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Some Properties of Compactness in Intuitionistic Fuzzy Topological Spaces

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Abstract. The purpose of this paper is to introduce and study the compactness in intuitionistic fuzzy topological spaces. Here we define two new notions of intuitionistic fuzzy compactness in intuitionistic fuzzy topological space and find their relation. Also we find the relationship between intuitionistic general compactness and intuitionistic fuzzy compactness. Here we see that our notions satisfy hereditary and productive property. Finally we observe that our notions preserve under one-one, onto and continuous mapping.

Keywords: Fuzzy set, intuitionistic fuzzy set, intuitionistic fuzzy topological space, intuitionistic fuzzy compactness.

AMS Mathematics Subject Classification (2010): 54A40, 03E72, 03F55

1. Introduction

As a generalization of fuzzy sets introduced by Zadeh [1] after then the concept of intuitionistic fuzzy set was introduced by Atanassov [2] as a sense of generalized this concept and introduced intuitionistic fuzzy sets which take into account both the degrees of membership and non-membership subject to the condition that their sum does not exceed 1. Fuzzy topology was first defined by Chang [17]. Later Coker [3,4,5,6] introduced the basic definitions and properties of intuitionistic fuzzy topological spaces and fuzzy compactness in intuitionistic fuzzy topological spaces. Islam et al. [7, 8], Lee et al. [10, 11], Ahmed et al. [12,13,14,15,16], Ming et al. [21], Talukder et al. [22] and Tamilmani [23] subsequently initiated a study of intuitionistic fuzzy topological spaces by using intuitionistic fuzzy sets. In this paper, we define two notions of compactness in intuitionistic fuzzy topological spaces and some of their features.

2. Notations and preliminaries

Through this paper, X will be a nonempty set, T is a topology, t is a fuzzy topology, $\mathcal{T}$ is an intuitionistic topology and τ is an intuitionistic fuzzy topology. λ and μ are fuzzy sets, $A=(\mu_A,\nu_A)$ is intuitionistic fuzzy set. Particularly $\underline{0}$ and $\underline{1}$ we denote constant fuzzy sets taking values 0 and 1 respectively.

Definition 2.1. [17] Let X be a non empty set. A family t of fuzzy sets in X is called a fuzzy topology on X if the following conditions hold.

(1) $\underline{0}, \underline{1} \in t$,
(2) $\lambda \cap \mu \in t$ for all $\lambda, \mu \in t$,
(3) $\cup \lambda_j \in t$ for any arbitrary family $\{\lambda_j \in t,\ j \in J\}$.

Definition 2.2. [3] Suppose X is a non empty set. An intuitionistic set A on X is an object having the form $A = (X, A_1, A_2)$ where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of member of A while A_2 is called the set of non-member of A. In this paper, we use the simpler notation $A = (A_1, A_2)$ instead of $A = (X, A_1, A_2)$ for an intuitionistic set.

Remark 2.1. Every subset A of a nonempty set X may obviously be regarded as an intuitionistic set having the form $A = (A, A^c)$ where $A^c = X - A$.

Definition 2.3. [3] Let the intuitionistic sets A and B in X be of the forms $A = (A_1, A_2)$ and $B = (B_1, B_2)$ respectively. Furthermore, let $\{A_j, j \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_j = \left(A_j^{(1)}, A_j^{(2)}\right)$. Then

(a) $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$,
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (A_2, A_1)$, denotes the complement of A,
(d) $\cap A_j = (\cap A_j^{(1)}, \cup A_j^{(2)})$,
(e) $\cup A_j = (\cup A_j^{(1)}, \cap A_j^{(2)})$,
(f) $\phi_\sim = (\phi, X)$ and $X_\sim = (X, \phi)$.

Definition 2.4. [5] Let X be a non empty set. A family $\mathcal{T}$ of intuitionistic sets in X is called an intuitionistic topology on X if the following conditions hold.

(1) $\phi_\sim, X_\sim \in \mathcal{T}$,
(2) $A \cap B \in \mathcal{T}$ for all $A, B \in \mathcal{T}$,
(3) $\cup A_j \in \mathcal{T}$ for any arbitrary family $\{A_j \in \mathcal{T}, j \in J\}$.

The pair $(X, \mathcal{T})$ is called an intuitionistic topological space (ITS, in short), members of $\mathcal{T}$ are called intuitionistic open sets (IOS, in short) in X and their complements are called intuitionistic closed sets (ICS, in short) in X.

