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Fuzzy α_q Open Sets and Fuzzy β_q Open Sets in Fuzzy Quad Topological Space

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Abstract. The main aim of this paper is to introduce fuzzy α_q open sets and fuzzy β_q open sets in fuzzy q-topological spaces along with their several properties and characterization. We introduce fuzzy α_q continuous function and fuzzy quad β_q continuous function and obtain some of their basic properties.

Keywords: fuzzy α_q open sets, fuzzy α_q continuous function, fuzzy β_q open sets, fuzzy β_q continuous function.

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1. Introduction

In 1965, Njastad [6] introduced pre semi open sets (α open sets). In 1990, Jelic [3] introduced the concept of α open sets in bitopological spaces. In 1986, Andrijevic [1] was introduced semi-pre-open sets (β open sets) in topological spaces. Khedr et al. [4] generalize the notion of semi pre-open sets to bitopological spaces and semi pre continuity in bitopological space. Tri-topological space was first initiated by Kovar Martin [5]. Palaniammal [7] studied tri topological space and introduced α open sets and β open sets in tri topological space. Hameed and Abid Moh. Yahya [2] defined open set in tri topological space. We [8,9,10] introduced semi-open sets and pre-open sets and studied α_T open set and β_T open set in tri topological space. We [4,10] introduced fuzzy α_T open set and fuzzy β_T open set in tri topological space and fuzzy semi-open set and fuzzy pre-open set in quad topological space.

The purpose of the present paper is to introduce fuzzy α_q open sets, fuzzy β_q open sets, fuzzy α_q continuity, fuzzy β_q continuity and their fundamental properties in quad topological space.

2. Preliminaries

Definition 2.1. [7] Let X be a nonempty set and τ_1, τ_2, τ_3 and τ_4 are general topologies on X . Then a subset A of space X is said to be quad-open (q-open) set if $A \subset \tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4$ and its complement is said to be q-closed and set X with four topologies called q-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 2.2. [7] Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a quad topological space and let $A \subset X$. The intersection of all q-closed sets containing A is called the q-closure of A and denoted by $q-clA$.We will denote the q-interior (respectively q-closure) of any subset, say of A by $q-\text{int} A$ ($q-clA$), where $q-clA$ is the union of all q-open sets contained in A, and $q-clA$ is the intersection of all q-closed sets containing A.

Definition 2.3. [7] Let X be a non-empty set τ_1 , τ_2 τ_3 and τ_4 are fuzzy topologies on *X*. Then a fuzzy subset χ_λ of space *X* is said to be fuzzy q-open if $A \prec \tau_1 \vee \tau_2 \vee \tau_3 \vee \tau_4$ and its complement is said to be fuzzy q-closed and set *X* with four fuzzy topologies called fuzzy q-topological spaces $(X, \tau_1, \tau_2, \tau_3, \tau_4)$.

Definition 2.4. [8] Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space then a fuzzy subset χ_λ of *X* is said to be fuzzy q-semi-open set if $\chi_\lambda \le qcl(q\,\text{int}\,\chi_\lambda)$ complement of fuzzy q-semi-open set is called fuzzy q-semi-closed set. The collection of all fuzzy q-semi-open sets of *X* are denoted by *FqSO(X).*

Definition 2.5. [8] Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy q-topological space then a fuzzy subset χ_λ of *X* is said to be fuzzy q-pre-open set if $\chi_\lambda \le q\,\text{int}(qcl\,\chi_\lambda)$. Complement of fuzzy q-pre-open set is called fuzzy q-pre-closed set. The collection of all fuzzy q-pre-open sets of *X* is denoted by F*qPO*(*X*).

3. Fuzzy α_q open sets and fuzzy β_q open sets in fuzzy quad topological space

Definition 3.1. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space then a fuzzy subset χ_λ of X is said to be fuzzy α_q open set if

$$\chi_\lambda \le Fqp\sin t(Fqpscl(Fqp\sin t(\chi_\lambda)))$$

and complement of fuzzy quad α_q open set is fuzzy α_q closed. The collection of all fuzzy α_q open sets of X is denoted by $F_q PSO(X)$.

