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Analysis of Two-Echelon Inventory System with Two Demand Classes with Partial Backlogging

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Abstract. This paper deals with a continuous review two-echelon inventory system with two demand classes. Two echelon inventory systems consist of one retailer (lower echelon) and one distributor (upper echelon) handling a single finished product. The demand at retailer is of two types. The First type of demand is usual single unit and the second type is of bulk or packet demand. The arrival distribution for single and packet demands are assumed to be independent Poisson with rates $\lambda_1(>0)$ and λ_d (>0) respectively. The operating policy at the lower echelon for the (s, S) that is whenever the inventory level drops to 's' on order for Q = (S-s) items is placed, the ordered items are received after a random time which is distributed as exponential with rate $\mu>0$. We assume that the demands occurring during the stock-out period are lost. The retailer replenishes the stock from the distributor which adopts (0, M) policy. The objective is to minimize the anticipated total cost rate by simultaneously optimizing the inventory level. The joint probability disruption of the inventory levels at retailer and the distributor are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

Keywords: Markov inventory system, two-echelon, two demand classes, cost optimization. partial backlogging

AMS Mathematics Subject Classification (2010): 90B05

1. Introduction

Inventory management is one of the most important business processes during the operation of a manufacturing company as it relates to purchases, sales and logistic activities. It concerned with the control of stocks throughout the whole supply chain. Inventory control is concerned with maintaining the correct level of stock and recording its movement. It deals mainly with historic data. Inventory is generally considered to comprise in three main areas which are raw materials, work in progress and finished goods, where these are held and in what quantities, and how they are managed will vary significantly from one organization to another. The activities of inventory management involves are identifying inventory requirements, setting targets, providing replenishment techniques and options, monitoring item usages, reconciling the inventory balances, and reporting inventory status. In order to have clear inventory management, a company should not only focus on logistic management but also on sales and purchase management. Inventory management and control is not only the responsibility of the accounting department and the warehouse, but also the responsibility of the entire organization. Actually, there are many departments involved in the inventory management and control process, such as sales, purchasing, production, logistics and accounting. All these departments must work together in order to achieve effective inventory controls.

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [22] and Veinott [23]. Sivazlian [20] discussed the stationary characteristics of a multi commodity single period inventory system. The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain) when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris [6]. Clark and Scarf [4] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968. Hadley, G and Whitin [5], Naddor [12] analyses various inventory Systems. HP's (Hawlett Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [21] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [7] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthy et al., [8] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [9]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

In the literature of stochastic inventory models, there are two different assumptions about the excess demand unfilled from existing inventories: the backlog assumption and the lost sales assumption. The former is more popular in the literature partly because historically the inventory studies started with spare parts inventory management problems in military applications, where the backlog assumption is realistic. However in many other business situations, it is quite often that demand that cannot be satisfied on time is lost. This is particularly true in a competitive business environment. For example in many retail establishments, such as a supermarket or a department store, a customer chooses a competitive brand or goes to another store if his/her preferred brand is out of stock.

All these papers deal with repairable items with batch ordering. A Complete review was provided by Beamon[2]. Axsater[1] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

A developments in two-echelon models for perishable inventory may be found in Nahimas [13, 14, 15, 16]. Yadavalli et al. [24. 25] considers two commodity inventory system under continuous review with lost sales. Again continuous review Perishable inventory with instantaneous replenishment in two echelon system was considered by Krishnan [10] and Rameshpandy et al. [17, 18, 19].

The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model description

In this model, a three level supply chain consisting of a single product, one manufacturing facility, one Distribution centre (DC) and one retailer. We assume that demands to the Retailer is of two types which follows independent Poisson process with parameter $\lambda_1(>0)$ and $\lambda_q(>0)$. The replenishment of Q items from DC to retailer follows exponentially distributed with parameter $\mu(>0)$. The retailer follows (s, S) policy and the distributor follow (0, nQ) policy for maintaining their inventories. The demand occurring during stock-out period are assumed to be lost. Even though we have adopted two different policies in the Supply Chain, the distributors policy is depends upon the retailers policy. The model minimizes the total cost incurred at all the locations. The system performance measures and the total cost are computed in the steady state.