Definition 2.5. [2] Let X be a non empty set. An intuitionistic fuzzy set A (IFS, in short) in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where μ_A and ν_A are fuzzy sets in X denote the degree of membership and the degree of non-membership respectively subject to the condition $\mu_A(x) + \nu_A(x) \leq 1$.
Throughout this paper, we use the simpler notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ for IFSs.

Definition 2.6. [2] Let X be a nonempty set and IFSs A, B in X be given by $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ respectively, then

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,
(c) $\bar{A} = (\nu_A, \mu_A)$,
(d) $A \cap B = (\mu_A \cap \mu_B,\ \nu_A \cup \nu_B)$,
(e) $A \cup B = (\mu_A \cup \mu_B,\ \nu_A \cap \nu_B)$.

Definition 2.7. [4] Let $\{A_j = \left(\mu_{A_j}, \nu_{A_j}\right), j \in J\}$ be an arbitrary family of IFSs in X. Then
(a) $\cap A_j = (\cap \mu_{A_j},\ \cup \nu_{A_j})$,
(b) $\cup A_j = (\cup \mu_{A_j},\ \cap \nu_{A_j})$,
(c) $0_\sim = (\underline{0}, \underline{1})$, $1_\sim = (\underline{1}, \underline{0})$.

Definition 2.8. [4] An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:
(1) $0_\sim, 1_\sim \in \tau$,
(2) $A \cap B \in \tau$, for all $A, B \in \tau$,
(3) $\cup A_j \in \tau$ for any arbitrary family $\{A_j \in \tau,\ j \in J\}$.

The pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS, in short), members of τ are called intuitionistic fuzzy open sets (IFOS, in short) in X and their complements are called intuitionistic fuzzy closed sets (IFCS, in short) in X.

Definition 2.9. [2] Let X and Y be two nonempty sets and $f: X \to Y$ be a function. If$A = \{(x, \mu_A(x),\ \nu_A(x)): x \in X\}$ and $B = \{(y, \mu_B(y),\ \nu_B(y)): y \in Y\}$ are IFSs in X and Y respectively, then the pre image of B under$f$, denoted by $f^{-1}(B)$ is the IFS in X defined by
$f^{-1}(B) = \left\{\left(x,\ (f^{-1}(\mu_B))(x), (f^{-1}(\nu_B))(x)\right): x \in X\right\} = \{(x, \mu_B(f(x)), \nu_B(f(x))): x \in X\}$ and the image of A under f, denoted by $f(A)$ is the IFS in Y defined by $f(A)$=$\left\{\left(y, (f(\mu_A))(y), (f(\nu_A))(y)\right): y \in Y\right\}$, where for each $y \in Y$

$$(f(\mu_A))(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_A(x) & if\ \ f^{-1}(y) \neq \phi \\ 0 & otherwise \end{cases}$$

$$(f(\nu_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \nu_A(x) & if\ \ f^{-1}(y) \neq \phi \\ 1 & otherwise \end{cases}$$

Definition 2.10. [18] Let $A = (x, \mu_A, \nu_A)$ and $B = (y, \mu_B, \nu_B)$ be IFSs in X and Y respectively. Then the product of IFSs A and B denoted by $A \times B$ is defined by

$$A \times B = \{(x, y), \mu_A \times \mu_B, \nu_A \times \nu_B)\} \text{ where } \left(\mu_A \dot{\times} \mu_B\right)(x, y) = \min\left(\mu_A(x), \mu_B(y)\right)$$

$$\text{and } \left(\nu_A \dot{\times} \nu_B\right)(x, y) = \max\left(\nu_A(x), \nu_B(y)\right) \text{ for all } (x, y) \in X \times Y.$$

Obviously $0 \leq (\mu_A \times \mu_B) + (\nu_A \times \nu_B) \leq 1$. This definition can be extended to an arbitrary family of IFSs.

Definition 2.11. [18] Let (X_j, τ_j), $j = 1,2$ be two IFTSs. The product topology $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is the IFT generated by $\left\{\rho_j^{-1}\left(U_j\right): U_j \in \tau_j, j = 1,2\right\}$, where $\rho_j: X_1 \times X_2 \to X_j$, $j = 1,2$ are the projection maps and IFTS $\{X_1 \times X_2, \tau_1 \times \tau_2\}$ is called the product IFTS of

(X_j, τ_j), $j = 1,2$. In this case, $\mathcal{S} = \{\rho_j^{-1}(U_j), j \in J: U_j \in \tau_j\}$ is a sub base and $\mathcal{B} = \{U_1 \times U_2: U_j \in \tau_j, j = 1,2\}$ is a base for $\tau_1 \times \tau_2$ on $X_1 \times X_2$.