Example 3.2. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.
Consider four fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,d\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{c,d\}}\}$,
$\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,c,d\}}\}$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Fuzzy q-open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Fuzzy α_q open sets of X are denoted by

$FqPSO(X). = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Definition 3.3. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space. Let $\chi_\lambda \prec \tilde{1}_X$. An element $\chi_{\{x\}} \le \chi_\lambda$ is called fuzzy α_q interior point of χ_λ, if there exist a fuzzy quad α_q open set χ_δ such that $\chi_{\{x\}} \le \chi_\delta \prec \chi_\lambda$. The set of all fuzzy quad α_q interior points of χ_λ is called the fuzzy α_q interior of χ_λ and is denoted by $Fqps\operatorname{int}(\chi_\lambda)$.

Theorem 3.4. Let $\chi_\lambda \prec \tilde{1}_X$ be a fuzzy quad topological space. $Fqps\operatorname{int}(\chi_\lambda)$ is equal to the union of all fuzzy α_q open sets contained in χ_λ.

Note 3.5: 1. $Fqps\operatorname{int}(\chi_\lambda) \prec \chi_\lambda$.
2. $Fqps\operatorname{int}(\chi_\lambda)$ is fuzzy α_q open sets.

Theorem 3.6. $Fqps\operatorname{int}(\chi_\lambda)$ is the largest fuzzy quad α_q open sets contained in χ_λ.

Theorem 3.7. χ_λ is fuzzy α_q open if and only if $\chi_\lambda = Fqps\operatorname{int}(\chi_\lambda)$

Theorem 3.8. $Fqps\operatorname{int}(\chi_\lambda \vee \chi_\delta) \succ Fqps\operatorname{int}(\chi_\lambda) \vee Fqps\operatorname{int}(\chi_\delta)$.

Definition 3.9. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space. Let $\chi_\lambda \prec \tilde{1}_X$. The intersection of all fuzzy α_q closed sets containing χ_λ is called a fuzzy quad α_q closure of χ_λ and is denoted as $Fqpscl(\chi_\lambda)$.

Note 3.10. Intersection of fuzzy α_q closed sets is fuzzy quad α_q closed set, $Fqpscl(\chi_\lambda)$ is a fuzzy quad α_q closed set.

Theorem 3.11. χ_λ is fuzzy α_q closed set if and only if $\chi_\lambda = Fqpscl(\chi_\lambda)$.

Theorem 3.12. Let χ_λ and χ_δ be fuzzy subsets of $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $\chi_{\{x\}} \le \tilde{1}_X$

a) χ_λ is fuzzy quad α_q closed if and only if $\chi_\lambda = Fqpscl(\chi_\lambda)$.

b) If $\chi_\lambda \prec \chi_\delta$, then $Fqps\operatorname{int}(\chi_\lambda) \prec Fqps\operatorname{int}(\chi_\delta)$.

c) $\chi_{\{x\}} \le Fqpscl(\chi_\lambda)$ If and only if $\chi_\lambda \wedge \chi_\delta \ne \tilde{0}_X$ for every fuzzy α_q open set χ_δ containing $\chi_{\{x\}}$.

Theorem 3.13. Let χ_λ be a fuzzy subsets of $(X,\tau_1,\tau_2,\tau_3,\tau_4)$, if there exist a fuzzy quad α_q open set χ_δ such that $\chi_\lambda \prec \chi_\delta \prec Fqpscl(\chi_\lambda)$, then χ_λ is fuzzy α_q open.

Theorem 3.14. In a fuzzy quad topological space $(X,\tau_1,\tau_2,\tau_3,\tau_4)$, the union of any two fuzzy α_q open sets is always a fuzzy α_q open set.

Proof: Let χ_λ and χ_δ be any two fuzzy α_q open sets in X .

Now $\chi_\lambda \vee \chi_\delta \le Fqpscl\left(Fqps\operatorname{int}(\chi_\lambda)\right) \vee Fqpscl\left(Fqps\operatorname{int}(\chi_\delta)\right)$

$\Rightarrow \chi_\lambda \vee \chi_\delta \le Fqpscl\left(Fqps\operatorname{int}\left(\chi_\lambda \vee \chi_\delta\right)\right)$. Hence $\chi_\lambda \vee \chi_\delta$ fuzzy α_q open sets.

Remark 3.15. The intersection of any two fuzzy α_q open sets may not be a fuzzy α_q open sets as show in the following example.

