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SD and k-SD Prime Cordial graphs

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Abstract. In this paper, we investigate the SD-Prime cordial labeling of Pl_n graph and k-SD-Prime cordial labeling of $(P_n \odot K_1) \cup K_{1,n,n}$ and $P_n \cup K_{1,n,n}$.

Keywords: SD-Prime cordial labeling, k-SD-Prime cordial labeling, k-SD-Prime cordial graph.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [5]. For standard terminology and notations related to number theory we refer to Burton [2] and graph labeling, we refer to Gallian [4]. The notion of prime labeling for graphs originated with Roger Entringer and was introduced in a paper by Tout et al. [12] in the early 1980s and since then it is an active field of research for many scholars. In [13], Vaidya et al. introduced the concept of k-prime labeling of graph. Sundaram et al. introduced the notion of prime cordial labeling in [11]. The concept of neighborhood-prime labeling of graph was introduced by Patel et al. [10]. Lawrence et al. introduced the notation of k-neighborhood-prime labeling of graph in [8]. Lau et al was introduced a variant of prime graph labeling of graph in [6]. In [7], Lau et al. introduced SD-prime cordial labeling and they discussed SD-prime cordial labeling for some standard graphs. In [9], Lourdusamy et al. investigated some new construction of SD-prime cordial graph. In [3], Delman et.al., introduced the concept of k-SD-prime cordial labeling of graph and discussed k-SD-prime cordial labeling of some standard graphs. In [1], Babujee defined a class of planar graph as graph obtained by removing certain edges from the corresponding complete graph. The class of planar graph so obtained is denoted by Pl_n. Here we discuss the SD-Prime cordial labeling of Pl_n graph, for $n \geq 3$ and k-SD-Prime cordial labeling of $(P_n \odot K_1) \cup K_{1,n,n}$, for $n \geq 2$ and $P_n \cup K_{1,n,n}$, for $n \geq 2$.

2. Basic definitions

Definition 2.1. A complete biparitite graph $K_{1,n}$ is called a star and it has n+1 vertices and n edges. $K_{1,n,n}$ is the graph obtained by the subdivision of the edges of the star $K_{1,n}$.

Definition 2.2. Let K_n be the complete graph on n vertices $V_n = \{1,2,...,n\}$. The class of graphs Pl_n has the vertex set V_n and the edge set

$$E_n = E(K_n)\backslash\{(k,l) : 3 \le k \le n-2,\ k+2 \le l \le n\}.$$

Definition 2.3. Comb is a graph obtained by joining a single pendant edge to each vertex of a path. In other words $P_n \odot K_1$ is a comb graph.

Definition 2.4. Let G = (V,E) be a graph with n vertices. A function $f : V(G) \to \{1,2,3,\dots,n\}$ is said to be a prime labeling, if it is bijective and for every pair of adjacent vertices u and v, $\gcd(f(u),f(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 2.5. A k-prime labeling of a graph G is an injective function $f : V \to \{k, k+1,\dots, k+|V|-1\}$ for some positive integer k that induces a function $f^+: E(G) \to N$ of the edges of G defined by $f^+(uv) = \gcd(f(u),f(v))$, $\forall e = uv \in E(G)$ such that $\gcd(f(u), f(v)) = 1$, $\forall e = uv \in E(G)$. The graph which admits a k-prime labeling is called a k-prime graph.

Definition 2.6. Let G = (V,E) be a graph with n vertices. A bijective function $f : V(G) \to \{1,2,3,...,n\}$ is said to be a neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd \{f(u): u \in N(v)\} = 1$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph.

Definition 2.7. Let G = (V(G),E(G)) be a graph with n vertices. A bijective function $f:V(G)\to\{k,k+1,\dots,k+n-1\}$ is said to be a k-neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd \{f(u) : u \in N(v)\} = 1$. A graph which admits k-neighborhood-prime labeling is called a k-neighborhood-prime graph.

Definition 2.8. Given a bijection $f : V(G) \to \{1,2,\dots,|n\}$, we associate 2 integers $S = f(u)+f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in E. The labeling f induces an edge labeling $f' : E(G) \to \{0,1\}$ such that for any edge uv in G, $f'(uv) = 1$ if $\gcd(S,D) = 1$ and 0 otherwise. We say f is SD-prime labeling if $f'(uv) = 1$ for all $uv \in E(G)$. Moreover, G is SD-prime if it admits SD-prime labeling.

