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Computing the F-ve-degree Index and its Polynomial of Dominating Oxide and Regular Triangulate Oxide Networks

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Abstract. In this paper, we introduce the F-ve-degree index of a molecular graph. Considering the F-ve-degree index, we define the F-ve-degree polynomial of a graph. We compute the F-ve-degree index and F-ve-degree polynomial of dominating oxide networks and regular triangulate oxide networks.

Keywords: F-ve-degree index, dominating oxide network, regular triangulate oxide network.

AMS Mathematics Subject Classification (2010): 05*C*05, 05*C*07, 05*C*35

1. Introduction

A molecular graph is a simple graph related to the structure of a chemical compound. Each vertex of this graph represents an atom of the molecule and its edges to the bonds between atoms. Chemical graph theory is a branch of Graph Theory whose focus of interest is to finding topological indices of molecular graphs, which correlate well with chemical properties of the chemical molecules. Several topological indices have been considered in Theoretical Chemistry, especially in QSPR/QSAR study, see [1].

Let G be a finite, simple connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The set of all vertices which adjacent to v is called open neighborhood of v and denoted by $N(v)$. The closed neighborhood set of v is the set $N[v] = N(v) \cup \{v\}$. Let S_v denote the sum of the degrees of all vertices adjacent to a vertex v. Chellali et al. [2] defined the ve-degree concept in graph theory as follows:

The ve-degree $d_{ve}(v)$ of a vertex v in a graph G is the number of different edges that incident to any vertex from the closed neighborhood of v.

The first ve-degree Zagreb beta index of a graph G is defined as

$$Ve_1(G) = \sum_{uv \in E(G)} \left[d_{ve}(u) + d_{ve}(v)\right].$$

The second ve-degree Zagreb index of a graph G is defined as

$$Ve_2(G) = \sum_{uv \in E(G)} d_{ve}(u) d_{ve}(v).$$

The above two ve-degree Zagreb indices were proposed by Ediz in [3]. Recently, some ve-degree topological indices were studied, for example, in [4, 5, 6, 7, 8, 9].

The forgotten topological index or F-index was studied by Furtula and Gutman in [10] and it is defined as

$$F(G) = \sum_{u \in V(G)} d_G(u)^3 = \sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2 \right].$$

The F-index was also studied, for example, in [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

Motivated by the definition of the F-index and its applications, we introduce the F-ve-degree index and F_1-ve-degree index of a graph G as follows:

The F-ve-degree index of a graph G is defined as

$$F_{ve}(G) = \sum_{uv \in E(G)} \left[d_{ve}(u)^2 + d_{ve}(v)^2 \right]. \tag{1}$$

The F_1-ve-degree index of a graph G is defined as

$$F_1ve(G) = \sum_{u \in V(G)} d_{ve}(u)^3.$$

Considering the F-ve-degree index, we propose the F-ve-degree polynomial of a graph G as

$$F_{ve}(G, x) = \sum_{uv \in E(G)} x^{\left[d_{ve}(u)^2 + d_{ve}(v)^2 \right]}. \tag{2}$$

We consider the families of dominating oxide networks and regular triangulate oxide networks [4]. In this paper, we obtain exact expressions for the F-ve-degree index and F-ve-degree polynomial of dominating oxide networks (*DOX*) and regular triangulate oxide networks (*RTOX*).

2. Results for dominating oxide networks *DOX*(*n*)

The family of dominating oxide networks is symbolized by *DOX*(*n*). The molecular structure of a dominating oxide network is presented in Figure 1.

In [4], Ediz obtained the partition of the edges with respect to their sum degree of end vertices of edges for dominating oxide networks in Table 1.

(S_u, S_v)	(8, 12)	(8, 14)	(12, 12)	(12, 14)	(14, 16)	(16, 16)
Number of edges	$12n$	$12n-12$	6	$12n-12$	$24n-24$	$54n^2-114n+60$

Table 1:

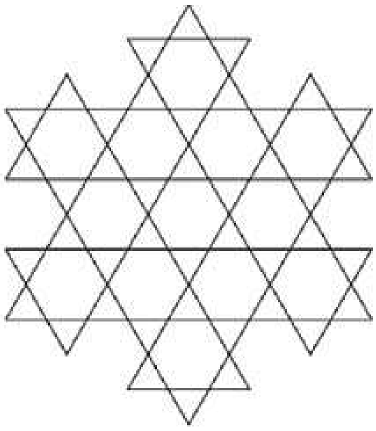

Figure 1: The structure of a dominating oxide network

Also he obtained the ve-degree partition of the end vertices of edges for dominating oxide networks in Table 2.

$(d_{ve}(u), d_{ve}(v))$	(7, 10)	(7, 12)	(10, 10)	(10, 12)	(12, 14)	(14, 14)
Number of edges	$12n$	$12n–12$	6	$12n–12$	$24n–24$	$54n^2– 114n+60$

Table 2: The ve-degree of the end vertices of edges for *DOX* networks

In the following theorem, we compute the F-ve-degree index of $DOX(n)$.

Theorem 1. The F-ve-degree index of a dominating oxide network $DOX(n)$ is given by
$F_{ve}(DOX(n)) = 21168n^2 – 29496n + 11316.$

Proof: Let G be the molecular graph of a dominating oxide network $DOX(n)$. By using equation (1) and Table 2, we deduce

$$F_{ve}(DOX(n)) = \sum_{uv\in E(G)}\left[d_{ve}(u)^2 + d_{ve}(v)^2\right]$$
$$= (7^2+10^2)12n+(7^2+12^2)(12n–12)+(10^2+10^2)6+(10^2+12^2)(12n – 12)$$
$$+(12^2+14^2)(24n – 24) +(14^2+14^2)(54n^2 –114n + 60)$$
$$= 21168n^2 – 29496n + 11316.$$

In the following theorem, we calculate the F-ve-degree polynomial of $DOX(n)$.