Example 3.16. Let $X = \{a,b,c,d\}$ be a non-empty fuzzy set.
Consider four fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,d\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{c,d\}}\}$,

$\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,c,d\}}\}$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies. Then Fuzzy q-open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Fuzzy α_q open set of *X* is denoted by

$FqPSO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Here $\chi_{\{a,d\}} \wedge \chi_{\{c,d\}} = \chi_{\{d\}} \notin FqPSO(X)$

Theorem 3.17. Let χ_λ and χ_δ be fuzzy subsets of X such that $\chi_\delta \le \chi_\lambda \le Fqpscl(\chi_\delta)$ .if χ_δ is fuzzy α_q open set then χ_λ is also fuzzy α_q open set.

Proof: Given χ_δ is fuzzy α_T open set. So, we have
$\chi_\delta \le Fqpscl\left(Fqpsint\chi_\delta\right) \le Fqpscl\left(Fqpsint\left(\chi_\lambda\right)\right)$. Thus
$Fqpscl(\chi_\delta) \le Fqpscl\left(Fqpsint\left(\chi_\lambda\right)\right)$. Hence χ_λ is also a fuzzy α_T open set.

Definition 3.18. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space then a fuzzy subset χ_λ of X is said to be fuzzy β_q open set if $\chi_\lambda \le Fqspint(Fqspcl(Fqspint(\chi_\lambda)))$ and complement of fuzzy β_q open set is fuzzy quad β_q closed. The collection of all fuzzy β_q open sets of X is denoted by $FqSPO(X)$.

Example 3.19. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.
Consider four fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,d\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{c,d\}}\}$,
$\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,c,d\}}\}$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.
Fuzzy q-open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$
Fuzzy β_q open sets of *X* are denoted by

$FqSPO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Definition 3.20. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ be a fuzzy quad topological space. Let $\chi_\lambda \prec \tilde{1}_X$. An element $\chi_{\{x\}} \le \chi_\lambda$ is called fuzzy β_q interior point of χ_λ, if there exist a fuzzy β_q open set χ_δ such that $\chi_{\{x\}} \le \chi_\delta \prec \chi_\lambda$. The set of all fuzzy quad β_q interior points of χ_λ is called the fuzzy β_q interior of χ_λ and is denoted by $Fqp\sin t(\chi_\lambda)$.

Theorem 3.21. Let $\chi_\lambda \prec \tilde{1}_X$ be a fuzzy quad topological space. $q - sp\,\mathrm{int}(\chi_\lambda)$ is equal to the union of all fuzzy quad β_q open sets contained in χ_λ.

Note 3.22. 1. $Fq\,\mathrm{spin}\,t(\chi_\lambda) \prec \chi_\lambda$.

2. $Fqp\sin t(\chi_\lambda)$ is fuzzy β_T open sets.

Theorem 3.23. $Fq\,\mathrm{spin}\,t(\chi_\lambda)$ is the largest fuzzy β_q open sets contained in χ_λ.

Theorem 3.24. χ_λ is fuzzy quad β_q open if and only if $\chi_\lambda = Fq\,\mathrm{spin}\,t(\chi_\lambda)$

Theorem 3.25. $Fq\,\mathrm{spin}\,t(\chi_\lambda \vee \chi_\delta) \succ Fq\,\mathrm{spin}\,t(\chi_\lambda) \vee Fq\,\mathrm{spin}\,t(\chi_\delta)$.

Definition 3.26. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ be a fuzzy quad topological space. Let $\chi_\lambda \prec \tilde{1}_X$. The intersection of all fuzzy quad β_q closed sets containing χ_λ is called a fuzzy quad β_q closure of χ_λ and is denoted as $Fq\,\mathrm{sp}\,cl(\chi_\lambda)$.

Note 3.27. Intersection of fuzzy quad β_q closed sets is fuzzy quad β_q closed set, $Fq\,\mathrm{sp}\,cl(\chi_\lambda)$ is a fuzzy β_q closed set.

Theorem 3.28. χ_λ is fuzzy quad β_q closed set if and only if $\chi_\lambda = Fq\,\mathrm{sp}\,cl(\chi_\lambda)$.

Theorem 3.29. Let χ_λ and χ_δ be fuzzy subsets of $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $\chi_{\{x\}} \le \tilde{1}_X$

a) χ_λ is fuzzy β_q closed if and only if $\chi_\lambda = Fq\,\mathrm{sp}\,cl(\chi_\lambda)$.

b) If $\chi_\lambda \prec \chi_\delta$, then $Fq\,\mathrm{sp}\,cl(\chi_\lambda) \prec Fq\,\mathrm{sp}\,cl(\chi_\delta)$.

c) $\chi_{\{x\}} \le Fq\,\mathrm{sp}\,cl(\chi_\lambda)$ If and only if $\chi_\lambda \wedge \chi_\delta \ne \tilde{0}_X$ for every fuzzy quad β_q open set χ_δ containing $\chi_{\{x\}}$.