Definition 2.9. Given a bijection $f : V(G) \to \{1,2,\dots,|V(G)|\}$, we associate two integers $S = f(u) + f(v)$ and $D = | f(u) - f(v) |$ with every edge uv in E(G). The labeling f induces an edge labeling $f^*: E(G) \to \{0, 1\}$ such that for any edge uv in E(G), $f^*(uv) = 1$ if $\gcd(S,D) = 1$ and 0 otherwise. Let $e_{f^*}(i)$ be the number of edges labeled with $i \in \{0, 1\}$. We say f is SD-prime cordial labeling if $| e_{f^*}(0)-e_{f^*}(1)| \le 1$. Moreover G is SD-prime cordial if it admits SD-prime cordial labeling.

3. Main theorems

Theorem 3.1. Pl_n is a SD-prime cordial graph, for $n \ge 3$.

Proof: Let $v_1,v_2,...,v_n$ be the vertices and $e_1,e_2,...,e_{3n-6}$ be the edges of Pl_n, where $e_i = v_iv_{i+1}$ for $1 \le i \le n-3$, $e_{i+n-3} = v_{n-1}v_i$ for $1 \le i \le n-2$, $e_{i+2n-5} = v_nv_i$ for $1 \le i \le n-2$ and $e_{3n-6} = v_{n-1}v_n$.

Let G = Pl_n. Then $|V(G)| = n$ and $| E(G) | = 3n-6$.

Define $f : V(G) \to \{1,2,\ldots,n\}$ as follows:

Case 1: $n \equiv 1,3 \pmod 4$.

$$g(v_i)=\begin{cases} i+2 & \text{if } i\equiv 0,1 \pmod 4 \text{ and } 1\le i\le n-2\\ i+3 & \text{if } i\equiv 2 \pmod 4 \text{ and } 1\le i\le n-2\\ i+1 & \text{if } i\equiv 3 \pmod 4 \text{ and } 1\le i\le n-2\\ 2 & \text{if } i=n-1\\ 1 & \text{if } i=n \end{cases}$$

Then induced edge labels are

$g^*(e_{2i-1}) = 0$, for $1 \le i \le \frac{n-3}{2}$

$g^*(e_{2i}) = 1$, for $1 \le i \le \frac{n-3}{2}$

$$g^*(e_{n-3+i})=\begin{cases} 1 & \text{if } i\equiv 1,2 \pmod 4 \text{ and } 1\le i\le n-2\\ 0 & \text{if } i\equiv 0,3 \pmod 4 \text{ and } 1\le i\le n-2 \end{cases}$$

$$g^*(e_{2n-5+i})=\begin{cases} 0 & \text{if } i\equiv 1,2 \pmod 4 \text{ and } 1\le i\le n-2\\ 1 & \text{if } i\equiv 0,3 \pmod 4 \text{ and } 1\le i\le n-2 \end{cases}$$

$g^*(e_{3n-6}) = 1$

In view of the above defined labeling pattern, we have $e_{f^*}(0)+1 = e_{f^*}(1) = \frac{3n-5}{2}$ and $|e_{f^*}(0) - e_{f^*}(1)| \le 1$.

Therefore the Pl_n is a SD-prime cordial graph, for $n \equiv 1,3 \pmod 4$.

Case 2: $n \equiv 0 \pmod 4$.

$$g(v_i)=\begin{cases} i+2 & \text{if } i\equiv 0,1 \pmod 4 \text{ and } 1\le i\le n-4\\ i+3 & \text{if } i\equiv 2 \pmod 4 \text{ and } 1\le i\le n-4\\ i+1 & \text{if } i\equiv 3 \pmod 4 \text{ and } 1\le i\le n-4\\ n-1 & \text{if } i=n-3\\ n & \text{if } i=n-2\\ 2 & \text{if } i=n-1\\ 1 & \text{if } i=n \end{cases}$$

Then induced edge labels are

$g^*(e_{2i-1}) = 0$, for $1 \le i \le \frac{n-4}{2}$

$g^*(e_{2i}) = 1$, for $1 \le i \le \frac{n-6}{2}$

$g^*(e_{n-4}) = 0$,

$g^*(e_{n-3}) = 1$,

$$g^*(e_{n-3+i})=\begin{cases} 1 & \text{if } i\equiv 1,2 \pmod 4 \text{ and } 1\le i\le n-4\\ 0 & \text{if } i\equiv 0,3 \pmod 4 \text{ and } 1\le i\le n-4 \end{cases}$$

$g^*(e_{2n-6}) = 1$,

$g^*(e_{2n-5}) = 0$,

$$g^*(e_{2n-5+i}) = \begin{cases} 0 & \text{if } i \equiv 1,2 (\text{mod } 4) \text{ and } 1 \le i \le n-4 \\ 1 & \text{if } i \equiv 0,3 (\text{mod } 4) \text{ and } 1 \le i \le n-4 \end{cases}$$

$g^*(e_{3n-8}) = 0$,
$g^*(e_{3n-7}) = 1$,
$g^*(e_{3n-6}) = 1$

In view of the above defined labeling pattern, we have $e_{f^*}(0) = e_{f^*}(1) = \frac{3n-6}{2}$ and $|e_{f^*}(0) - e_{f^*}(1)| \le 1$.

