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Certain Double Action on Bipolar Fuzzy Soft G-Modules

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Abstract. In this paper, we apply the notion of bipolar-valued fuzzy soft set to module theory. We introduce the concept of bipolar fuzzy soft G-modules, fuzzy soft d-ideals of modules and investigate several properties. We give relations between a bipolar fuzzy soft G-modules and bipolar fuzzy soft d-ideal. We provide a condition for bipolar fuzzy soft G-modules to be a bipolar fuzzy soft d-ideal. We also give characterizations of bipolar fuzzy soft ideal. We consider the concept of strongest bipolar fuzzy relations on bipolar fuzzy soft d-ideals of a module and discuss some related properties.

Keywords: G-module, Bipolar fuzzy soft G-module, d-ideals, t-cut, s-cut, fuzzy soft relation.

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1. Introduction

Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. There are several kinds of fuzzy sets extensions in the fuzzy set theory, for example, intuitionist fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1, 1]. Bipolar-Valued fuzzy sets have membership degrees that represent the degree of satisfaction to the property and its counter property. In a bipolar valued fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements some what satisfy the property, and the membership degrees on [-1,0) indicate that elements somewhat satisfy the implicit counter property. In the definition of bipolar-valued fuzzy sets, there are two kinds of representations so called canonical representation and reduced representation. In this paper, we use the canonical representation of bipolar valued fuzzy soft sets. We apply the notion of bipolar-valued fuzzy soft set to module theory. We introduce the concept of bipolar fuzzy soft G-modules/ (fuzzy soft d-ideals of modules and investigate several properties. We give relations between a bipolar fuzzy soft G-modules and bipolar fuzzy soft d-ideal. We provide a condition for bipolar fuzzy soft G-modules to be a bipolar fuzzy soft d-ideal. We also give characterizations of bipolar fuzzy soft ideal. We consider the concept of strongest bipolar fuzzy relations on bipolar fuzzy soft d-ideals of a module and discuss some related properties.

2. Preliminaries

In this section as a beginning, the concepts of G-module soft sets introduced by Molodsov and the notions of fuzzy soft set introduced by Maji et al. have been presented.

Definition 2.1. Let ‘S’ be a set. A fuzzy set in S is a function $\mu : S \rightarrow [0,1]$.

Definition 2.2. Let G be a finite group. A vector space M over a field K (a subfield of C) is called a G-module if for every $g \in G$ and $m \in M$, there exists a product (called the right action of G on M) $m.g \in M$ which satisfies the following axioms.

1. $m.1_G = m$ for all $m \in M$ (1_G being the identify of G)
2. $m.(g.h) = (m.g).h$, $m \in M$, $g, h \in G$
3. $(k_1 m_1 + k_2 m_2).G = k_1 (m_1.g) + k_2(m_2.g)$, $k_1, k_2 \in K$, $m_1, m_2 \in M$ & $g \in G$. In a similar manner the left action of G on M can be defined.

Definition 2.3. Let M and M* be G-modules. A mapping $\phi: M \rightarrow M^*$ is a G-module homomorphism if

1. $\phi(k_1 m_1 + k_2 m_2) = k_1 \phi(m_1) + k_2 \phi(m_2)$
2. $\phi(gm) = g\phi(m)$, $k_1, k_2 \in K$, $m, m_1, m_2 \in M$ & $g \in G$.

Definition 2.4. Let M be a G-module. A subspace N of M is a G - sub module if N is also a G-module under the action of G.

Let U be a universe set, E be a set of parameters, P(U) be the power set of U and $A \subseteq E$.

Definition 2.5. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U.

Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A, G_A, H_A, respectively.

Definition 2.6. The relative complement of the soft set F_A over U is denoted by F^r_A, where $F^r_A : A \rightarrow P(U)$ is a mapping given as $F^r_A(a) = U \setminus F_A(a)$, for all $a \in A$.

Definition 2.7. Let F_A and G_B be two soft sets over U such that $A \cap B \neq \phi$. The restricted intersection of F_A and G_B is denoted by $F_A \Cup G_B$, and is defined as $F_A \Cup G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 2.8. Let F_A and G_B be two soft sets over U such that $A \cap B \neq \phi$. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Definition 2.9. Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where $\psi(F_A) : B \rightarrow P(U)$ is a set valued function defined by $\psi(F_A)(b) = \cup\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \phi$. = 0 otherwise for all b ∈ B. Here, $\psi(F_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \to P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all a ∈ A. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

Definition 2.10. Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^{\star}(F_A)$ over U, where $\psi^{\star}(F_A):B \to P(U)$ is a set-valued function defined by

$\psi^{\star}(F_A)(b) = \cap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \phi$.