Theorem 3.30. Let χ_λ be a fuzzy subsets of $(X,\tau_1,\tau_2,\tau_3,\tau_4)$, if there exist a fuzzy quad β_q open set χ_δ such that $\chi_\lambda \prec \chi_\delta \prec Fq\,\mathrm{sp}\,cl(\chi_\lambda)$, then χ_λ is fuzzy β_q open.

Theorem 3.31. In a fuzzy quad topological space $(X,\tau_1,\tau_2,\tau_3,\tau_4)$, the union of any two fuzzy β_q open sets is always a fuzzy β_q open set.

Remark 3.32. The intersection of any two fuzzy β_q open sets may not be a fuzzy β_q open sets as show in the following example.

Example 3.33. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.
consider four fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,\mathrm{d}\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{\mathrm{b,d}\}}\}$,

$\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{\mathrm{a,c,d}\}}\}$

Fuzzy open sets in fuzzy q-topological space are union of all four fuzzy topologies.

Then fuzzy q-open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,\mathrm{d}\}}, \chi_{\{\mathrm{b,d}\}}, \chi_{\{\mathrm{a,c,d}\}}\}$

Fuzzy β_T open sets of X denoted by

F$qSO(X)$ = $\{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{b,d\}}, \chi_{\{a,c,d\}}\}$

Here $\chi_{\{a,d\}} \wedge \chi_{\{b,c,d\}} = \chi_{\{d\}} \notin F_q SPO(X)$.

Theorem 3.34. Let χ_λ and χ_δ be fuzzy subsets of X such that $\chi_\delta \le \chi_\lambda \le Fq\,\mathrm{sp}\,cl(\chi_\delta)$ if χ_δ is fuzzy β_q open set then χ_λ is also fuzzy β_q open set.

4. Fuzzy α_q continuity and fuzzy β_q continuity in fuzzy quad topological space

Definition 4.1. A fuzzy function f from a fuzzy quad topological space $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ into another fuzzy quad topological space $(Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ is called fuzzy α_q continuous if $f^{-1}(\chi_\lambda)$ is fuzzy quad α_q open set in X for each fuzzy quad open set χ_λ in Y.

Example 4.2. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.
Consider four fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,d\}}\}$, $\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{c,d\}}\}$,

$\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,c,d\}}\}$

Fuzzy open sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Then fuzzy q-open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$

Fuzzy α_q open set of X is denoted by

$F_q SPO(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,d\}}, \chi_{\{c,d\}}, \chi_{\{a,c,d\}}\}$.

Let $Y = \{1, 2, 3, 4\}$ be a non-empty fuzzy set.
Consider four fuzzy topologies on Y

$\tau'_1 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,4\}}\}$, $\tau'_2 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{4\}}\}$, $\tau'_3 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,2\}}\}$,

$\tau'_4 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,2,4\}}\}$

Fuzzy q-open sets of $Y = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}$.

Fuzzy q-semi-open set of Y is

$F_q SPO(Y) = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{4\}}, \chi_{\{1,2\}}, \chi_{\{1,4\}}, \chi_{\{1,2,4\}}\}$.

Consider the fuzzy function $f : I^X \to I^Y$ is defined as

$f^{-1}(\chi_{\{4\}}) = \chi_{\{a\}}$, $f^{-1}(\chi_{\{1,2\}}) = \chi_{\{c,d\}}$, $f^{-1}(\chi_{\{1,4\}}) = \chi_{\{a,d\}}$,
$f^{-1}(\chi_{\{1,2,4\}}) = \chi_{\{a,c,d\}}$, $f^{-1}(\tilde{0}_Y) = (\tilde{0}_X)$, $f^{-1}(\tilde{1}_Y) = (\tilde{1}_X)$.

Since the inverse image of each fuzzy q-open set in *Y* under *f* is fuzzy α_q open set in *X*. Hence *f* is fuzzy α_q continuous function.

Theorem 4.3. Let $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be a fuzzy α_q continuous open function .If χ_λ is a fuzzy α_q open set of X , then $f(\chi_\lambda)$ is fuzzy α_q open in Y .