Definition 2.12. [4] Let (X, τ) and (Y, δ) be IFTSs. A function $f: X \to Y$ is called continuous if $f^{-1}(B) \in \tau$ for all $B \in \delta$ and f is called open if $f(A) \in \delta$ for all $A \in \tau$.

Definition 2.13. [9] Let $A = (\mu_A, \nu_A)$ be a IFS in X and U be a non empty subset of X. The restriction of A to U is a IFS in U, denoted by $A|U$ and defined by $A|U = (\mu_A|U, \nu_A|U)$.

Definition 2.14. [7] Let (X, τ) be an intuitionistic fuzzy topological space and U is a non empty sub set of X then $\tau_U = \{A|U: A \in \tau\}$ is an intuitionistic fuzzy topology on U and (U, τ_U) is called sub space of (X, τ).

Definition 2.15. [9] Let $\alpha, \beta \in (0,1)$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $p^x_{(\alpha,\beta)}$ of X defined by $p^x_{(\alpha,\beta)} = \langle x, \mu_p, \nu_p \rangle$, for $y \in X$

$$\mu_p(y) = \begin{cases} \alpha & if \quad y = x \\ 0 & if \quad y \neq x \end{cases} \text{ and } \nu_p(y) = \begin{cases} \beta & if \quad y = x \\ 1 & if \quad y \neq x \end{cases}$$

In this case, x is called the support of $p^x_{(\alpha,\beta)}$. An IFP $p^x_{(\alpha,\beta)}$ is said to belong to an IFS A= $\langle x, \mu_A, \nu_A \rangle$ of X, denoted by $p^x_{(\alpha,\beta)} \in A$, if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Proposition 2.1. [9] An IFS A in X is the union of all IFP belonging to A.

Definition 2.16. [20] A collection $\mathcal{B}$of IFS on a set X is said to be basis (or base) for an IFT on X, if

(i) For every $p^x_{(\alpha,\beta)} \in X$, there exists $B \in \mathcal{B}$ such that $p^x_{(\alpha,\beta)} \in B$.

(ii) If $p^x_{(\alpha,\beta)} \in B_1 \cap B_2$, where $B_1, B_2 \in \mathcal{B}$ then $\exists B_3 \in \mathcal{B}$ such that $p^x_{(\alpha,\beta)} \in B_3 \subseteq B_1 \cap B_2$.

Definition 2.17. [4] Let (X, τ) and (Y, δ)be two IFTS's and let $f: X \to Y$ be a function. Then f is said to be fuzzy continuous iff the preimage of each IFS in $\emptyset$ is an IFS in τ.

Definition 2.18. [4] Let (X, τ) be an IFTS.

(a) If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle: i \in J\}$ of IFOS in X satisfy the condition $\cup \{\langle x, \mu_{G_i}, \nu_{G_i} \rangle: i \in J\} = 1_\sim$ then it is called a fuzzy open cover of X. A finite subfamily of fuzzy open cover $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle: i \in J\}$ of X, which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle: i \in J\}$.

(b) A family $\{\langle x, \mu_{K_i}, \nu_{K_i} \rangle: i \in J\}$ of IFCS's in X satisfies the finite intersection property iff every finite subfamily $\{\langle x, \mu_{K_i}, \nu_{K_i} \rangle: i = 1,2, \ldots, n\}$ of the family satisfies the condition $\bigcap_{i=1}^{n} \{\langle x, \mu_{K_i}, \nu_{K_i} \rangle\} \neq 0_\sim$.

Definition 2.19. [4] An IFTS (X, τ) is called fuzzy compact iff every fuzzy open cover of X has a finite subcover.

Definition 2.20. [4] (a) Let (X,τ) be an IFTS and A be an IFS in X. If a family $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle: i\in J\}$ of IFOS's in X satisfies the condition $A\subseteq\cup\{\langle x,\mu_{G_i},\nu_{G_i}\rangle: i\in J\}$, then it is called a fuzzy open cover of A. A finite subfamily of the fuzzy open cover $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle: i\in J\}$ of A, which is also a fuzzy open cover of A, is called a finite subcover of $\{\langle x,\mu_{G_i},\nu_{G_i}\rangle: i\in J\}$.
(b) An IFS $A=\langle x,\mu_{G_i},\nu_{G_i}\rangle$ in an IFTS (X,τ) is called fuzzy compact iff every fuzzy open cover of A has a finite subcover.