3. Analysis

Let $I_1(t)$ and $I_2(t)$ respectively denote the on hand inventory level in the retailer node and the number of items in the Distribution centre at time t. From the assumptions on the input and output processes, clearly,

$$I(t) = \{ (I_1(t), I_2(t) \ : t \geq 0 \}$$

is a Markov process with state space

$$E = \{(i, j) \ / \ i = S, S-1, S-2, \ldots s, s-1, \ldots Q, Q-1, \ 1, 0, -1, -2, \ldots -b \ ; \ j = Q, 2Q, \ldots nQ\}.$$

The infinitesimal generator of this process

$$A = (a(i, j : l, m)), \ _{(i, j), (l, m)) \in \ E}$$

can be obtained from the following arguments.

- The primary arrival of unit demand to the retailer node makes a transition in the Markov process from (i, j) to (i-1 , j, k) with intensity of transition $\lambda_{1.}$
- The arrival of packet demand to the retailer node makes a transition in the Markov process from (i, j) to (i-q , j) with intensity of transition ë$_{q.}$
- The replenishment of an inventory at retailer node makes a transition in the Markov process from (i, j) to (i+Q, k − Q) with rate of transition µ .

Then, the infinitesimal generator has the following structure:

$$R=\begin{bmatrix} A & B & 0 & \cdots & 0 & 0 \\ 0 & A & B & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & A & B \\ B & 0 & 0 & \cdots & 0 & A \\ & & & & & \end{bmatrix}$$

The entries of R are given by

$$[R]_{pxq}=\begin{cases} A & p=q; \quad q=nQ, \quad (n-1)Q,...Q \\ B & p=q+Q; \quad q=(n-1)Q,....Q \\ B & p=q-(n-1)Q \quad q=nQ \\ 0 & otherwise \end{cases}$$

The elements in the sub matrices of A and B are

$$[A]=\begin{cases} \lambda_1 \ \ if & i+1=j \quad i=S,S-1.......1 \\ \lambda_q \ \ if & i+Q=j \quad i=Q,Q+1......1 \\ -(\lambda_1+\lambda_q) \ if & i=j \quad i=S,S-1.....Q \\ -\lambda_1 \ \ if & i=j \quad i=(s+1),....(Q-1) \\ -(\lambda_1+\lambda_q+\mu) \ if & i=j \quad i=1,2,........s \\ -\mu \ \ if & i=j \quad i=0 \\ 0 & otherwise \end{cases}$$

$$[B]=\begin{cases} \mu \ \ if & i+\mu=j \quad i=S,S-1.....1,0 \\ 0 & otherwise \end{cases}$$

3.1. Transient analysis

Let $I_1(t)$ and $I_2(t)$ respectively denote the on hand inventory level in the retailer node and the Distribution centre at time t. From the assumptions on the input and output processes, clearly,

$$I(t) = \{ (I_1(t), I_2(t) \ : t \geq 0 \}$$

is a Markov process with state space

E = {(i, j) / i = S,S-1,S-2,…s,s-1,…Q. Q-1…1,0,-1,-2,….-b; j= Q,2Q,….nQ}.

Theorem 3.1.1. The vector process I(t) = { (I_1(t), I_2(t) : $t \geq 0$ is a continuous time Markov Chain with state space E = {(i, j,) / i = S,S-1,S-2,…s,s-1,…1,0,-1,-2…-b: j= Q,2Q,….nQ}.

Proof: The stochastic process I(t) = { (I_1(t), I_2(t) : $t \geq 0$ }has a discrete state space with order relation ' $\leq$ ' that (i, j) $\leq$ (l, m) if and only if $i \leq l$, $j \leq m$.

To prove that {I(t) : $t \geq 0$} is a Markov chain, first we do a transformation for state space E to E′ such that (i, j) $\rightarrow$ i + j $\in$ E′, where

$$E' = \{Q ,Q + 1, ...,Q + S, ..., nQ + 1,..... nQ + S \}.$$

Now we may realize that {I(t) : $t \geq 0$} is a stochastic process with discrete state space E′.

The joint distribution of random variables

$$\{I(t_1), I(t_2), ..., I(t_n)\} \text{ and } \{I(t_1+ \tau), I(t_2 + \tau), ..., I(t_n + \tau)\}$$

with$\tau > 0$ (an arbitrary real number) are equal.