Proof: First, let χ_λ be fuzzy α_q open set in X . There exist a fuzzy quad open set χ_δ in X such that $\chi_\lambda \prec \chi_\delta \prec Fq\,\mathrm{sp}\,cl(\chi_\lambda)$ .since f is fuzzy quad open function then $f(\chi_\delta)$ is fuzzy quad open in Y .Since f is fuzzy quad continuous function, we have $f(\chi_\lambda) \prec f(\chi_\delta) \prec f(Fq\,\mathrm{sp}\,cl(\chi_\lambda)) \prec Fq\,\mathrm{sp}\,cl(f(\chi_\lambda))$. This shows that $f(\chi_\lambda)$ is fuzzy α_q open in Y .

Theorem 4.4. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological space. Then function $f: X \to Y$ is fuzzy quad α_q continuous function if and only if the inverse image of every α_q open set in Y is fuzzy quad open set X .

Proof: (Necessary): Let $f : I^X \to I^Y$ be a fuzzy α_q continuous function and χ_δ be any fuzzy α_q open set in Y .Then $\tilde{1}_Y - \chi_\delta$ is fuzzy α_q closed in Y . Since f is fuzzy α_q continuous function, $f^{-1}(\tilde{1}_Y - \chi_\delta) = \tilde{1}_X - f^{-1}(\chi_\delta)$ is fuzzy α_q closed in X and hence $f^{-1}(\chi_\delta)$ is fuzzy α_q open in X .

(Sufficiency): Assume that $f^{-1}(\chi_\lambda)$ is fuzzy α_q open in X for each fuzzy quad open set χ_λ in Y . Let χ_λ be a fuzzy quad closed set in Y .Then $\tilde{1}_Y - \chi_\lambda$ is fuzzy α_q open in Y .By assumption $f^{-1}(\tilde{1}_Y - \chi_\lambda) = \tilde{1}_X - f^{-1}(\chi_\lambda)$ is fuzzy α_q open in X which implies that $f^{-1}(\chi_\lambda)$ is fuzzy quad α_q closed in $(X,\tau_1,\tau_2,\tau_3,\tau_4)$. Hence f is fuzzy α_q continuous function.

Theorem 4.5. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological space. Then function $f: X \to Y$ is fuzzy quad α_q continuous open function. If χ_λ is an α_q open set of Y then $f^{-1}(\chi_\delta)$ is fuzzy α_q open in X .

Proof: Let χ_λ be fuzzy quad α_q open in X .There exist a fuzzy α_q open set χ_δ such that $\chi_\delta \prec \chi_\lambda \prec (Fq\,\mathrm{ps}\,cl(\chi_\delta))$ .Since f is fuzzy quad continuous function, we have $f(\chi_\delta) \prec f(\chi_\lambda) \prec f(Fq\,\mathrm{ps}\,cl(\chi_\delta)) \prec Fqpscl(f(\chi_\delta))$ by the proof of first part $f(\chi_\delta)$ is fuzzy quad α_q open in X .Therefore, $f(\chi_\lambda)$ is fuzzy quad α_q open in Y .

Theorem 4.6. The following are equivalent for a fuzzy function $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$

a) f is fuzzy α_q continuous function ;
b) the inverse image of each fuzzy quad closed set of Y is fuzzy quad α_q closed in X ;
c) For each $\chi_{\{x\}} \le \tilde{1}_X$ and each fuzzy quad open set χ_λ in χ_δ containing $f(\chi_{\{x\}})$ there exist a fuzzy α_q open set χ_α of X containing $\chi_{\{x\}}$ such that $f(\chi_\alpha) \prec \chi_\lambda$;
d) $Fqpscl(f^{-1}(\chi_\lambda)) \prec f^{-1}(Fqpscl(\chi_\lambda))$ For every fuzzy subset χ_λ of Y .
e) $f(Fqpscl(\chi_\delta)) \prec Fqpscl(f(\chi_\delta))$ For every subset χ_δ of X .

Theorem 4.7. Let X and Y are two quad topological spaces. Then $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is fuzzy α_q continuous function if one of the followings holds:

I. $f^{-1}(Fqps\,\mathrm{int}(\chi_\lambda)) \le Fqps\,\mathrm{int}(f^{-1}(\chi_\lambda))$, for every fuzzy quad open set χ_λ in Y .