Definition 2.21. [19] An IFTS (X,τ) is called (α,β)-intuitionistic fuzzy compact (resp., (α,β)-intuitionistic fuzzy nearly compact and (α,β)-intuitionistic fuzzy almost compact) if and only if for every family $\{G_i: i\in J\}$ in $\{G: G\in\zeta^X,\tau(G)>\langle\alpha,\beta\rangle\}$ such that $\bigcup_{i\in J}G_i=1_\sim$, where $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, there exists a finite subset J_0 of J such that $\bigcup_{i\in J_0}G_i=1_\sim$ (resp., $\bigcup_{i\in J_0}int_{\alpha,\beta}\left(cl_{\alpha,\beta}(G_i)\right)=1_\sim$ and $\bigcup_{i\in J_0}cl_{\alpha,\beta}(G_i)=1_\sim$).

Definition 2.22. [19] Let (X,τ) be an IFTS and A be an IFS in X. A is said to be (α,β)-intuitionistic fuzzy compact if and only if every family $\{G_i: i\in J\}$ in $\{G: G\in\zeta^X,\tau(G)>\langle\alpha,\beta\rangle\}$ such that $A\subseteq\bigcup_{i\in J_0}G_i$, where $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$.

Definition 2.23. [19] A family $\{K_i: i\in J\}$ in $\{K: K\in\zeta^X,\tau^*(K)>\langle\alpha,\beta\rangle\}$, where $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$ has the finite intersection property (FIP) if and only if for any finite subset J_0 of J, $\bigcap_{i\in J_0}K_i\ne 0_\sim$.

Definition 2.24. [19] An IFTS (X,τ) is called (α,β)-intuitionistic fuzzy regular if and only if for each IFS A in X such that $\tau(A)>\langle\alpha,\beta\rangle$, where $\alpha\in I_0,\beta\in I_1$ with $\alpha+\beta\le 1$, can be written as $A=\cup\{B: B\in\zeta^X,\tau(B)\ge\tau(A), cl_{\alpha,\beta}(B)\subseteq A\}$.

3. Compactness in intuitionistic fuzzy topological space

In this section we define two new definitions of intuitionistic fuzzy compactness (IF-compact) in intuitionistic fuzzy topological space (IFTS, in short) and established several properties of such notions.

Definition 3.1. Let (X,τ) be an intuitionistic fuzzy topological space. A family $\{(\mu_{G_i},\nu_{G_i}): i\in J\}$ of IFOS in X is called open cover of X if $\cup\mu_{G_i}=1$ and $\cap\nu_{G_i}=0$. If every open cover of X has a finite subcover then X is said to be intuitionistic fuzzy compact (IF-compact, in short).

Definition 3.2. A family $\{(\mu_{G_i},\nu_{G_i}): i\in J\}$ of IFOS in X is called (α,β)-level open cover of X if $\cup\mu_{G_i}\ge\alpha$ and $\cap\nu_{G_i}\le\beta$ with $\alpha+\beta\le 1$. If every (α,β)-level open cover of X has a finite subcover then X is said to be (α,β)-level IF-compact.

Theorem 3.1. Let (X,T) be a topological space and (X,τ) be its corresponding IFTS, where $\tau = \left\{(1_{A_j}, 1_{A_j}^{c}), j \in J : A_j \in T\right\}$. Then (X,T) is compact if (X,τ) is IF-compact.

Proof: Let (X,T) be compact. Consider $\{G_i | i \epsilon J\}$be the open cover of X,

i.e. $\cup G_i = X$ (i)

Since X is compact then $\exists G_{i1}, G_{i2}, \dots, G_{in} \in T$ such that

$G_{i1} \cup G_{i2} \cup \dots \cup G_{in} = X$ (ii)

Now it is clear that $(1_{G_i}, 1_{G_i}{}^{c}) \in T$ (by the definition).

Also we have, $\cup\, (1_{G_i}, 1_{G_i}{}^{c}) = (\cup\, 1_{G_i}, \cap\, 1_{G_i}{}^{c})$

$= (1_{\cup G_i}, 1_{\cap G_i}{}^{c})$

$= (1_X, 1_{\cap G_i}{}^{c})$

But we have, $1_X + 1_{\cap G_i}{}^{c} \leq 1$ then it must be $1_{\cap G_i}{}^{c} = 0$. Therefore we get, $\cup\, (1_{G_i}, 1_{G_i}{}^{c}) = (1_X, 0)$.