= 0 otherwise for all b ∈ B. Here, $\psi^{\star}(F_A)$ is called the soft anti image of F_A under ψ.

Theorem 2.11. Let F_H and T_K be soft sets over U, F^r_H, T^r_K be their relative soft sets, respectively and ψ be a function from H to K. then,

i) $\psi^{-1}(T^r_K) = (\psi^{-1}(T_K))^r$,

ii) $\psi(F^r_H) = (\psi^{\star}(F_H))^r$ and $\psi^{\star}(F^r_H) = (\psi(F_H))^r$.

Definition 2.12. Let F_A be a soft set over U and a be a subset of U. Then upper α-inclusion of F_A, denoted by $F_A^{\supseteq\alpha}$, is defined as $F_A^{\supseteq\alpha} = \{x \in A / F(x) \supseteq \alpha\}$. Similarly, $F_A^{\subseteq\alpha} = \{x \in A \mid F(x) \subseteq \alpha\}$is called the lower α -inclusion of F_A. A nonempty subset U of a vector space V is called a subspace of V if U is a vector space on F. From now on, V denotes a vector space over F and if U is a subspace of V, then it is denoted by U < V.

3. Bipolar fuzzy soft G-modules

Definition 3.1. Let 'G' be a non-empty set. A bipolar-Valued Fuzzy set A in G is an object having the form $A = \{(x, \mu_A^+(x), \mu_A^-(x) \ / \ x \varepsilon G\}$ where $\mu_A^+: G \to [0,1]$ and $\mu_A^-: G \to [-1,0]$ are mapping. The positive membership degree $\mu_A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to 'A' and the negative membership degree $\mu_A^-(x)$ denotes the satisfaction degree of x to some implicit counter property of A.

Definition 3.2. Let U be an initial universe, E be the set of parameters, A is subset of E. Define F: A → BFU , where BFU is the collection of all bipolar fuzzy subsets of U. Then (F,A) is said to be a bipolar fuzzy soft set over a universe U. It is defined by $(F,A) = \{ (x, \mu_e^+(x), \mu_e^-(x) : \text{for all } x \varepsilon U \text{ and } e \varepsilon A\}$.

Example 3.3. Let $U = \{ c_1,c_2,c_3,c_4 \}$ be the set of four cars under consideration and $E = \{ e_1 = \text{costly}, e_2 = \text{beautiful}, e_3 = \text{fuel efficient}, e_4 = \text{modern technology} \}$ be the set of parameters and $A = \{e_1,e_2,e_3\}$ is subset of E. Then

$$(F,A) = \left\{ \begin{array}{l} F(e_1) = \{ (c_1, 0.3, -0.4), (c_2, 0.3, -0.5), (c_3,0.1,-0.2), (c_4, 0.7, -0.6)\} \\ F(e_2) = \{(c_1, 0.2, -0.6), (c_2, 0.1, -0.7), (c_3,0.3,-0.7), (c_4, 0.5, -0.6)\} \\ F(e_3) = \{(c_1, 0.1, -0.3), (c_2, 0.3, -0.5), (c_3,0.7,-0.2), (c_4, 0.3, -0.7)\} \end{array} \right\}$$

Definition 3.4. Let U be a universe and E a set of attributes. Then,(U,E) is the collection of all bipolar fuzzy soft sets on U with attributes from E and is said to be bipolar fuzzy soft class.

Definition 3.5. A bipolar fuzzy soft set (F,A) is said to be a null bipolar fuzzy soft set denoted by empty set ϕ, if for all e ε A , F(e) = ϕ .

Definition 3.6. A bipolar fuzzy soft set (F,A) is said to be an absolute bipolar fuzzy soft set, if for all e ε A , F(e) = BFU.

Definition 3.7. The complement of a bipolar fuzzy soft set (F,A) is denoted $(F,A)^c$ and is denoted by $(F,A)^c = \{ (x, 1- \mu_A^+ (x), 1- \mu_A^-(x) ; x \varepsilon U\}$.