II. $Fqpscl(f^{-1}(\chi_\lambda)) \le f^{-1}(Fqpscl(\chi_\lambda))$, for every fuzzy quad open set χ_λ in Y .

Proof: Let χ_λ be any fuzzy quad open set in Y and if condition (i) is satisfied then $f^{-1}(Fqps\,\mathrm{int}(\chi_\lambda)) \le Fqps\,\mathrm{int}(f^{-1}(\chi_\lambda))$.

We get $f^{-1}(\chi_\lambda) \le Fqps\,\mathrm{int}(f^{-1}(\chi_\lambda))$. Therefore $f^{-1}(\chi_\lambda)$ is a fuzzy α_q open set in X . Hence f is a fuzzy α_q continuous function. Similarly we can prove (ii).

Theorem 4.8. A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is called fuzzy quad α_q open continuous function if and only if

$$f(Fqps\,\mathrm{int}(\chi_\lambda)) \le Fqps\,\mathrm{int}(f(\chi_\lambda)),$$

for every fuzzy quad open set χ_λ in X .

Proof: Suppose that f is a fuzzy quad α_q open continuous function.

Since $Fqps\,\mathrm{int}(\chi_\lambda) \le \chi_\lambda$ so, $f(Fq\,\mathrm{ps}\,\mathrm{int}(\chi_\lambda)) \le f(\chi_\lambda)$.

By hypothesis $f(Fq\operatorname{int}(\chi_\lambda))$ is a fuzzy quad α_q open set and $Fqps\operatorname{int}(f(\chi_\lambda))$ is largest fuzzy quad α_q open set contained in $f(\chi_\lambda)$ so
$f(Fqps\operatorname{int}(\chi_\lambda)) \le Fqps\operatorname{int}(f(\chi_\lambda))$.

Conversely, suppose χ_λ is a fuzzy quad open set in X .So $f(Fqps\operatorname{int}(\chi_\lambda)) \le Fqps\operatorname{int}(f(\chi_\lambda))$.

Now since $\chi_\lambda = Fqps\operatorname{int}(\chi_\lambda)$ so $f(\chi_\lambda) \le Fqps\operatorname{int}(f(\chi_\lambda))$ .Therefore $f(\chi_\lambda)$ is a fuzzy quad α_q open set in Y and f is fuzzy quad α_q open continuous function.

Theorem 4.9. A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is called fuzzy α_q open continuous function if and only if $f(Fqps\operatorname{int}(\chi_\lambda)) \le Fqps\operatorname{int}(f(\chi_\lambda))$, for every fuzzy quad open set χ_λ in X .

Theorem 4.10. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is α_q continuous function if and only if $\mathrm{f}^{-1}(\chi_\lambda)$ is α_q closed in X whenever χ_λ is fuzzy quad closed in Y.

Theorem 4.11. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is α_q continuous function if and only if $\mathrm{f}(Fq\,\mathrm{pscl}\,\chi_\lambda) \prec Fq\,\mathrm{pscl}(\chi_\lambda), \forall \chi_\lambda \prec \tilde{1}_X$

Theorem 4.12. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is α_q continuous function if and only if $Fq\,\mathrm{pscl}\, f^{-1}(\chi_\lambda) \prec f^{-1}(Fq\,\mathrm{pscl}(\chi_\lambda)),\ \forall \chi_\lambda \prec X$

Theorem 4.13. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is α_q continuous open function if and only if $f^{-1}(Fq\,\mathrm{psint}(\chi_\lambda)) \prec Fq\,\mathrm{psint}(f^{-1}(\chi_\lambda)),\ \forall \chi_\lambda \prec X$

Definition 4.14. A fuzzy function f from a fuzzy quad topological space $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ into another fuzzy quad topological space $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is called fuzzy β_q continuous if $f^{-1}(\chi_\lambda)$ is fuzzy β_q open set in X for each fuzzy quad open set χ_λ in Y.

Example 4.15. Let $X = \{a, b, c, d\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on X, $\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}\}$, $\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a,\mathrm{d}\}}\}$

$\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{\mathrm{b,d}\}}\}$, $\tau_4 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{\mathrm{a,b,d}\}}\}$

Fuzzy open-sets in fuzzy q-topological spaces are union of all four fuzzy topologies.