Also by (ii) we get, $(1_X, 0) = (1_{G_{i1} \cup G_{i2} \cup \dots \cup G_{in}}, 0)$

$= (\bigcup_{j=1}^{n} 1_{G_{ij}}, 0)$

$= \cup\, (1_{G_{ij}}, 0)$

Hence it is clear that the IFTS (X,τ) is IF-compact.

Corollary 3.1. Let (X,T) be a topological space and (X,τ) be its corresponding IFTS, where $\tau = \left\{(1_{A_j}, 1_{A_j}^{c}), j \in J : A_j \in T\right\}$. Then (X,T) is compact if (X,τ) is (α,β)-level IF-compact.

Proof: Here it is clear that for any $\alpha, \beta \epsilon I$ with $\alpha + \beta \leq 1 \Rightarrow 1 \geq \alpha$ and $\beta \geq 0$. So, (X,τ) is (α,β)-level IF-compact.

Theorem 3.2. Let $(X,\mathcal{T})$ be an intuitionistic topological space and (X,τ) be its corresponding IFTS, where $\tau = \left\{1_{A_j} = \left(1_{A_{j1}}, 1_{A_{j2}}\right), j \in J : A_j = \left(A_{j1}, A_{j2}\right) \in \mathcal{T}\right\}$. Then $(X,\mathcal{T})$ is intuitionistic compact if (X,τ) is IF-compact.

Proof: Let $(X,\mathcal{T})$ be an intuitionistic compact space, we shall prove that (X,τ) is IF-compact. Consider $\{1_{A_k}\}$ be an open cover of τ, i.e. $\cup\, 1_{A_k}$= (1, 0), where (1, 0) is intuitionistic fuzzy set.

Now $1_{A_k} = \left(1_{A_{k1}}, 1_{A_{k2}}\right) \Rightarrow \cup\, 1_{A_k} = \cup \left(1_{A_{k1}}, 1_{A_{k2}}\right)$

$\Rightarrow \cup\, 1_{A_k} = \left(\cup\, 1_{A_{k1}}, \cap\, 1_{A_{k2}}\right)$

$\Rightarrow 1_{\cup A_k} = \left(1_{\cup A_{k1}}, 1_{\cap A_{k2}}\right)$

$\Rightarrow 1_{\cup A_k} = (1_X, 0)$

$\Rightarrow 1_{\cup A_k} = (1, 0)$

By the given definition $\{A_k \in \mathcal{T}\}$, $k \in \Lambda$ is the open cover of X, since $\cup\, A_k$= (X, $\emptyset$). But we have $(X,\mathcal{T})$ is compact then $\exists\, A_{k_{i1}}, A_{k_{i2}}, \dots, A_{k_{in}} \in \mathcal{T}$ such that $\bigcup_{j=1}^{n} A_{k_{ij}} = (X, \emptyset)$

$\Rightarrow \bigcup_{j=1}^{n} (A_{k_{ij_1}}, A_{k_{ij_2}}) = (X, \emptyset)$

$\Rightarrow (\bigcup_{j=1}^{n} A_{k_{ij_1}}, \bigcap_{j=1}^{n} A_{k_{ij_2}}) = (X, \emptyset)$

$\Rightarrow (1_{\bigcup_{j=1}^{n} A_{k_{ij_1}}}, 1_{\bigcap_{j=1}^{n} A_{k_{ij_2}}}) = (1_X, 1_\emptyset)$

$\Rightarrow (1_{\bigcup_{j=1}^{n} A_{k_{ij_1}}}, 1_{\bigcap_{j=1}^{n} A_{k_{ij_2}}}) = (1, 0)$

Hence (X, τ) is IF-compact.

Theorem 3.3. Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is bijective, open and continuous. Then (Y, δ) is IF-compact $\Rightarrow$ (X, τ) is IF-compact.

Proof: Let $A_i = (\mu_i, \nu_i) \in \tau$ with $\cup A_i$ = (1, 0). Now $A_i \in \tau$ =>f(A_i) $\in \delta$ with $\cup f(A_i)$ = (1, 0).