In particular the conditional probability

$$\Pr\{I_n = k |I_{n-1} = j, I_{n-2} = i, ...I_0 = 1\} = \Pr \{I_n = k | I_{n-1} = j\}$$

due to the single step transition of states in E.

Hence {I(t) : $t \geq 0$} is a continuous time Markov Chain.

Define the transition probability function

$$P_{i,j,k}(l, m : t) = \Pr \{(I_1(t), I_2(t)) = (l, m) | (I_1(0), I_2(0)) = (i, j)\}$$

The corresponding transition matrix function is given by

$$P(t) = (P_{i,j}(l, m: t))_{(i,j)\ (l,m) \in E}$$

which satisfies the Kolmogorov- forward equation

$$P'(T) = P(T)R$$

where A is the infinitesimal generator.

From the above equation, together with initial condition P(0) = I, the solution can be expressed in the form

$$P(t) = P(0)e^{Rt} = e^{Rt}$$

where the matrix expansion in power series form is $e^{Rt} = I + \sum_{n=1}^{\infty} \frac{A^n t^n}{n!}$

Case (i) : suppose that the Eigen values of R are all distinct. Then from the spectral theorem of matrices, we have

$$R = HDH^{-1}$$

where H is the non-singular (formed with the right Eigen vectors of R and D is the diagonal matrix having its diagonal elements the eigen values of R. Now 0 is an Eigen value of R and if $d_i \neq 0$, i = 1, 2, ...,m are the distinct eigen values then

$$D = \begin{pmatrix} 0 & 0 & .. & .. & 0 \\ 0 & d_1 & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & d_{m-1} & 0 \\ 0 & .. & .. & .. & d_m \end{pmatrix}$$

Then we have $D^n = \begin{pmatrix} 0 & 0 & .. & .. & 0 \\ 0 & (d_1)^n & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & (d_{m-1})^n & 0 \\ 0 & .. & .. & .. & (d_m)^n \end{pmatrix}$

and

$$R^n = HD^nH^{-1}$$

Using R^n in P(t) we have the explicit solution of P(t) as $P(t) = He^{Dt}H^{-1}$
where

$$e^{Dt} = \begin{pmatrix} 1 & 0 & .. & .. & 0 \\ 0 & e^{d_1 t} & .. & .. & ... \\ ... & ... & .. & .. & ... \\ 0 & 0 & .. & e^{d_{m-1}t} & 0 \\ 0 & .. & .. & .. & e^{d_m t} \end{pmatrix}$$

Case (ii): Suppose the Eigen values of R are all not distinct, we can find a canonical representation as $R = SZS^{-1}$. From this the transition matrix P(t) can be obtained in a modified form (Medhi [11]).

3.2. Steady state analysis

Since the state space is finite and irreducible, hence the stationary probability vector Π for the generator A always exists and satisfies $\Pi R = 0$ $\Pi e = 1$.
The vector Π can be represented by

$$\Pi = (\Pi_i^{<Q>}, \Pi_i^{<2Q}, \Pi_i^{<3Q>}, ..., \Pi_i^{<nQ>}) \text{ Where , } 0 \le i \le S$$

Now the structure of R shows, the model under study is a finite birth death model in the Markovian environment. Hence we use the Gaver algorithm for computing the limiting probability vector. For the sake of completeness we provide the algorithm here.

3.3. Performance measures

In this section the following system performance measures in steady state for the proposed inventory system is computed.

(a) Mean inventory level

Let I_R denote the expected inventory level in the steady state at retailer node and I_D denote the expected inventory level at distribution centre.