Definition 3.8. A bipolar fuzzy soft set A (μ_A^+, μ_A^-) of S is called a bipolar fuzzy soft G-modules of S provided that for all x,y,z,a,b ε S;
(BFSGM1) $\mu_A^+(ax+by) \geq$ min $\mu_A^+(x), \mu_A^+(y) \}$, $\mu_A^-(ax+by) \leq \max \{ \mu_A^-(x), \mu_A^-(y) \}$,
(BFSGM2) $\mu_A^+(\alpha x) \geq \mu_A^+(x)$, , $\mu_A^-(\alpha x) \leq \mu_A^-(y) \}$

Definition 3.9. For a bipolar fuzzy set 'A' and (β,α) ε [-1.0] x [0,1], we define
$A_t^x = \{x\varepsilon X \,/\, \mu_A^+(x) \geq \alpha\}$, $A_S^- = \{x\varepsilon X \,/\, \mu_A^+(x) \geq \alpha\}$ which are called the positive α-cut and negative β-cut of A respectively.

Definition 3.10. A bipolar fuzzy soft set 'A' in X is called a bipolar fuzzy soft d-ideal of X if it satisfies;
$(BPFSDI_1)$ $\mu_A^+(x) \geq T\{ \mu_A^+(ax+by), \mu_A^+(y)\}$
$(BPFSDI_2)$ $\mu_A^-(x) \leq S\{ \mu_A^-(ax+by), \mu_A^-(y)\}$
$(BPFSDI_3)$ $\mu_A^+(e) \geq \mu_A^+(x)$ and $\mu_A^-(e) \geq \mu_A^-(x)$ and for all x,y ε X.

Definition 3.11. Let λ and μ be two fuzzy subsets in X. The Cartesian Product of $\lambda^+\times\mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^+\times\mu^+(x,y) = T\{ \lambda^+(x), \mu^+(y)\}$ and
$\lambda^+\times\mu^+ : X \times X \rightarrow [0,1]$ is defined by $\lambda^+\times\mu^+ (x,y) = S\{ \lambda^-(x), \mu^-(y)\}$ for all x,y ε X.

Definition 3.12. Let $f : X \rightarrow Y$ be a mapping of modules and 'μ' be a bipolar fuzzy soft set of y. The map μ^f is the pre image of μ_1 and μ_2 under f. so $\mu_1^{+f}(x) = \mu^{+f}(x)$, $\mu_2^{-f}(x) = \mu^{-f}(x)$

Definition 3.13. Let 'A' be a bipolar fuzzy soft set in a X, the strongest bipolar fuzzy soft relation on X that is fuzzy relation on A is μ_A given by,
$\mu_A^+(x,y) =T\{A^+(x), A^+(y)\}$ $\mu_A^-(x,y) =S\{A^-(x), A^-(y)\}$ for all x,y ε X.

4. Main results

Proposition 4.1. If ϕ is a bipolar fuzzy soft G-modules of X, then $\mu_\varphi^+(e) \geq \mu_\varphi^+(x)$ and $\mu_\varphi^-(e) \leq \mu_\varphi^-(x)$ for all x ε X.

Proof: Let x ε X, then
$\mu_\varphi^+(e) = \mu_\varphi^+(x\, x^{-1}) \geq T \{ \mu_\varphi^+(x), \mu_\varphi^+(x^{-1})\} \geq T \{ \mu_\varphi^+(x), \mu_\varphi^+(x)\} \geq \mu_\varphi^+(x)$
and $\mu_\varphi^-(e) = \mu_\varphi^-(x\, x^{-1}) \leq S \{ \mu_\varphi^-(x), \mu_\varphi^-(x^{-1})\} \leq S \{ \mu_\varphi^-(x), \mu_\varphi^-(x)\} \leq \mu_\varphi^-(x)$

This completes the proof.

Proposition 4.2. Let 'ϕ' be a bipolar fuzzy soft G-modules of X, then the following assertions are valid.
(i) $(\forall\ \alpha\ \varepsilon\ [0,1]\ (\phi_\alpha^+ \neq \phi \Rightarrow \phi_t^+$ is a group of X)
(ii) $(\forall\ \beta\ \varepsilon\ [-1,0]\ (\phi_\beta^- \neq \phi \Rightarrow \phi_\beta^-$ is a group of X)
Proof: Let $t\ \varepsilon\ [0,1]$ be such that $\phi_t^+ \neq \phi$. If $x,y\ \varepsilon\ \phi_t^+$, then $\mu_\phi^+(x) \geq t$ and $\mu_\phi^+(y) \geq t$ It follows that $\mu_\phi^+(ax+by) \geq T\ \{\ \mu_\phi^+(x), \mu_\phi^-(y)\} \geq t$.

