm{O}(\mathrm{X})$. The complement of δ^*-open set is called a δ^*-closed set.

Definition 2.17. [27] Let $(\mathcal{U}, \mathcal{T}, E)$, $(\mathcal{U}, \mathcal{P}, E)$ and $(\mathcal{U}, \mathcal{Q}, E)$ be the three soft topological spaces on $\mathcal{U}$. Then $(\mathcal{U}, \mathcal{T}, \mathcal{P}, \mathcal{Q}, E)$ is called a soft tritopological space. The three soft topological spaces $(\mathcal{U},\mathcal{T},E)$, $(\mathcal{U},\mathcal{P},E)$ and $(\mathcal{U},\mathcal{Q},E)$ are independently satisfy the axioms of soft topological space.

3. Fuzzy soft tritopological spaces

In this section the definition of fuzzy soft tritopological spaces is initiated. And we have study some its properties.

Definition 3.1. Let $(\mathcal{U},E,\tau_1)$, $(\mathcal{U},E,\tau_2)$ and $(\mathcal{U},E,\tau_3)$ be the three fuzzy soft topological spaces on $\mathcal{U}$. Then a space equipped with three fuzzy soft topologies, i.e. triple of fuzzy soft topologies on the same set is called a fuzzy soft tritopological space and denoted by $(\mathcal{U},E,\tau_1,\tau_2,\tau_3)$. Where the three fuzzy soft topological spaces are independently satisfy the axioms of fuzzy soft topological space.
The members of $\tau_j(\,j=1,2,3)$ are called $\tau_j(\,j=1,2,3)$-fuzzy soft open sets and the complements of $\tau_j(\,j=1,2,3)$- fuzzy soft open sets are called $\tau_j(\,j=1,2,3)$-fuzzy soft closed sets.

Definition 3.2. Let $(\mathcal{U},E,\tau_1,\tau_2,\tau_3)$ be a fuzzy soft tritopological space and let $\mathcal{S}_E$ be a fuzzy soft set over $\mathcal{U}$. Then:

i. the $\tau_j(\,j=1,2,3)$-fuzzy soft closure of $\mathcal{S}_E$ denoted by $\tau_j(\,j=1,2,3)cl(\mathcal{S}_E)$, is the intersection of all $\tau_j(\,j=1,2,3)$ fuzzy soft closed supersets of $\mathcal{S}_E$.
ii. the $\tau_j(\,j=1,2,3)$-fuzzy soft interior of $\mathcal{S}_E$ denoted by $\tau_j(\,j=1,2,3)int(\mathcal{S}_E)$, union of all $\tau_j(\,j=1,2,3)$ -fuzzy soft open sets contained in $\mathcal{S}_E$.

Example 3.3. Let $\mathcal{U}=\{\mathrm{u}_1,\mathrm{u}_2,\mathrm{u}_3,\mathrm{u}_4\}$, $\mathrm{E}=\{x_1,x_2\}$,
$\tau_1=\{0_E,1_E,f_{\mathrm{E}}\}$, $\tau_2=\{0_E,1_E,g_{1\mathrm{E}},g_{2\mathrm{E}},g_{3\mathrm{E}}\}$ and $\tau_3=\{0_E,1_E,h_{1\mathrm{E}},h_{2\mathrm{E}}\}$,
where $f_{\mathrm{E}},g_{1\mathrm{E}},g_{2\mathrm{E}},g_{3\mathrm{E}},h_{1\mathrm{E}}$ and $h_{2\mathrm{E}}$ are fuzzy soft sets over $\mathcal{U}$. Where defined as follows;