Then fuzzy β_q open sets of $X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,\mathrm{d}\}}, \chi_{\{\mathrm{b,d}\}}, \chi_{\{\mathrm{a,b,d}\}}\}$

Fuzzy open set of X is denoted by

$$FqSP(X) = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{a\}}, \chi_{\{a,\mathrm{d}\}}, \chi_{\{\mathrm{b,d}\}}, \chi_{\{\mathrm{a,b,d}\}}\}$$

Let $Y = \{1, 2, 3, 4\}$ be a non-empty fuzzy set.

Consider four fuzzy topologies on Y

$\tau'_1 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,4\}}\}$, $\tau'_2 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{4\}}\}$, $\tau'_3 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,2\}}\}$,

$\tau'_4 = \{\tilde{1}_Y, \tilde{0}_Y, \chi_{\{1,2,4\}}\}$

Fuzzy q-open sets of $Y = \{\tilde{1}_Y, \tilde{0}_\mathrm{Y}, \chi_{\{4\}}, \chi_{\{1,\ 2\}}, \chi_{\{1,\ 4\}}, \chi_{\{1,2,4\}}\}$.

Fuzzy β_q open set of Y is denoted by

$$FqSPO(Y) = \{\tilde{1}_Y, \tilde{0}_\mathrm{Y}, \chi_{\{4\}}, \chi_{\{1,\ 2\}}, \chi_{\{1,\ 4\}}, \chi_{\{1,2,4\}}\} .$$

Consider the fuzzy function $f : I^X \to I^Y$ is defined as

$f^{-1}(\chi_{\{4\}}) = \chi_{\{\mathrm{a}\}}$, $f^{-1}(\chi_{\{1,2\}}) = \chi_{\{\mathrm{b,d}\}}$, $f^{-1}(\chi_{\{1,4\}}) = \chi_{\{\mathrm{a,d}\}}$,

$f^{-1}(\chi_{\{1,2,4\}}) = \chi_{\{\mathrm{a,b,d}\}}$, $f^{-1}(\tilde{0}_Y) = (\tilde{0}_X)$, $f^{-1}(\tilde{1}_Y) = (\tilde{1}_X)$.

Since the inverse image of each fuzzy q-open set in Y under f is fuzzy β_q open set in X. Hence f is fuzzy β_q continuous function.

Theorem 4.16. Let $f : (X, \tau_1, \tau_2, \tau_3, \tau_4) \to (Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ be a fuzzy quad β_q continuous open function .If χ_λ is a fuzzy quad β_q open set of X , then $f(\chi_\lambda)$ is fuzzy quad β_q open in Y .

Theorem 4.17. Let $(X, \tau_1, \tau_2, \tau_3, \tau_4)$ and $(Y, \tau'_1, \tau'_2, \tau'_3, \tau'_4)$ be two fuzzy quad topological space. Then function $f : X \to Y$ is fuzzy quad β_q continuous function if and only if the inverse image of every fuzzy β_q open set in Y is fuzzy quad open set in X .

Theorem 4.18. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological space. Then function $f:X\to Y$ is fuzzy continuous open function. If χ_λ is a fuzzy β_q open set of Y then $f^{-1}(\chi_\delta)$ is fuzzy β_q open in X.

Theorem 4.19. The following are equivalent for a fuzzy function
$f:(X,\tau_1,\tau_2,\tau_3,\tau_4)\to(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$

f) f is fuzzy quad β_q continuous function ;

g) the inverse image of each fuzzy β_q closed set of Y is fuzzy β_q closed in X ;

h) For each $\chi_{\{x\}}\le\tilde{1}_X$ and each fuzzy quad open set χ_λ in χ_δ containing $f(\chi_{\{x\}})$ there exist a fuzzy quad β_q open set χ_α of X containing $\chi_{\{x\}}$ such that $f(\chi_\alpha)\prec\chi_\lambda$;

i) $Fqspcl(f^{-1}(\chi_\lambda))\prec f^{-1}(Fqspcl(\chi_\lambda))$ for every fuzzy subset χ_λ of Y.

j) $f(Fqspcl(\chi_\delta))\prec Fqspcl(f(\chi_\delta))$ for every fuzzy subset χ_δ of X .

Theorem 4.20. Let X and Y are two fuzzy quad topological spaces. Then $f:(X,\tau_1,\tau_2,\tau_3,\tau_4)\to(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is fuzzy β_q continuous function if one of the followings holds:

(i) $f^{-1}(Fqsp\operatorname{int}(\chi_\lambda))\le Fqsp\operatorname{int}(f^{-1}(\chi_\lambda))$, for every fuzzy quad open set χ_λ in Y.