Corollary 4.3. If ϕ is a bipolar fuzzy soft G-modules of X, then the sets $\phi^+_{\mu\varphi+(e)}$ and $\phi^-\ \mu_\varphi^-(e)$ are group of X.
Proof: Straight forward.

Proposition 4.4. Let $\phi = (X, \mu_\phi^+, \mu_\phi^-\ )$ be a bipolar fuzzy soft d-ideal of X. If the inequality $xy \leq z$ holds in X, then

$$\mu_\phi^+(x) \geq T\ \{\ \mu_\phi^+(y), \mu_\phi^+(z)\ \}$$
$$\mu_\phi^-(x) \leq\ S\ \{\ \mu_\phi^+(y), \mu_\phi^-(z)\ \}$$

Proof: Let $x, y, z\ \varepsilon$ X be such that $xy \leq z$, then $(xy)z = 0$, and so
$\mu_\phi^+(x) \geq T\ \{\ \mu_\phi^+(ax+by), \mu_\phi^+(y)\} \geq T\ \{T\ \{\ \mu_\phi^+(ax+by)z, \mu_\phi^+(z)\}, \mu_\phi^+(y)\ \} =$
$T\ \{T\ \{\ \mu_\phi^+(e), \mu_\phi^+(z)\}, \mu_\phi^+(y)\} = T\ \{\ \mu_\phi^+(y), \mu_\phi^+(z)\ \}$and
$\mu_\phi^-(x) \leq S\ \{\ \mu_\phi^-(\alpha x), \mu_\phi^-(y)\}\ v\ \chi \leq S\ \{S\ \{\ \mu_\phi^-(\alpha x)z, \mu_\phi^-(z)\}, \mu_\phi^-(y)\ \}$
$= S\ \{S\ \{\ \mu_\phi^-(e), \mu_\phi^-(z)\}, \mu_\phi^-(y)\} = S\ \{\ \mu_\phi^-(y), \mu_\phi^-(z)\ \}$
This completes the proof.

Proposition 4.5. Let ϕ be a$(\psi, \chi\ )$ -bipolar fuzzy soft d-ideal of X. If the inequality $x \leq y$ holds in X, then $\mu_\phi^+(x) \geq \mu_\phi^+(y)$ and $\mu_\phi^-(x) \leq \mu_\phi^-(y)$
Proof: Let $x, y\ \varepsilon$ X be such that $x \leq y$, then $\mu_\phi^+(x) \geq T\ \{\ \mu_\phi^+(ax+by), \mu_\phi^+(y)\}$
$= T\{\ \mu_\phi^+(e), \mu_\phi^+(y)\} = \mu_\phi^+(y)\ \mu_\phi^-(x)$
$\leq S\ \{\ \mu_\phi^-(\alpha x), \mu_\phi^-(y)\}$
$= T\{\ \mu_\phi^-(e), \mu_\phi^-(y)\} = \mu_\phi^-(y)$
This completes the proof.

Proposition 4.6. In a group X, every bipolar fuzzy soft d-ideal of X is bipolar fuzzy soft G-modules of X.
Proof: Let 'ϕ' be a bipolar fuzzy soft d-ideal of a group X. Since $xy \leq x$ for all $x,y\ \varepsilon$ X, it follows from Proposition (4.5) that
$\mu_\phi^+(ax+by) \geq T\ \{\ \mu_\phi^+(x)$ and $\mu_\phi^-(x) \leq \mu_\phi^-(x)$, so from Proposition 4.1
(BPFSGM1) $\mu_\phi^+(ax+by) \geq T\ \{\ \mu_\phi^+(x)\ v\ \chi \geq T\{\mu_\phi^+(ax+by), \mu_\phi^+(y)\} = T\{\ \mu_\phi^+(x), \mu_\phi^+(y)\}$
and (BPFSGM2) $\mu_\phi^-(\ ax+by\ ) \leq \mu_\phi^-(x) \leq S\ \{\ \mu_\phi^-(ax+by), \mu_\phi^-(y)\} \leq S\{\ \mu_\phi^-(x), \mu_\phi^-(y)\}$
$\mu_\phi^+(x^{-1}) \geq T\ \{\mu_\phi^+(\alpha x), \mu_\phi^+(x)\} = T\{\ \mu_\phi^+(e), \mu_\phi^+(y)\} \geq \mu_\phi^+(x)$, $\mu_\phi^-(x^{-1}) \leq S\ \{\ \mu_\phi^-(\alpha x), \mu_\phi^-(y)\} \leq S\{\ \mu_\phi^-(e), \mu_\phi^-(y)\} \leq \mu_\phi^-(x)$. Hence ϕ is bipolar fuzzy soft G-modules. The converse of the theorem is not true in general.
Proposition 4.7. Let 'ϕ' be a bipolar fuzzy soft G-modules of a module X such that Proposition 4.2 holds for all $x, y, z\ \varepsilon$ X satisfying the inequality $xy\ \varepsilon\ z$ then ϕ is a bipolar fuzzy soft d-ideal of X.