$f_{\mathrm{E}}=\{(x_1,\{0.5/u_1,0.0/u_2,0.7/u_3,0.0/u_4\}),(x_2,\{0.0/u_1,0.0/u_2,0.3/u_3,0.0/u_4\})\}$
$g_{1\mathrm{E}}=\{(x_1,\{0.5/u_1,0.0/u_2,0.7/u_3,0.0/u_4\}),(x_2,\{0.5/u_1,0.0/u_2,0.7/u_3,0.0/u_4\})\}$
$g_{2\mathrm{E}}=\{(x_1,\{0.0/u_1,0.4/u_2,0.0/u_3,0.5/u_4\}),(x_2,\{0.0/u_1,0.1/u_2,0.0/u_3,0.0/u_4\})\}$
$g_{3\mathrm{E}}=\{(x_1,\{0.0/u_1,0.4/u_2,0.3/u_3,0.5/u_4\}),(x_2,\{0.3/u_1,0.1/u_2,0.0/u_3,0.6/u_4\})\}$
$h_{1\mathrm{E}}=\{(x_1,\{0.3/u_1,0.0/u_2,0.0/u_3,0.6/u_4\}),(x_2,\{0.0/u_1,0.0/u_2,0.0/u_3,0.7/u_4\})\}$ and
$h_{2\mathrm{E}}=\{(x_1,\{0.0/u_1,0.0/u_2,0.0/u_3,0.6/u_4\}),(x_2,\{0.0/u_1,0.0/u_2,0.0/u_3,0.0/u_4\})\}$

Then τ_1, τ_2 an τ_3 are three fuzzy soft topologies over $(\mathcal{U},E)$. Therefore $(\mathcal{U},E,\tau_1,\tau_2,\tau_3)$ is a fuzzy soft tritopological space.

Proposition 3.4. If τ_1,τ_2 and τ_3 are three fuzzy soft topologies on $(\mathcal{U},E)$ then $\tau_1\sqcap\tau_2\sqcap\tau_3$ is a fuzzy soft topology on $(\mathcal{U},E)$ but $\tau_1\sqcup\tau_2\sqcup\tau_3$ is not necessarily a fuzzy soft topology on $(\mathcal{U},E)$.

Proof: Since τ_1, τ_2 and τ_3 are three fuzzy soft topologies on $(\mathcal{U},E)$, $0_E,1_E\in\tau_1$, $0_E,1_E\in\tau_2$ and $0_E,1_E\in\tau_3$ thus $0_E,1_E\in\tau_1\sqcap\tau_2\sqcap\tau_3$.
Let $f_A,\ g_B,\ h_C\in\tau_1\sqcap\tau_2\sqcap\tau_3$ then $f_A,\ g_B,\ h_C\in\tau_1$, $f_A,\ g_B,\ h_C\in\tau_2$ and $f_A,\ g_B,\ h_C\in\tau_3$, Since τ_1, τ_2 and τ_3 are fuzzy soft topologies on $(\mathcal{U},E)$, it follows that

$f_A \sqcap g_B \sqcap h_C \in \tau_1$, $f_A \sqcap g_B \sqcap h_C \in \tau_2$ and $f_A \sqcap g_B \sqcap h_C \in \tau_3$, this implies that $f_A \sqcap g_B \sqcap h_C \in \tau_1 \sqcap \tau_2 \sqcap \tau_3$

Let $f_{Aj} \in \tau_1 \sqcap \tau_2 \sqcap \tau_3$, $\forall j \in \Lambda$, where Λ is some index set, and since τ_1, τ_2 and τ_3 are three fuzzy soft topologies on $(\mathcal{U}, E)$, then $\sqcup_{j\in\Lambda} f_{Aj} \in \tau_1$, $\sqcup_{j\in\Lambda} f_{Aj} \in \tau_2$ and $\sqcup_{j\in\Lambda} f_{Aj} \in \tau_3$ follows that $\sqcup_{j\in\Lambda} f_{Aj} \in \tau_1 \sqcap \tau_2 \sqcap \tau_3$.

Therefore $\tau_1 \sqcap \tau_2 \sqcap \tau_3$ is a fuzzy soft topology on $(\mathcal{U}, E)$.

And clearly from the example below that union of τ_1 , τ_2 and τ_3 may not be a fuzzy soft topology on $(\mathcal{U}, E)$.