(ii) $Fqspcl(f^{-1}(\chi_\lambda))\le f^{-1}(Fqspcl(\chi_\lambda))$, for every fuzzy quad open set χ_λ in Y.

Proof: Let χ_λ be any fuzzy quad open set in Y and if condition (i) is satisfied then $f^{-1}(Fqsp\operatorname{int}(\chi_\lambda))\le Fqsp\operatorname{int}(f^{-1}(\chi_\lambda))$.

We get $f^{-1}(\chi_\lambda)\le Fqsp\operatorname{int}(f^{-1}(\chi_\lambda))$.Therefore $f^{-1}(\chi_\lambda)$ is a fuzzy β_q open set in X . Hence f is a fuzzy β_q continuous function. Similarly we can prove (ii).

Theorem 4.21. A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3,\tau_4)\to(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is called fuzzy β_q open continuous function if and only if

$$f(Fqsp\operatorname{int}(\chi_\lambda))\le Fqsp\operatorname{int}(f(\chi_\lambda)),$$

for every fuzzy quad open set χ_λ in X .

Proof: Suppose that f is a fuzzy β_q open continuous function.

Since $Fqsp\operatorname{int}(\chi_\lambda)\le\chi_\lambda$ so, $f(Fqsp\operatorname{int}(\chi_\lambda))\le f(\chi_\lambda)$.

By hypothesis $f(Fqsp\operatorname{int}(\chi_\lambda))$ is a fuzzy β_q pen set and $Fqsp\operatorname{int}(f(\chi_\lambda))$ is largest fuzzy quad β_q open set contained in $f(\chi_\lambda)$ so $f(Fqsp\operatorname{int}(\chi_\lambda))\le Fqsp\operatorname{int}(f(\chi_\lambda))$.

Conversely, suppose χ_λ is a fuzzy quad open set in X .
So $f(Fqsp\text{int}(\chi_\lambda)) \leq Fqsp\text{int}(f(\chi_\lambda))$.
Now since $\chi_\lambda = Fqsp\text{int}(\chi_\lambda)$ so $f(\chi_\lambda) \leq Fqsp\text{int}(f(\chi_\lambda))$. Therefore $f(\chi_\lambda)$ is a fuzzy β_q open set in Y and f is fuzzy β_q open continuous function.

Theorem 4.22. A fuzzy function $f:(X,\tau_1,\tau_2,\tau_3,\tau_4) \to (Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ is called fuzzy β_q open continuous function if and only if $f(Fqsp\text{int}(\chi_\lambda)) \leq Fqsp\text{int}(f(\chi_\lambda))$, for every fuzzy quad open set χ_λ in X .

Theorem 4.23. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is β_q continuous function if and only if $\mathrm{f}^{-1}(\chi_\lambda)$ is β_q closed in X whenever χ_λ is fuzzy quad closed in Y.

Theorem 4.24. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is β_q continuous function if and only if $\mathrm{f}(Fq\,\mathrm{spcl}\,\chi_\lambda) \prec Fq\,\mathrm{spcl}(\chi_\lambda), \forall \chi_\lambda \prec \tilde{1}_X$

Theorem 4.25. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is β_q continuous function if and only if $Fq\,\mathrm{spcl}(f^{-1}(\chi_\lambda)) \prec f^{-1}(Fq\,\mathrm{spcl}(\chi_\lambda)),\ \forall \chi_\lambda \prec \tilde{1}_X$

Theorem 4.26. Let $(X,\tau_1,\tau_2,\tau_3,\tau_4)$ and $(Y,\tau'_1,\tau'_2,\tau'_3,\tau'_4)$ be two fuzzy quad topological spaces. Then $f: X \to Y$ is β_q continuous open function if and only if $f^{-1}(Fq\,\mathrm{spint}(\chi_\lambda)) \prec Fq\,\mathrm{spint}(f^{-1}(\chi_\lambda)),\ \forall \chi_\lambda \prec X$

5. Conclusion

In this paper, we studied fuzzy α_q open sets and fuzzy β_q open sets in fuzzy quad topological space. We also studied fuzzy α_q continuous function and fuzzy β_q continuous function in fuzzy quad topological space and some of their fundamental properties.

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