Proof: Recall from Proposition 4.1; that $\mu_\varphi^{+}(e) \geq \mu_\varphi^{+}(x)$ and $\mu_\varphi^{-}(e) \leq \mu_\varphi^{-}(x)$ for all $x \varepsilon X$. Since $x(xy) \leq y$ for all $x, y \varepsilon X$, it follows that Proposition 4.2,

$\mu_\phi^{+}(x) \geq T\{\mu_\phi^{+}(ax+by), \mu_\phi^{+}(y)\}$ and

$\mu_\phi^{-}(x) \leq S\{\mu_\phi^{-}(\alpha x), \mu_\phi^{-}(y)\}$

Hence ϕ is a bipolar fuzzy soft d-ideal of X.

Proposition 4.8. Let λ and μ be bipolar fuzzy soft d-ideal of X, then $\lambda\times\mu$ is also bipolar fuzzy soft d-ideal of X.

Proof: For any $(x_1, x_2), (y_1, y_2) \varepsilon X \times X$, we have

(BFd$_1$) $(\lambda^{+} \times \mu^{+})(x_1, x_2) = T\{\lambda^{+}(x_1), \mu^{+}(x_2)\}$

$\geq T\{T\{\lambda^{+}(x_1, y_1), \lambda^{+}(y_1)\}, T\{\mu^{+}(x_2, y_2), \mu^{+}(y_2)\}\}$

$= T\{T\{\lambda^{+}(x_1, y_1), \mu^{+}(x_2, y_2)\}, T\{\lambda^{+}(y_1), \mu^{+}(y_2)\}\}$

$= T\{(\lambda^{+} \times \mu^{+})((x_1, x_2), (y_1, y_2)\}$

$(\lambda^{-}\times\mu^{-})(x_1, x_2) = S\{\lambda^{-}(x_1), \mu^{-}(x_2)\}$

$\leq S\{S\{\lambda^{-}(x_1,y_1), \lambda^{-}(y_1)\}, S\{\mu^{-}(x_2,y_2), \mu^{-}(y_2)\}$

$= S\{S\{\lambda^{-}(x_1,y_1), \mu^{-}(x_2,y_2)\}, S\{\lambda^{-}(y_1), \mu^{-}(y_2)\}$

$= S\{(\lambda^{-}\times\mu^{-})(x_1, x_2)(y_1, y_2), (\lambda^{-}\times\mu^{-})(y_1, y_2)\}$

$(\lambda^{+}\times\mu^{+})(x_1^{-1}, x_2^{-1})$

$= T\{\lambda^{+}(x_1^{-1}), \mu^{+}(x_2^{-1})\} \geq T\{T\{\lambda^{+}(x_1,y_1), \lambda^{+}(y_1)\}, T\{\mu^{+}(x_2,y_2), \mu^{+}(y_2)\}$

$= T\{T\{\lambda^{+}(x_1,y_1), \mu^{+}(x_2,y_2)\}, T\{\lambda^{+}(y_1), \mu^{+}(y_2)\}$

$= T\{(\lambda^{+}\times\mu^{+})(x_1, x_2)(y_1, y_2), (\lambda^{+}\times\mu^{+})(y_1, y_2)\}$

$(\lambda^{-}\times\mu^{-})(x_1^{-1}, x_2^{-1}) = S\{\lambda^{-}(x_1^{-1}), \lambda^{-}(x_2^{-1})\}$

$\leq S\{S\{\lambda^{-}(x_1,y_1), \lambda^{-}(y_1)\}, S\{\mu^{-}(x_2,y_2), \mu^{-}(y_2)\}$

$= S\{S\{\lambda^{-}(x_1,y_1), \mu^{-}(x_2,y_2)\}, S\{\lambda^{-}(y_1), \mu^{-}(y_2)\}$

$\leq S\{(\lambda^{-}\times\mu^{-})(x_1, x_{2,} y_1, y_2), (\lambda^{-}\times\mu^{-})(y_1, y_2)\}$

Hence $\lambda\times\mu$ is bipolar fuzzy soft d-ideal of X.