Example 3.5. Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3, x_4\}$ where

$$f_E = \{(x_1, \{0.4/u_1, 0.1/u_2, 0.0/u_3\}), (x_2, \{0.6/u_1, 0.5/u_2, 0.2/u_3\}), (x_3, \{0.0/u_1, 0.0/u_2, 0.0/u_3\}), (x_4, \{0.2/u_1, 0.6/u_2, 0.3/u_3\})\}$$

$$g_E = \{(x_1, \{0.0/u_1, 0.0/u_2, 0.0/u_3\}), (x_2, \{0.7/u_1, 0.2/u_2, 0.1/u_3\}), (x_3, \{0.0/u_1, 0.0/u_2, 0.0/u_3\}), (x_4, \{0.5/u_1, 0.3/u_2, 0.9/u_3\})\}$$

$$h_E = \{(x_1, \{0.1/u_1, 0.1/u_2, 0.0/u_3\}), (x_2, \{0.1/u_1, 0.5/u_2, 0.1/u_3\}), (x_3, \{0.0/u_1, 0.0/u_2, 0.1/u_3\}), (x_4, \{0.2/u_1, 0.4/u_2, 0.3/u_3\})\}$$

Consider the three fuzzy soft topologies τ_1, τ_2 and τ_3 on $(\mathcal{U}, E)$ as fellows;

$\tau_1 = \{0_E, 1_E, f_E\}$, $\tau_2 = \{0_E, 1_E, g_E\}$ and $\tau_3 = \{0_E, 1_E, h_E\}$. Thus

$\tau_1 \sqcup \tau_2 \sqcup \tau_3 = \{0_E, 1_E, f_E, g_E, h_E\}$. Then,

$$f_E \sqcap g_E = \{(x_1, \{0.0/u_1, 0.0/u_2, 0.0/u_3\}), (x_2, \{0.6/u_1, 0.2/u_2, 0.1/u_3\}), (x_3, \{0.0/u_1, 0.0/u_2, 0.0/u_3\}), (x_4, \{0.2/u_1, 0.3/u_2, 0.3/u_3\})\}$$

Thus $f_E, g_E \in \tau_1 \sqcup \tau_2 \sqcup \tau_3$ but $f_E \sqcap g_E \notin \tau_1 \sqcup \tau_2 \sqcup \tau_3$. Therefore $\tau_1 \sqcup \tau_2 \sqcup \tau_3$ is not a fuzzy soft topology on $(\mathcal{U}, E)$. However $\tau_1 \sqcap \tau_2 \sqcap \tau_3 = \{0_E, 1_E\}$ is a fuzzy soft topology on $(\mathcal{U}, E)$.

4. Some kinds of fuzzy soft open sets in fuzzy soft tritopological spaces

Definition 4.1. Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a fuzzy soft tritopological space and Γ_E is a fuzzy soft set in $\mathcal{U}$, then:

(i) Γ_E is called a fuzzy soft $\tau_1\tau_2\tau_3$-open set if $\Gamma_E = f_E \sqcup g_E \sqcup h_E$, where $f_E \in \tau_1$, $g_E \in \tau_2$ and $h_E \in \tau_3$. The complement of fuzzy soft $\tau_1\tau_2\tau_3$-open set is called fuzzy soft $\tau_1\tau_2\tau_3$-closed. The family of all fuzzy soft $\tau_1\tau_2\tau_3$-open sets is denoted by $FS.\tau_1\tau_2\tau_3.\mathrm{O}(\mathcal{U})$. And the family of all fuzzy soft $\tau_1\tau_2\tau_3$-closed sets is denoted by $FS.\tau_1\tau_2\tau_3.\mathrm{C}(\mathcal{U})$.

(ii) Γ_E is called a fuzzy soft $\tau_1\tau_2\tau_3$-pre-open set iff $\Gamma_E \sqsubseteq \mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\Gamma_E))$. The complement of fuzzy soft $\tau_1\tau_2\tau_3$-pre-open set is said to be fuzzy soft $\tau_1\tau_2\tau_3$-pre-closed.

(iii) Γ_E is called a fuzzy soft $\tau_1\tau_2\tau_3$-semi-open set iff $\Gamma_E \sqsubseteq \mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\Gamma_E))$. The complement of fuzzy soft $\tau_1\tau_2\tau_3$-semi-open set is said to be fuzzy soft $\tau_1\tau_2\tau_3$-semi-closed.

(iv) Γ_E is called a fuzzy soft $\tau_1\tau_2\tau_3$-α-open set (or fuzzy soft tri-α-open set) if $\Gamma_E \sqsubseteq \mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\Gamma_E)))$. The complement of fuzzy soft $\tau_1\tau_2\tau_3$- α -open set is said to be fuzzy soft $\tau_1\tau_2\tau_3$- α -closed.