Therefore the Pl_n is a SD-prime cordial graph, $n \equiv 0 \pmod 4$.

Case 3: $n \equiv 2 \pmod 4$.

$$g(v_i) = \begin{cases} i+2 & \text{if } i \equiv 0,1 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \\ i+3 & \text{if } i \equiv 2 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \\ i+1 & \text{if } i \equiv 3 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \\ 2 & \text{if } i = n-1 \\ 1 & \text{if } i = n \end{cases}$$

Then induced edge labels are

$g^*(e_{2i-1}) = 0$, for $1 \le i \le \frac{n-3}{2}$

$g^*(e_{2i}) = 1$, for $1 \le i \le \frac{n-3}{2}$

$$g^*(e_{n-3+i}) = \begin{cases} 1 & \text{if } i \equiv 1,2 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \\ 0 & \text{if } i \equiv 0,3 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \end{cases}$$

$$g^*(e_{2n-5+i}) = \begin{cases} 0 & \text{if } i \equiv 1,2 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \\ 1 & \text{if } i \equiv 0,3 (\text{mod } 4) \text{ and } 1 \le i \le n-2 \end{cases}$$

$g^*(e_{3n-6}) = 1$

In view of the above defined labeling pattern, we have $e_{f^*}(0) = e_{f^*}(1) = \frac{3n-6}{2}$ and $|e_{f^*}(0) - e_{f^*}(1)| \le 1$.

Therefore the Pl_n is a SD-prime cordial graph, for $n \equiv 2 \pmod 4$.

Therefore the Pl_n is a SD-prime cordial graph, $n \ge 3$.

Theorem 3.2: The disconnected graph $(P_n \odot K_1) \cup K_{1,m,m}$ is k-SD-prime cordial graph, for $n,m \ge 2$.

Proof: Let $P_n \odot K_1$ be a comb graph. Let $v_1, v_2, ..., v_{2n}$ be the vertices and $e_1, e_2, ..., e_{2n-1}$ be the edges of $P_n \odot K_1$. Let $u, u_1, u_2, ..., u_{2m}$ be the vertices and $s_1, s_2, ..., s_{2m}$ be the edges of $K_{1,m,m}$.

Let G be the disconnected graph $(P_n \odot K_1) \cup K_{1,m,m}$.

Then $|V(G)| = 2n+2m+1$ and $|E(G)| = 2n+2m-1$.

Define $g : V(G) \to \{k, k+1, \ldots, k+2n+2m\}$ as follows:

$$g(v_i) = \begin{cases} k+2i-2, & \text{if } 1 \le i \le n \\ k+2i-2n-1, & \text{if } n+1 \le i \le 2n \end{cases}$$

$g(u) = k+2n$

$$g(u_i)=\begin{cases} k+2n+2i, & \text{if } 1\le i\le m \\ k+2n-2m+2i+1, & \text{if } m+1\le i\le 2m \end{cases}$$

Then induced edge labels are

$$g^*(e_i)=\begin{cases} 0, & \text{if } 1\le i\le n-1 \\ 1, & \text{if } n\le i\le 2n-1 \end{cases}$$

$$g^*(s_i)=\begin{cases} 0, & \text{if } 1\le i\le m \\ 1, & \text{if } m+1\le i\le 2m \end{cases}$$

In view of the above defined labeling pattern, we have $e_{f*}(0)+1 = e_{f*}(1) = n+m$ and $| e_{f*}(0) - e_{f*}(1)| \le 1$.

Therefore the disconnected graph $(P_n \odot K_1)\cup K_{1,m,m}$ is k-SD-prime cordial graph, for $n,m \ge 2$.

Theorem 3.3. The disconnected graph $P_n\cup K_{1,m,m}$ is k-SD-prime cordial graph, for $n,m \ge 2$.

Proof: Let P_n be a path graph. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_{n-1}$ be the edges of P_n. Let $u,u_1,u_2,...,u_{2m}$ be the vertices and $s_1,s_2, ...,s_{2m}$ be the edges of $K_{1,m,m}$.

Let G be the disconnected graph $P_n \cup K_{1,m,m}$.

Then $|V(G)| = n+2m+1$ and $| E(G) | = n+2m-1$.