For $y \in Y$, $f(A_i)$(y) = $(y, f(\mu_{A_i})(y), f(\nu_{A_i})(y))$,

where $f(\mu_{A_i})(y) = \sup_{x \in f^{-1}(y)} \mu_{A_i}(x) = \mu_{A_i}(x)$.

Similarly we get, $f(\nu_{A_i})(y) = \nu_{A_i}(x)$.Now $\cup f(A_i) = \cup (f(\mu_{A_i}), f(\nu_{A_i})) = (\cup f(\mu_{A_i}), \cap f(\nu_{A_i}))$,i.e.$\cup f(\mu_{A_i})$(y) = $\cup \mu_{A_i}(x)$ = 1 and $\cap f(\nu_{A_i})$(y) = $\cap \nu_{A_i}(x)$ = 0, so $\cup f(A_i)$= (1, 0). Since f is an open then $\{f(A_i)\}$ is an open cover of Y. Again Y is compact then there exist $f(A_{1i}), f(A_{2i}), \ldots, f(A_{ni}) \in \delta$ such that $\bigcup_{j=1}^{n} f(A_{ji}) = (1, 0)$ => $f(\bigcup_{j=1}^{n} A_{ji}) = (1, 0)$ => $f^{-1}(f(\bigcup_{j=1}^{n} A_{ji})) = f^{-1}(1, 0)$ => $f^{-1}(1, 0) \subseteq \bigcup_{j=1}^{n} A_{ji}$ (since from Chang $\mu \subseteq f^{-1}(f(\mu))$). Therefore $\bigcup_{j=1}^{n} A_{ji}$ = (1, 0). Hence (X, τ) is IF-compact.

Theorem 3.4. Let (X, τ) and (Y, δ) be IFTSs and $f: X \to Y$ is one-one, onto and continuous. Then (X, τ) is IF-compact $\Rightarrow$ (Y, δ) is IF-compact.

Proof: Let $A_i = (\mu_i, \nu_i) \in \delta$ with $\cup A_i$ = (1, 0). Since δ is a topology so $\cup A_i \in \delta$ =>$f^{-1}(\cup A_i) \in \tau$ with $f^{-1}(\cup A_i)$ = (1, 0) (as f is continuous) =>$\cup f^{-1}(A_i)$ = (1, 0). But $\cup f^{-1}(A_i)$ =$\cup f^{-1}(\mu_i, \nu_i)$=$\cup (f^{-1}(\mu_i), f^{-1}(\nu_i)) \in \tau$ with $\cup(f^{-1}(\mu_i), f^{-1}(\nu_i))$ = (1, 0). Since (X, τ) is IF-compact then $\exists A_{i1}, A_{i2}, \ldots, A_{im} \in \delta$ where $(f^{-1}(\mu_{i1}), f^{-1}(\nu_{i1}))$, $(f^{-1}(\mu_{i2}), f^{-1}(\nu_{i2}))$, …, $(f^{-1}(\mu_{im}), f^{-1}(\nu_i m)) \in \tau$ such that

$(f^{-1}(\mu_{i1}), f^{-1}(\nu_{i1})) \cup (f^{-1}(\mu_{i2}), f^{-1}(\nu_{i2})) \cup \ldots \cup (f^{-1}(\mu_{im}), f^{-1}(\nu_i m))$ = (1, 0)

=> $\bigcup_{j=1}^{m}(f^{-1}(\mu_{ij}), f^{-1}(\nu_{ij}))$ = (1, 0)

=> $(\bigcup_{j=1}^{m} f^{-1}(\mu_{ij}), \bigcap_{j=1}^{m} f^{-1}(\nu_{ij}))$ = (1, 0)

=>$f(\bigcup_{j=1}^{m} f^{-1}(\mu_{ij}), \bigcap_{j=1}^{m} f^{-1}(\nu_{ij})) = f(1, 0)$

=>$(\bigcup_{j=1}^{m} f(f^{-1}(\mu_{ij})), \bigcap_{j=1}^{m} f(f^{-1}(\nu_{ij})))$ = (1, 0),

since f is one-one and onto, so $f(1, 0) = (1, 0)$.

Therefore $(\bigcup_{j=1}^{m} \mu_{ij}, \bigcap_{j=1}^{m} \nu_{ij})$ = (1, 0), i.e. $\bigcup_{j=1}^{m}(\mu_{ij}, \nu_{ij})$ = (1, 0).

Hence (Y, δ) is IF-compact.