$$I_R = \sum_{j=Q}^{nQ} \sum_{i=0}^{S} i \Pi^{<<i,j>>}$$

$$I_D = \sum_{i=0}^{S} \sum_{j=Q}^{nQ} j \Pi^{<<i,j>>}$$

(c) Mean reorder rate

The mean reorder rate at retailer node and distribution center is given by

$$r_R = \lambda_1 \sum_{j=Q}^{nQ} \Pi^{<<s+1,j>>} + \lambda_q \sum_{j=Q}^{nQ} \Pi^{<<s+q,j>>}$$

$$r_D = \mu \sum_{i=0}^{s} \Pi^{<<i,Q>>}$$

(d) Shortage rate

Shortage occurs only at retailer node and the shortage rate for the retailer is denoted by α_R and which is given by

$$\alpha_R = \lambda_1 \sum_{j=Q}^{nQ} \Pi^{<<-b,j>>} + \lambda_q \sum_{j=Q}^{nQ} \sum_{i=0}^{q-1} \Pi^{<<i,j>>}$$

4. Cost analysis

In this section, the cost structure for the proposed models by considering the minimization of the steady state total expected cost per time is analyzed

The long run expected cost rate for the model is defined to be

$$TC(s,Q) = h_R\ I_R + h_D\ I_D + k_R\ r_R + k_D\ r_D + g_R\ \alpha_R$$

where, h_R- denote the inventory holding cost/ unit / unit time at retailer node

h_D- denote the inventory holding cost/ unit / unit time at distribution centre

k_R- denote the setup cost/ order at retailer node

k_{D-} denote the setup cost/ order at distribution node

g_R- denote the shortage cost/ unit shortage at retailer node

Although the convexity of the cost function TC(s,Q), is not proved, our any experience with considerable number of numerical examples indicates that TC(s,Q) for fixed Q(> s+1) appears to be locally convex in s. For large number of parameter, the calculation of TC(s,Q) revealed a convex structure.

Hence, a numerical search procedure is adopted to obtain the optimal value s for each S. Consequently, the optimal Q(= S-s) and M (= nQ) are obtained. A numerical example with sensitivity analysis of the optimal values by varying the different cost parameters is presented below

5. Numerical illustration

In the section, the problem of minimizing the long run total expected cost per unit time under the following cost structure is considered for discussion. The optimum values of the system parameters s is obtained and the sensitive analysis is also done for the system.

The results we obtained in the steady state case may be illustrated through the following numerical example

S = 20, M = 80, $\lambda_1 = 3$, $\lambda_q = 2$, $\mu = 3$ $h_R = 1.1$, $h_D = 1.2$, $k_R = 1.5, k_d = 1.3$

$g_R = 2.1$, $g_D = 2.2$, $b = 3$

The cost for different reorder level are given by

S	2	3	4	5*	6	7	8
Q	18	17	16	15	14	13	12
TC(s,Q)	82.6753	75.5903	64.679	62.7863*	67.3499	78.8514	88.3332

Table:1. Total expected cost rate as a function s and Q

For the inventory capacity S, the optimal reorder level s* and optimal cost *TC(s, Q)* are indicated by the symbol *. The Convexity of the cost function is given in the graph.

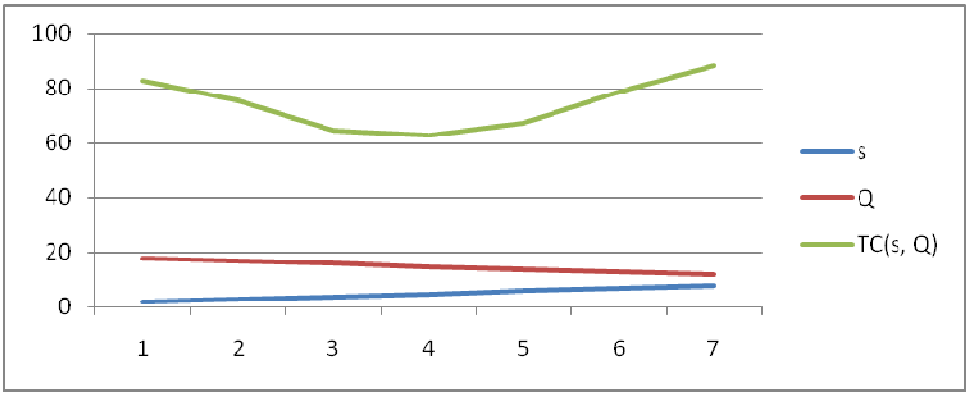

Figure 1: Total expected cost rate as a function TC(s. Q), s and Q

5.1. Sensitivity analysis

The effect of changes in Demand rate at retailer node and distributor node.