Define $g : V(G) \to \{k,k+1,\ldots,k+n+2m\}$ as follows:

Case 1: $n \equiv 1,3 \pmod 4$.

$$g(v_i)=\begin{cases} k+i-1 & \text{if } i\equiv 0,1 \pmod 4 \text{ and } 1\le i\le n \\ k+i & \text{if } i\equiv 2 \pmod 4 \text{ and } 1\le i\le n \\ k+i-2 & \text{if } i\equiv 3 \pmod 4 \text{ and } 1\le i\le n \end{cases}$$

$g(u) = k+n$

$$g(u_i)=\begin{cases} k+n+2i, & \text{if } 1\le i\le m \\ k+n-2m+2i-1, & \text{if } m+1\le i\le 2m \end{cases}$$

Then induced edge labels are

$g^*(e_{2i-1}) = 0, \quad \text{if } 1 \le i \le \frac{n-1}{2}$

$g^*(e_{2i}) = 1, \quad \text{if } 1 \le i \le \frac{n-1}{2}$

$$g^*(s_i)=\begin{cases} 0, & \text{if } 1\le i\le m \\ 1, & \text{if } m+1\le i\le 2m \end{cases}$$

In view of the above defined labeling pattern, we have

$$e_{f*}(0) = e_{f*}(1) = \frac{n+2m-1}{2} \text{ and } | e_{f*}(0) - e_{f*}(1)| \le 1.$$

Therefore $P_n\cup K_{1,m,m}$ is k-SD-prime cordial graph, for $n \equiv 0,1,3 \pmod 4$.

Case 2: $n \equiv 0 \pmod 4$.

$$g(v_i)=\begin{cases} k+i-1 & \text{if } i\equiv 0,1(\text{mod } 4) \text{ and } 1\le i\le n \\ k+i & \text{if } i\equiv 2(\text{mod } 4) \text{ and } 1\le i\le n \\ k+i-2 & \text{if } i\equiv 3(\text{mod } 4) \text{ and } 1\le i\le n \end{cases}$$

g(u) = k+n

$$g(u_i)=\begin{cases} k+n+2i, & \text{if } 1\le i\le m \\ k+n-2m+2i-1, & \text{if } m+1\le i\le 2m \end{cases}$$

Then induced edge labels are

$g^*(e_{2i-1}) = 0$, if $1 \le i \le \frac{n}{2}$

$g^*(e_{2i}) = 1$, if $1 \le i \le \frac{n-2}{2}$

$$g^*(s_i)=\begin{cases} 0, & \text{if } 1\le i\le m \\ 1, & \text{if } m+1\le i\le 2m \end{cases}$$

In view of the above defined labeling pattern, we have $e_{f^*}(0) = e_{f^*}(1)+1 = \frac{n+2m}{2}$ and $| e_{f^*}(0) - e_{f^*}(1)| \le 1$.

Therefore $P_n \cup K_{1,m,m}$ is k-SD-prime cordial graph, for n ≡ 0 (mod 4).

Case 3: n ≡ 2 (mod 4).

$$g(v_i)=\begin{cases} k+i-1 & \text{if } i\equiv 0,1(\text{mod } 4) \text{ and } 1\le i\le n-1 \\ k+i & \text{if } i\equiv 2(\text{mod } 4) \text{ and } 1\le i\le n-1 \\ k+i-2 & \text{if } i\equiv 3(\text{mod } 4) \text{ and } 1\le i\le n-1 \\ k+n-1 & \text{if } i=n \end{cases}$$

f(u) = k+n

$$g(u_i)=\begin{cases} k+n+2i, & \text{if } 1\le i\le m \\ k+n-2m+2i-1, & \text{if } m+1\le i\le 2m \end{cases}$$

Then induced edge labels are

$f^*(e_{2i-1}) = 0$ if $1 \le i \le \frac{n-2}{2}$

$f^*(e_{2i}) = 1$ if $1 \le i \le \frac{n-2}{2}$

$f^*(e_{2n-1}) = 1$

$$g^*(s_i)=\begin{cases} 0, & \text{if } 1\le i\le m \\ 1, & \text{if } m+1\le i\le 2m \end{cases}$$

In view of the above defined labeling pattern, we have $e_{f^*}(0)+1 = e_{f^*}(1) = \frac{n+2m}{2}$ and $| e_{f^*}(0) - e_{f^*}(1)| \le 1$.

Therefore $P_n \cup K_{1,n,n}$ is k-SD-prime cordial graph, for n ≡ 2 (mod 4).

Hence the disconnected graph $P_n \cup K_{1,n,n}$ is k-SD-prime cordial graph, for n,m ≥ 2.

4. Conclusions

In this paper, we presented the SD-Prime cordial labeling of Pl_n graph, for $n \geq 3$ and k-SD-Prime cordial labeling of $(P_n \odot K_1) \cup K_{1,n,n}$, for $n \geq 2$ and $P_n \cup K_{1,n,n}$, for $n \geq 2$.

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