Proposition 4.9. Let $f : X \rightarrow Y$ be a homomorphism of groups. If 'μ' is a bipolar fuzzy soft d-ideal of y, then μ^{f} is bipolar fuzzy soft d-ideal of X.

Proof: For any $x \varepsilon X$, we have

$\mu^{+f}(x) = \mu^{+}(f(x)) \geq \mu^{+}(e) = \mu^{+}(f(e)) = \mu^{+f}(e)$

$\mu^{-f}(x) = \mu^{-}(f(x)) \leq \mu^{-}(e) = \mu^{-}(f(e)) = \mu^{-f}(e)$

Let $x, y \varepsilon X$

$T\{\mu^{+f}(xy), \mu^{+f}(y)\} = T\{\mu^{+}(f(xy), \mu^{+}(f(y)\} = T\{\mu^{+}(f(x).f(y)), \mu^{+}(f(y)\}$

$\leq \mu^{+}f(x) = \mu^{+f}(x)$.

$S\{\mu^{-f}(xy), \mu^{-f}(y)\} = S\{\mu^{-}f(xy), \mu^{-}(f(x)\} = S\{\mu^{-}(f(x),f(x)), \mu^{-}(f(x)\}$

$\geq \mu^{-}(f(x) = \mu^{-f}(x)$

Hence μ^{f} is bipolar fuzzy soft d-ideal of X.

Proposition 4.10. Let $f : X \rightarrow Y$ be an epimorphism of groups. If μ^{f} is bipolar fuzzy soft d-ideal of X, then μ is bipolar fuzzy soft d-ideal of Y.

Proof: Let $y \varepsilon Y$, there exists $x \varepsilon X$ such that $f(x) = y$, then

$\mu^{+}(y) = \mu^{+}(f(x)) = \mu^{+f}(x) \leq \mu^{+f}(e) = \mu^{+}(f(e) = \mu^{+}(e)$

$\mu^{-}(y) = \mu^{-}(f(x)) = \mu^{-f}(x) \geq \mu^{-f}(e) = \mu^{-}(f(e) = \mu^{-}(e)$

Let $x, y \varepsilon Y$, then there exists $a, b \varepsilon X$, such that $f(a) = x$ and $f(b) = y$. It follows that

$\mu^{+}(x) = \mu^{+}(f(a) \cap \psi = \mu^{+f}(a)$ and $\mu^{-}(x) = \mu^{-}(f(a) \cap \psi = \mu^{-f}(a)$

$\geq T\{ \mu^{+f}(ab), \mu^{+f}(b)\} = T\{ \mu^{+}(f(ab), \mu^{+}(f(b)\} = T\{ \mu^{+}(f(a).f(b)), \mu^{+}(f(b)\}$
$= T\{ \mu^{+}(xy), \mu^{+}(y)\}$
Also
$\leq S\{ \mu^{-f}(ab), \mu^{-f}(b)\} = S\{ \mu^{-}(f(ab), \mu^{-}(f(b)\} \vee \chi = S\{ \mu^{-}(f(a).f(b)), \mu^{-}(f(b)\}$
$= S\{ \mu^{-}(xy), \mu^{-}(y)\}$
Hence μ is a bipolar fuzzy soft d-ideal of y.