	λ_q= 8	λ_q= 10	λ_q= 12	λ_q= 14	λ_q= 16
λ_1= 12	45.073539	46.419603	47.214733	47.629158	47.785721
λ_1= 14	46.271834	47.617898	48.413028	48.827452	48.984016
λ_1= 16	47.136498	48.482562	49.277692	49.692117	49.848680
λ_1= 18	47.793130	49.139194	49.934324	50.348749	50.505312
λ_1= 20	48.325884	49.671948	50.467078	50.881502	51.038066

Table 2: Total expected cost rate as a function when demand increases

The graph of the demand rate variation is given below and it describes, if the demand rate increases then the total cost also increases.

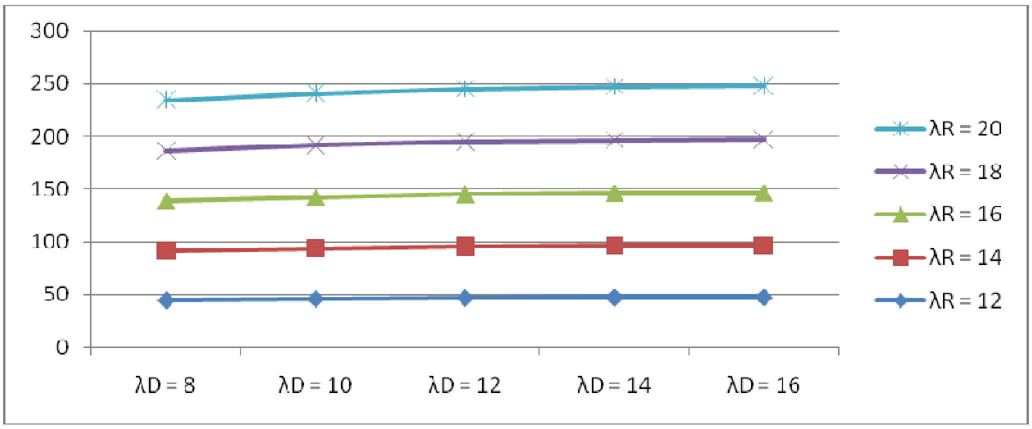

Figure 2: TC(s, Q) for different demand rates

	s=2	s=4	s=6	s=8	s=10
S=45	111.998164	118.761642	124.799813	130.664478	136.843108
S=50	118.699318	124.359054	129.299781	133.996082	138.773110
S=55	123.090318	127.562168	131.434202	135.090008	138.765317
S=60	125.576395	129.128145	132.243072	135.192545	138.137225
S=65	126.984419	130.065937	132.851239	135.511516	138.145444

Table 3: Total expected cost rate as a function when s and S increases

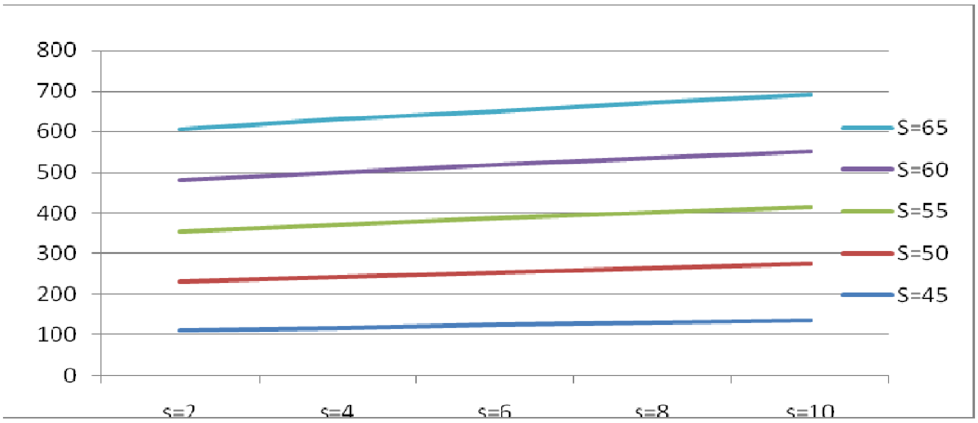

Figure 3: TC(s, Q) for different s and S values

From the graph it is identified that the total cost increases when the s and S increases.