Theorem 3.5. Let (X, τ)be an IFTS and (V, τ_V) be a subspace of (X, τ) with (X, τ) is IF-compact. Let $f: (X, \tau) \to (V, \tau_V)$ be continuous, open and onto, then (V, τ_V) is IF-compact.

Proof: Let $\mathcal{M} = \{B_i : i \epsilon J\}$ be an open cover of (V, τ_V) with $\cup B_i = (1_V, 0)$. By the definition of subspace topology, let $B_i = U_i | V$, where $U_i \epsilon \tau$. Since f is continuous then $f^{-1}(B_i) \epsilon\, \tau$ implies that $f^{-1}(U_i | V) \epsilon\, \tau$.

As, (X, τ) is IF-compact then $\bigcup_{i \epsilon J} f^{-1}(U_i | V)(x) = (1_X, 0)$. Thus we see that, $\{f^{-1}(U_i | V) : i \in J\}$ is an open cover of (X, τ). Hence there exist

$$f^{-1}(U_{i1} | V), f^{-1}(U_{i2} | V), \ldots, f^{-1}(U_{in} | V) \in \{f^{-1}(U_i | V)$$

such that

$$\cup_{k=1}^{n} f^{-1}(U_{ik}|V) = (1_X, 0).$$

Put $B_{ik} = U_{ik}|V$, then it is clear that $B_{ik} \in \tau_V$ with $\cup_{k=1}^{n} f^{-1}(B_{ik}) = (1_X, 0)$

$$=> f\left(\bigcup_{k=1}^{n} f^{-1}(B_{ik})\right) = f(1_X, 0)$$

$$=> \bigcup_{k=1}^{n} f(f^{-1}(B_{ik})) = (f(1_X), 0)$$

$=> \cup_{k=1}^{n} B_{ik} = (1_V, 0)$ as f is open.
Hence (V, τ_V) is IF-compact.

Theorem 3.6. Show that the following statements are equivalent:
(i) X is IF-compact,
(ii) For every $\{F_i\}$ where $F_i = (\nu_{F_i,}\mu_{F_i})$ of closed subset of X with $\cap F_i = (0, 1)$ implies $\{F_i\}$ contains finite subclass $\{F_{i1}, F_{i2}, \dots, F_{im}\}$ with $F_{i1} \cap F_{i2} \cap \dots \cap F_{im} = (0, 1)$.
Proof: (i) => (ii). Suppose $\cap F_i = (0, 1)$ then by De Morgan's law

$$(\cap F_i)^c = ((0, 1))^c$$
$$=> \cup {F_i}^c = (1, 0)$$
$$=> \cup (\nu_{F_i,}\mu_{F_i})^c = (1, 0)$$
$$=> \cup (\mu_{F_i}, \nu_{F_i}) = (1, 0)$$
$$=> (\cup \mu_{F_i}, \cap \nu_{F_i}) = (1, 0).$$

So, $\{{F_i}^c\}$, $({F_i}^c = (\mu_{F_i}, \nu_{F_i}))$ is an open cover of X. Since X is IF-compact hence $\exists {F_{i1}}^c, {F_{i2}}^c, \dots, {F_{im}}^c \epsilon \{{F_i}^c\}$ such that ${F_{i1}}^c \cup {F_{i2}}^c \cup \dots \cup {F_{im}}^c = (1, 0)$. Then

$(0, 1) = (1, 0)^c = ({F_{i1}}^c \cup {F_{i2}}^c \cup \dots \cup {F_{im}}^c)^c$
$= ({F_{i1}}^c)^c \cap ({F_{i2}}^c)^c \cap \dots \cap ({F_{im}}^c)^c$ (By De Morgan's law)
$= F_{i1} \cap F_{i2} \cap \dots \cap F_{im}$,

so we have shown that (i) => (ii).
(ii)=>(i). Let $\{G_i\}$be an open cover of X where $G_i = (\mu_{G_i}, \nu_{G_i})$, i.e. $\cup_i G_i = (1, 0)$.
By De Morgan's law,

$$(0, 1) = (1, 0)^c = (\cup_i G_i)^c = \cap_i {G_i}^c.$$

Since each G_i is open, so $\{{G_i}^c\}$ is a class of closed sets and by (ii) $\exists {G_{i1}}^c, {G_{i2}}^c, \dots, {G_{im}}^c \epsilon \{{G_i}^c\}$ such that

$${G_{i1}}^c \cap {G_{i2}}^c \cap \dots \cap {G_{im}}^c = (0, 1).$$

So by De Morgan's law

$$(1, 0) = (0, 1)^c = ({G_{i1}}^c \cap {G_{i2}}^c \cap \dots \cap {G_{im}}^c)^c = G_{i1} \cup G_{i2} \cup \dots \cup G_{im},$$

hence X is IF-compact. So, we have shown that (ii)=>(i).