Proposition 4.11. Let 'A' be a bipolar fuzzy soft set in a module X and μ_A be the strongest bipolar fuzzy soft relation on X, then A is a bipolar fuzzy soft d-ideal of X if and only if μ_A is a bipolar fuzzy soft d-ideal of X × X.
Proof: Suppose that 'A' is a bipolar fuzzy soft d-ideal of X, then
$\mu_A^{+}(e, e) = T\{ A^{+}(e), A^{+}(e)\}$
$\geq T\{ A^{+}(x), A^{+}(y)\} = \mu_A^{+}(x, y)$ for all $(x, y) \varepsilon X \times X$.
$\mu_A^{-}(e, e) = S\{ A^{-}(e), A^{-}(e)\} \leq S\{ A^{-}(x), A^{-}(y)\} = \mu_A^{-}(x, y)$ for all $(x, y) \varepsilon X \times X$.
For any $x = (x_1, x_2)$ and
$y = (y_1, y_2) \varepsilon X \times X$.
$\mu_A^{+}(x) = \mu_A^{+}(x_1, x_2)$
$= T\{ A^{+}(x_1), A^{+}(x_2)\} \geq T\{T\{A^{+}(x_1,y_1), A^{+}(y_1)\}, T\{A^{+}(x_2, y_2), A^{+}(y_2)\} \}$
$= T\{ T\{A^{+}(x_1, y_1), A^{+}(x_2, y_2)\}, T\{A^{+}(y_1), A^{+}(y_2) \} \}$
$= T\{ \mu_A^{+}(x_1, y_1), (x_2, y_2)), \mu_A^{+}(y_1, y_2)\} = T\{ \mu_A^{+}(xy), \mu_A^{+}(y)\}$
$\mu_A^{-}(x) = \mu_A^{-}(x_1, x_2)$
$= S\{ A^{-}(x_1), A^{-}(x_2)\} \leq S\{S\{A^{-}(x_1, y_1), A^{-}(y_1)\}, S\{A^{-}(x_2, y_2), A^{-}(y_2)\} \}$
$= S\{S\{A^{-}(x_1, y_1), A^{-}(x_2, y_2)\}, S\{A^{-}(y_1), A^{-}(y_2) \} \}$
$= S\{ \mu_A^{-}(x_1, y_1), (x_2, y_2)), \mu_A^{-}(y_1, y_2)\} = S\{ \mu_A^{-}(xy), \mu_A^{-}(y)\}$
Hence μ_A is a bipolar fuzzy soft d-ideal of X × X. Conversely, suppose that μ_A is a bipolar fuzzy soft d-ideal of X × X. Then,
$T\{A^{+}(e), A^{+}(e)\} = \mu_A^{+}(e, e)$
$\geq \mu_A^{+}(x, y) = T\{A^{+}(x), A^{+}(y)\} \ \forall(x, y) \varepsilon X \times X$.
$S\{ A^{-}(e), A^{-}(e)\} = \mu_A^{-}(e, e) \leq \mu_A^{-}(x, y) = S\{ A^{-}(x), A^{-}(y)\}$
for any $x = (x_1, y_1)$ and $y = (y_1, y_2) \varepsilon X \times X$., we have
$T\{A(x_1), A(x_2)\} = \mu_A(x_1, x_2) \geq T\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\}$
$= T\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} = T\{ T\{A(x_1, y_1), A(x_2, y_2)\}, T\{A(y_1), A(y_2)\}$
$= T\{ T\{ A(x_1, y_1), A(y_1), T\{A(x_2, y_2), A(y_2)\}$
Putting $x_1 = x_2 = 0$, we have
$\mu_A(x_1) \geq T\{\mu_A(x_1, y_1), \mu_A(y_1)\}$
Likewise, $\mu_A(x_1y_1) \geq T\{\mu_A(x_1), \mu_A(x_2)\}$
$S\{A(x_1), A(x_2)\} = \mu_A(x_1, x_2) \leq S\{\mu_A((x_1, x_2), (y_1, y_2)), \mu_A(y_1, y_2)\}$
$= S\{\mu_A(x_1y_1, x_2y_2)), \mu_A(y_1, y_2)\} = S\{ S\{A(x_1, y_1), A(x_2, y_2)\}, S\{A(y_1), A(y_2)\}$
$= S\{ S\{ A(x_1, y_1), A(y_1), S\{A(x_2, y_2), A(y_2)\}$
Putting $x_1 = x_2 = 0$, we have
$\mu_A(x_1) \leq S\{\mu_A(x_1, y_1), \mu_A(y_1)\}$
Likewise, $\mu_A(x_1y_1) \leq S\{\mu_A(x_1), \mu_A(x_2)\}$. Hence A is a bipolar fuzzy soft d-ideal of X.
Proposition 4.12. Let ϕ be a bipolar fuzzy soft set in X, then ϕ is a bipolar fuzzy soft d-ideal of X if and only if it satisfies the following assertions.
$(\forall \alpha \varepsilon [0,1] \ (\phi_t^{+} \neq \phi \Rightarrow \phi_t^{+}$ is an ideal of X)
$(\forall \beta \varepsilon [-1,0] \ (\phi_s^{-} \neq \phi \Rightarrow \phi_\beta^{-}$ is an ideal of X)