Theorem 2. The F-ve-degree polynomial of a dominating oxide network $DOX(n)$ is given by
$F_{ve}(DOX(n), x) = 12nx^{149} + (12n – 12)x^{193} + 6x^{200}+ (12n – 12)x^{244} + (24n – 24)x^{340} + (54n^2 – 114n + 60)x^{392}.$

Proof: Let G be the molecular graph of a dominating oxide network $DOX(n)$. By using equation (2) and Table 2, we derive

$$F_{ve}(DOX(n),x)=\sum_{uv\in E(G)} x^{\left[d_{ve}(u)^2+d_{ve}(v)^2\right]}$$

$$=12nx^{(7^2+10^2)}+(12n-12)x^{(7^2+12^2)}+6x^{10^2+10^2}(12n-12)x^{(10^2+12^2)}$$
$$+(24n-24)x^{(12^2+14^2)}+(54n^2-114n+60)x^{(14^2+14^2)}$$
$$=12nx^{149}+(12n-12)x^{193}+6x^{200}+(12n-12)x^{244}$$
$$+(24n-24)x^{340}+(54n^2-114n+60)x^{392}.$$

3. Results for regular triangulate oxide networks *RTOX*(*n*)

The family of regular triangulate oxide network is denoted by *RTOX*(*n*), $n\geq 3$. The molecular structure of a regular triangulate oxide network is shown in Figure 2.

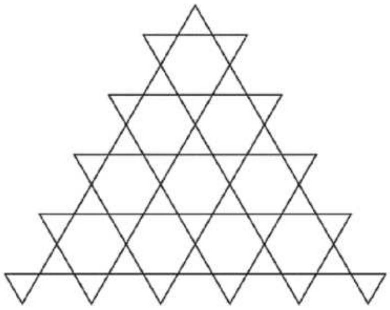

Figure 2: The structure of a regular triangulate oxide network *RTOX*(5)

Ediz [4] obtained the partition of the edges with respect to their sum degree of end vertices of edges for regular triangulate oxide networks in Table 3.

(S_u, S_v)	(6,6)	(6,12)	(8,12)	(8,14)	(12,12)	(12,14)	(14,14)	(14,16)	(16, 16)
Number of edges	2	4	4	$6n-8$	1	6	$6n-9$	$6n-12$	$3n^2-12n+12$

Table 3:

Also he obtained the ve-degree partition of the end vertices of edges for regular triangulate oxide networks in Table 4.

$(d_{ve}(u),d_{ve}(v))$	(5,5)	(5,10)	(7,10)	(7,12)	(10,10)	(10,12)	(12,12)	(12,14)	(14,14)
Number of edges	2	4	4	$6n-8$	1	6	$6n-9$	$6n-12$	$3n^2-12n+12$

Table 4: The ve-degree of the end vertices of edges for *RTOX* networks

In the following theorem, we determine the F-ve-degree index of *TROX*(*n*).

Theorem 3. The *F*-ve-degree index of a regular triangulate oxide network *TROX*(*n*) is
$F_{ve}(TROX(n)) = 1176n^2 + 222n - 652$.

Proof: Let G be the molecular graph of a regular triangulate oxide network $TROX(n)$. By using equation (1) and Table 4, we deduce

$$F_{ve}(TROX(n)) = \sum_{uv\in E(G)} \left[d_{ve}(u)^2 + d_{ve}(v)^2 \right]$$

$$= (5^2+5^2)2+(5^2+10^2)4+(7^2+10^2)4+(7^2+12^2)(6n-8)+(10^2+10^2) + (10^2+12^2)6 + (12^2+12^2)(6n-9) + (12^2+14^2)(6n-12) + (14^2+14^2)(3n^2-12n+12)$$

$$= 1176n^2 + 222n - 652.$$

In the following theorem, we calculate the F-ve-degree polynomial of $TROX(n)$.

Theorem 4. The F-ve-degree polynomial of a regular triangulate oxide network $TROX(n)$ is given by

$$F_{ve}(TROX(n), x) = 2x^{50} + 4x^{125} + 4x^{149} + (6n-8)x^{193} + x^{200} + 6x^{244} + (6n-9)x^{288} + (6n-12)x^{340} + (3n^2-12n+12)x^{392}.$$

Proof: Let G be the molecular graph of a regular triangulate oxide network $TROX(n)$. By using equation (2) and Table 4, we obtain

$$F_{ve}(TROX(n), x) = \sum_{uv\in E(G)} x^{\left[d_{ve}(u)^2 + d_{ve}(v)^2 \right]}$$

$$= 2x^{(5^2+5^2)} + 4x^{(5^2+10^2)} + 4x^{(7^2+10^2)} + (6n-8)x^{(7^2+12^2)} + x^{(10^2+10^2)}$$
$$+6x^{(10^2+12^2)} + (6n-9)x^{(12^2+12^2)} + (6n-12)x^{(12^2+14^2)} + (3n^2-12n+12)x^{(14^2+14^2)}$$
$$= 2x^{50} + 4x^{125} + 4x^{149} + (6n-8)x^{193} + x^{200} + 6x^{244}$$
$$+(6n-9)x^{288} + (6n-12)x^{340} + (3n^2-12n+12)x^{392}.$$

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