	h_D=0.04	h_D=0.08	h_D=0.12	h_D=0.16	h_D=0.20
h_R=0.002	30.8517816	31.3422	31.8325	32.3229	32.8133
h_R=0.004	30.8598022	31.3502	31.8405	32.3309	32.8213
h_R=0.006	30.8678228	31.3582	31.8486	32.3389	32.8293
h_R=0.008	30.8758424	31.3662	31.8566	32.3469	32.8373
h_R=0.010	30.883863	31.3742	31.8646	32.355	32.8453

Table 4: Total expected cost rate when h_R and h_D increases

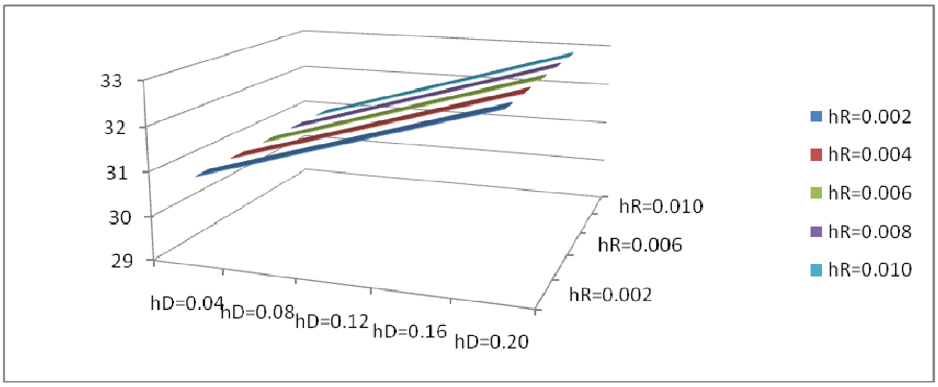

Figure 4: TC(s, Q) for different h_R and h_D values

As is to be expected the graph shows that the total cost increases when $h_R$and$h_D$increases

	g_R=0.2	g_R=0.4	g_R=0.6	g_R=0.8	g_R=1.0
k_D=10	23.104477	23.619900	24.135323	24.650745	25.166168
k_D=15	24.431902	24.947325	25.462748	25.978170	26.493593
k_D=20	25.759327	26.274750	26.790173	27.305595	27.821018
k_D=25	27.086752	27.602175	28.117598	28.633021	29.148443
k_D=30	28.414177	28.929600	29.445023	29.960446	30.475868

Table 5: Total expected cost rate when g_R and k_D increases

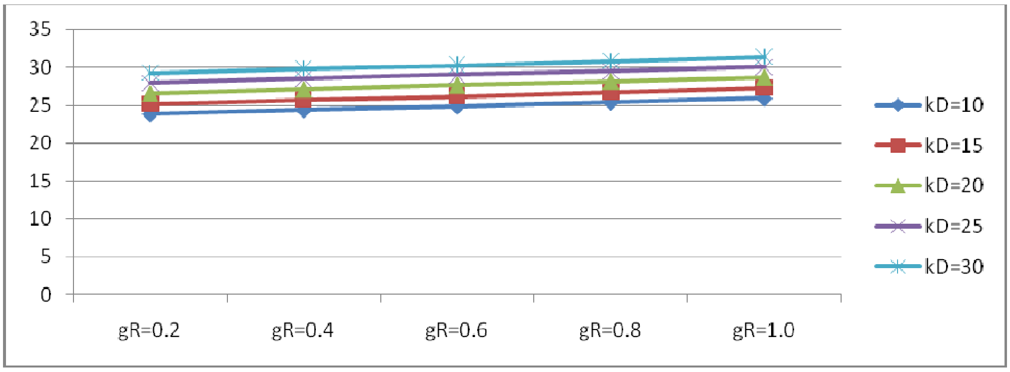

Figure 5: TC(s, Q) for different g_R and k_D values

6. Conclusion

In this model a three level supply chain consisting of a single product, one manufacturing facility, one Distribution centre (DC) and one retailer.. The model is analyzed within the framework of Markov processes.. Various system performance measures are derived and the long-run expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values on the total expected cost rate. It would be interesting to analyze the problem discussed in this paper by relaxing the assumption of exponentially distributed lead-times to a class of arbitrarily distributed lead-times. Once this is done, the general model can be used to generate various special eases.