Proof: Assume that ϕ is a bipolar fuzzy soft d-ideal of X. Let $(s,t) \varepsilon [-1, 0] \varepsilon [0,1]$ be such that $\phi_t^+ \neq \phi$ and $\phi_s^- \neq \phi$.
Obviously, $e \varepsilon \phi_t^+ \cap \phi_s^-$.
Let x, y ε X be such that $xy \varepsilon \phi_t^+$ and $y \varepsilon \phi_t^+$, and
Let a, b ε X be such that $ab \varepsilon \phi_s^-$ and $b \varepsilon \phi_s^-$, then
$\mu_\phi^+(xy) \geq t$, $\mu_\phi^+(y) \geq t$ $\mu_\phi^-(ab) \leq s \vee \chi$ and $\mu_\phi^-(b) \leq s$.
It follows from Proposition 4.1
$\mu_\phi^+(x) \geq T\{ \mu_\phi^+(xy), \mu_\phi^+(y)\} \geq t$ and
$\mu_\phi^-(a) \leq S\{ \mu_\phi^-(ab), \mu_\phi^-(b) \leq s$.
so that $x \varepsilon \phi_t^+$ and $a \varepsilon \phi_s^-$. Therefore ϕ_t^+ and ϕ_s^- are ideals of X.

Conversely, suppose that the condition (corollary) is valid. For any x ε X, let $\mu_\varphi^+(x) = t$ and $\mu_\varphi^-(x) = s$. then $x \varepsilon \phi_t^+ \cap \phi_s^-$, and so ϕ_t^+ and ϕ_s^- are non-empty. Since ϕ_t^+ and ϕ_s^- are ideal of X, $e \varepsilon \phi_t^+ \cap \phi_s^-$. Hence $\mu_\phi^+(e) \cap \psi \geq t = \mu_\phi^+(x)$ and $\mu_\phi^-(e) \cap \psi \leq s = \mu_\phi^-(x) \vee \chi$ for all x ε X.

If there exists $x^1, y^1, a^1, b^1 \varepsilon X$ such that $\mu_\phi^+(x^1) \leq T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\}$
and $\mu_\phi^-(a^1) \geq S\{ \mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\}$ then by taking
$t_0 = ½ \{ \mu_\phi^+(x^1) + T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\}$
$S_0 = ½ \{ \mu_\phi^-(a^1) + S\{ \mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\}$
We have,
$\mu_\phi^+(x^1) < t_0 \leq T\{ \mu_\phi^+(x^1y^1), \mu_\phi^+(y^1)\}$
$\mu_\phi^-(a^1) < s_0 \leq S\{\mu_\phi^-(a^1b^1), \mu_\phi^-(b^1)\}$
Hence $x^1 \notin \phi_{t0}^+$, $x^1, y^1 \varepsilon \phi_{t0}^+$, $y^1 \varepsilon \phi_{t0}^+$, $a^1 \notin \phi_{s0}^-$ and $b^1 \varepsilon \phi_{s0}^-$. This is a contradiction and thus ϕ is a bipolar fuzzy soft d-ideal of X.

5. Conclusion

Lee [6] introduces the notion of bipolar fuzzy sub-algebra and bipolar fuzzy ideals of BCK/BCI-algebra. In this paper, we provide a condition for a bipolar fuzzy soft G-modules and bipolar fuzzy soft d-ideal. We give relations between a bipolar fuzzy soft G-modules and bipolar fuzzy soft d-ideal. We consider the concept of strongest bipolar fuzzy soft relation and discuss some related properties.

REFERENCES

1. M.Akram and K.H.Dar, On fuzzy d-algebras, *Punjab University Journal of Mathematics*, 37 (2005) 61-76.
2. D.Dubois and H.Prade, *Fuzzy sets and Systems; theory and applications*, Acaemic Press, (1980).
3. Y.B.Jun and S.Z.Song, Sub algebra’s and Closed ideals of Belt-algebras based on bipolar-valued fuzzy sets, *Sci. Math. Jpn* (submitted).
4. K.M.Lee, Bipolar-valued fuzzy sets and their operations, *Proc. Int. Conf. on. Intelligent Technologies*, Bongkok, Thailand, (2000) 307-312.
5. K.M.Lee, Composition of interval valued fuzzy sets, Intuitionistic fuzzy sets and bipolar fuzzy sets, *J. Fuzzy Logic Intelligent Systems*, 14(2) (2004) 125-129.
6. R.Nagarajan and A.Solairaju, Characterizations of bipolar Q-fuzzy groups in terms of Bipolar Q-fuzzy d-ideals, (submitted).
7. H.J.Zimmermann, *Fuzzy set theory and its applications*, Klower-Publishing, (1985).