(v) Γ_E is called a fuzzy soft δ^*-open set iff $\Gamma_E \sqsubseteq \mathrm{FS}.\tau_1\mathrm{int}(\mathrm{FS}.\tau_2\mathrm{cl}(\mathrm{FS}.\tau_3\mathrm{int}(\Gamma_E)))$. The complement of fuzzy soft δ^*-open set is called a fuzzy soft δ^*-closed set.

Definition 4.2. Let $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ be a fuzzy soft tritopological space, and Γ_E is a fuzzy soft set in $\mathcal{U}$, then:

(i) The fuzzy soft $\tau_1\tau_2\tau_3$-closure of Γ_E denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3 cl(\Gamma_E)$ is defined by:

$$\mathrm{FS}.\tau_1\tau_2\tau_3 cl(\Gamma_E) = \sqcap\{g_E:\ \Gamma_E \sqsubseteq g_E,\ and\ g_E \text{ is fuzzy soft } \tau_1\tau_2\tau_3\text{-closed}\}$$

(ii) The fuzzy soft $\tau_1\tau_2\tau_3$-interior of Γ_E, denoted by $\mathrm{FS}.\tau_1\tau_2\tau_3 int(\Gamma_E)$ is defined by:

$$\mathrm{FS}.\tau_1\tau_2\tau_3 int(\Gamma_E) = \sqcup\{h_E:\ h_E \sqsubseteq \Gamma_E,\ and\ h_E \text{ is fuzzy soft } \tau_1\tau_2\tau_3\text{-open}\}$$

Example 4.3. Let $\mathcal{U} = \{u_1, u_2\}$, $E = \{x_1, x_2\}$,
$\tau_1 = \{0_E, 1_E, f_{1E}, f_{2E},\}$, $\tau_2 = \{0_E, 1_E, g_{1E}, g_{2E}\}$ and $\tau_3 = \{0_E, 1_E, h_E\}$,
Where $f_{1E}, f_{2E}, f_{3E}, f_{4E}, g_{1E}, g_{2E}, g_{3E}$ and h_E are fuzzy soft sets over $\mathcal{U}$, defined as follows;

$$f_{1E} = \{(x_1, \{0.4/u_1, 0.6/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$$
$$f_{2E} = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.1/u_1, 0.2/u_2\})\}$$
$$g_{1E} = \{(x_1, \{0.2/u_1, 0.4/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$$
$$g_{2E} = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.3/u_1, 0.8/u_2\})\}$$
$$\text{and}\quad h_E = \{(x_1, \{0.3/u_1, 0.0/u_2\}), (x_2, \{0.0/u_1, 0.2/u_2\})$$

Then τ_1, τ_2 and τ_3 are three fuzzy soft topologies over $(\mathcal{U}, E)$, Therefore $(\mathcal{U}, E, \tau_1, \tau_2, \tau_3)$ is a fuzzy soft tritopological space.

It is clear that the family of all fuzzy soft $\tau_1\tau_2\tau_3$-open sets are:
$FS.\tau_1\tau_2\tau_3.\mathrm{O}(\mathcal{U}) = \{0_E, 1_E, f_{1E}, f_{2E}, g_{1E}, g_{2E}, h_E, t_E, b_E\} = \tau_1 \cup \tau_2 \cup \tau_3 \cup \{t_E, b_E\}$,
where

$$t_E = f_{1E} \sqcup g_{2E} = \{(x_1, \{0.4/u_1, 0.6/u_2\}), (x_2, \{0.3/u_1, 0.8/u_2\})\} \text{ and}$$
$$b_E = f_{2E} \sqcup g_{1E} = \{(x_1, \{0.3/u_1, 0.4/u_2\}), (x_2, \{0.2/u_1, 0.5/u_2\})\}$$