REFERENCES

1. S.Axsater, Exact and approximate evaluation of batch ordering policies for two level inventory systems, *Oper. Res.*, 41 (1993) 777-785.
2. B.M.Beamon, Supply chain design and analysis: models and methods, *International Journal of Production Economics*, 55(3) (1998) 281-294.
3. E.Cinlar, *Introduction to Stochastic 'Processes*, Prentice Hall, Engle-wood Cliffs, NJ, 1975.
4. A.J.Clark and H.Scarf, Optimal policies for a multi- echelon inventory problem, *Management Science*, 6(4) (1960) 475-490.
5. G.Hadley and T.M.Whitin, *Analysis of inventory systems*, Prentice-Hall, Englewood Cliff. (1963).
6. F.Harris, *Operations and costs, factory management series*, A.W.Shah Co., Chicago, 48-52, (1915).

7. S.Kalpakam and G.Arivarigan, A coordinated multicommodity (*s*, *S*) inventory system, *Mathematical and Computer Modelling*, 18 (1993) 69-73.
8. A.Krishnamoorthy, R.I.Basha and B.Lakshmy, Analysis of a two commodity problem, *International Journal of Information and Management Sciences*, 5(1) (1994) 127-136.
9. A.Krishnamoorthy and T.V.Varghese, A two commodity inventory problem, *International Journal of Information and Management Sciences*, 5(3) (1994) 55-70.
10. K.Krishnan, *Stochastic Modeling in Supply Chain Management System*, unpublished Ph.D., Thesis, Madurai Kamaraj University, Madurai, (2007).
11. J.Medhi, *Stochastic processes*, Third edition, New Age International Publishers, New Delhi, (2009).
12. E.Naddor, *Inventory System*, John Wiley and Sons, New York, (1966).
13. S.Nahmias, Perishable inventory theory; a review, *Opsearch*, 30 (1982) 680-708.
14. S.Nahmias and S.S.Wang, A heuristic lot size reorder point model for decaying inventories, *Management Science*, 25 (1979) 90-97.
15. S.Nahmias, On the equivalence of three approximate continuous review models, *Naval Research Logistics Quarterly*, 23 (1974) 31 - 36.
16. S.Nahmias, Optimal ordering policies for perishable inventory-II, *Operations Research*, 23 (1975) 735 - 749.
17. M.Rameshpandi, C.Periyasamy and K.Krishnan, Analysis of two-echelon perishable inventory system with direct and retrial demands, *IOSR Journal of Mathematics*, 10 (5) (2014) 51-57.
18. M.Rameshpandi, C.Periyasamy and K.Krishnan, Two Echelon inventory system with finite population and retrial demands, Proceedings of the National conference on Mathematical Modelling, (2014) 128-135.
19. M.Rameshpandi and K.Krishnan, Stochastic analysis of perishable inventory system, Proceedings of the National seminar on Recent Trends in Applied Mathematics (RTAM-2015) (2015) 32-39.
20. B.D.Sivazlian, Stationary analysis of a multi-commodity inventory system with interacting set-up costs, *SIAM Journal of Applied Mathematics*, 20(2) (1975) 264-278.
21. B.D.Sivazlian and L.E.Stanfel, *Analysis of systems in Operations Research*, First edition, Prentice Hall, (1974).
22. A.F.Veinott and H.M.Wagner, Computing optimal (*s*,*S*) inventory policies, *Management Science*, 11 (1965) 525-552.
23. A.F.Veinott, The status of mathematical inventory theory, *Management Science*, 12 (1966) 745-777.
24. V.S.S.Yadavalli, N.Anbazhagan and G.Arivarignan, A two commodity continuous review inventory system with lost sales, *Stochastic Analysis and Applications*, 22 (2004) 479-497.
25. V.S.S.Yadavalli, W.C.De and S.Udayabaskaran, A substitutable two-product inventory system with joint ordering policy and common demand, *Applied Mathematics and Computation*, 172(2) (2006) 1257-1271.