Theorem 3.7. Let the IFTS's (X_1, τ_1) and (X_2, τ_2) be IF-compact. Then the product IFT $\tau_1 \times \tau_2$ on $X_1 \times X_2$ is IF-compact.
Proof: Consider, (X_1, τ_1) and (X_2, τ_2) is IF-compact. Let $A_i = (\mu_{A_i,}\nu_{A_i})\epsilon\tau_1$ with $\cup A_i = (1, 0)$ and $B_i = (\mu_{B_i,}\nu_{B_i})\epsilon\tau_2$ with $\cup B_i = (1, 0)$.

Now

$$A_i \times B_i = (\mu_{A_i}, \nu_{A_i}) \times (\mu_{B_i}, \nu_{B_i}) = (\mu_{A_i} \times \mu_{B_i}, \nu_{A_i} \times \nu_{B_i})$$

where

$$\left(\mu_{A_i} \dot{\times} \mu_{B_i}\right)(x, y) = \min\left(\mu_{A_i}(x), \mu_{B_i}(y)\right), \text{ where } x \epsilon X_1,\ y \epsilon X_2$$
$$= \min(1, 1) = 1.$$

Similarly,

$$(\nu_{A_i} \dot{\times} \nu_{B_i})(x, y) = \max(\nu_{A_i}(x), \nu_{B_i}(y))$$
$$= \max(0, 0) = 0.$$

So, $A_i \times B_i = (1, 0)$. But by the definition of product topology, $A_i \times B_i \,\epsilon\, \tau_1 \times \tau_2$, i.e. $\{A_i \times B_i\}$ is a family of intuitionistic fuzzy open set in $X_1 \times X_2$. Choose $\cup (A_i \times B_i) = (1, 0)$. Since (X_1, τ_1) is IF-compact, then $\{A_i\}$ has finite subclass $\{A_{ij}\}$ such that $\bigcup_{j=1}^{n} A_{ij} = (1, 0)$.

Similarly, since (X_2, τ_2) is IF- compact, then $\{B_i\}$ has finite subclass $\{B_{ik}\}$ such that $\bigcup_{k=1}^{m} B_{ik} = (1, 0)$. Therefore

$$\bigcup_{j=1}^{n} A_{ij} \times \bigcup_{k=1}^{m} B_{ik} = (1, 0)$$

$$=> \bigcup_{j=1}^{n} \left(\mu_{A_{ij}}, \nu_{A_{ij}}\right) \times \bigcup_{k=1}^{m} (\mu_{B_{ik}}, \nu_{B_{ik}}) = (1, 0)$$

$=> \left(\bigcup_{j=1}^{n} \mu_{A_{ij}},\ \bigcap_{j=1}^{n} \nu_{A_{ij}}\right) \times \left(\bigcup_{k=1}^{m} \mu_{B_{ik}},\ \bigcap_{k=1}^{m} \nu_{B_{ik}}\right) = (1, 0).$

Hence there exist four cases:

Case-I: If $\bigcup_{j=1}^{n} \mu_{A_{ij}} = 1$, $\bigcup_{k=1}^{m} \mu_{B_{ik}} = 1$

Case-II: If $\bigcup_{j=1}^{n} \mu_{A_{ij}} = 1$, $\bigcap_{k=1}^{m} \nu_{B_{ik}} = 0$

Case-III: If $\bigcap_{j=1}^{n} \nu_{A_{ij}} = 0$, $\bigcup_{k=1}^{m} \mu_{B_{ik}} = 1$

Case-IV: If $\bigcap_{j=1}^{n} \nu_{A_{ij}} = 0$,$\bigcap_{k=1}^{m} \nu_{B_{ik}} = 0$

Here from four cases, we see that the product topology $(X_1 \times X_2, \tau_1 \times \tau_2)$ is IF-compact.

4. Conclusion

In this paper, theorems 3.1 and 3.2 show that our definitions are appropriate for intuitionistic fuzzy compactness. Here we see that our two notions satisfy hereditary and productive property. Also we observe that our notions preserve under one-one, onto and continuous mapping.

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