Now, we find the fuzzy soft $\tau_1\tau_2\tau_3$-closed sets:
$FS.\tau_1\tau_2\tau_3.\mathrm{C}(\mathcal{U}) = \{1_E, 0_E, f_{1E}^c, f_{2E}^c, g_{1E}^c, g_{2E}^c, h_E^c, t_E^c, b_E^c\}$ Where defined as follows;

$$f_{1E}^c = \{(x_1, \{0.6/u_1, 0.4/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$$
$$f_{2E}^c = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.9/u_1, 0.8/u_2\})\}$$
$$g_{1E}^c = \{(x_1, \{0.8/u_1, 0.6/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$$
$$g_{2E}^c = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.7/u_1, 0.2/u_2\})\}$$
$$h_E^c = \{(x_1, \{0.7/u_1, 1.0/u_2\}), (x_2, \{1.0/u_1, 0.8/u_2\})$$
$$t_E^c = \{(x_1, \{0.6/u_1, 0.4/u_2\}), (x_2, \{0.7/u_1, 0.2/u_2\})\} \text{ and}$$
$$b_E^c = \{(x_1, \{0.7/u_1, 0.6/u_2\}), (x_2, \{0.8/u_1, 0.5/u_2\})\}$$

If we take the fuzzy soft set Γ_E which defined as:
$\Gamma_E = \{(x_1, \{0.1/u_1, 0.9/u_2\}), (x_2, \{0.8/u_1, 0.2/u_2\})$
the fuzzy soft $\tau_1\tau_2\tau_3$-closed sets which contains Γ_E is only 1_E
and $\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\Gamma_E) = 1_E$. Then, the fuzzy soft $\tau_1\tau_2\tau_3$-open sets which containing in $1_E$is only 1_E. Thus $\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\Gamma_E))$=$\mathrm{s}.\tau_1\tau_2\tau_3\mathrm{int}(1_E) = 1_E$

Hence Γ_E is a fuzzy soft $\tau_1\tau_2\tau_3$-pre-open set since $\Gamma_E \sqsubseteq 1_E$.

And if we take the fuzzy soft open set $f_{1\mathrm{E}}$ in the fuzzy soft topology τ_1 above, it is clear that :
$\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(f_{1\mathrm{E}}))$=$\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(f_{1\mathrm{E}}) = b_E^c$, Hence $f_{1\mathrm{E}}$ is a fuzzy soft $\tau_1\tau_2\tau_3$-semi-open set since $f_{1\mathrm{E}} \sqsubseteq b_E^c$. Also $f_{1\mathrm{E}}$ is a fuzzy soft $\tau_1\tau_2\tau_3$-α-open set since $\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{cl}(\mathrm{FS}.\tau_1\tau_2\tau_3\mathrm{int}(f_{1\mathrm{E}}))) = f_{1\mathrm{E}}$

Now, the fuzzy soft set δ_E which is defined as:
$\delta_\mathrm{E} = \{(x_1, \{0.3/u_1, 0.1/u_2\}), (x_2, \{0.1/u_1, 0.2/u_2\})$ is a fuzzy soft δ^*-open set sinceFS.$\tau_1\mathrm{int}(\mathrm{FS}.\tau_2\mathrm{cl}(\mathrm{FS}.\tau_3\mathrm{int}(\delta_\mathrm{E})))$ = $\mathrm{FS}.\tau_1\mathrm{int}(\mathrm{FS}.\tau_2\mathrm{cl}((h_\mathrm{E})))$ =
$\mathrm{FS}.\tau_1\mathrm{int}(g_{2E}^c)$ = $f_{1\mathrm{E}}$
Hence $\delta_\mathrm{E} \sqsubseteq f_{1\mathrm{E}}$. Therefore the fuzzy soft set δ_E is a fuzzy soft δ^*-open set.

5. Conclusion

Fuzzy soft tritopology is a new and promising domain which can lead to the development of new mathematical models that will significantly contribute to the applications in natural sciences such as and decision making problems, biomathematics and information systems .The concept of fuzzy soft tritopological spaces is initiated in this paper. Some basic notions of classical and generalized concepts have been studied. the purpose of this paper is just to initiate the concept, and there is a lot of scope for the researchers to make their investigations in this field, i.e. this is a beginning of some new generalized structure and the concept like separation axioms and a new kinds of continuous functions and another basic concepts can be studied. Also can be study the relationships among some types of fuzzy soft open sets in fuzzy soft tritopological spaces.

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