

Face Sum Divisor Cordial Graphs

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Abstract. In this paper, we investigate the face sum divisor cordial labeling of switching of any vertex in cycle C_n , switching of a pendent vertex in path P_n and $S'(K_{1,n})$.

Keywords: Sum divisor cordial labeling, face sum divisor cordial labeling, switching of a vertex.

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1. Introduction

We begin with simple, finite, planar, undirected graph. A (p,q) planar graph G means a graph $G = (V,E)$, where V is the set of vertices with $|V| = p$, E is the set of edges with $|E| = q$ and F is the set of interior faces of G with $|F| =$ number of interior faces of G . For standard terminology and notations related to graph theory we refer to Harary [4] while for number theory we refer to Burton [2]. A graph labeling is the assignment of unique identifiers to the edges and vertices of a graph. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [3]. In [1], Cahit introduced the concept of cordial labeling of graph. Varatharajan et al. [7] introduced the concept of divisor cordial labeling of graphs. The concept of sum divisor cordial labeling was introduced by Lourdasamy et al. [6]. Lawrence et al. introduced the concept of face product cordial labeling of graphs in [5]. Motivated by the concept of face product cordial labeling and sum divisor cordial labeling, we introduce new type of labeling which is called a face sum divisor cordial labeling of graph. The present work is focused on some new families of face sum divisor cordial labeling of switching of a pendent vertex in path P_n , switching of any vertex in cycle C_n and $S'(K_{1,n})$. We will provide brief summary of definitions and other information which are necessary for the present investigations.

2. Basic definitions

Definition 2.1. Let a and b be two integers. If a divides b means that there is a positive integer k such that $b = ka$. It is denoted by $a|b$. If a does not divide b , then we denote $a \nmid b$.

Definition 2.2. Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ be a bijection. For each edge uv , assign the label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0

otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph with a divisor cordial labeling is called a divisor cordial graph.

Definition 2.3. Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ be a bijection. For each edge uv , assign the label 1 if $2|(f(u)+f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 2.4. A vertex switching G_v of a graph G is obtained by taking a vertex v of G , removing the entire edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G .

Definition 2.5. For a graph G , the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 2.6. A complete bipartite graph $K_{1,n}$ is called a star and it has $n+1$ vertices and n edges.

Definition 2.7. A face sum divisor cordial labeling of a graph G with vertex set V is a bijection f from $V(G)$ to $\{1, 2, \dots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise and for face f is assigned the label 1 if 2 divides $f(u_1)+f(u_2)+\dots+f(u_k)$ and 0 otherwise, where u_1, u_2, \dots, u_k are vertices corresponding to the face. Also the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1 and the number of faces labeled with 0 and the number of faces labeled with 1 differ by at most 1. A graph with a face sum divisor cordial labeling is called a face sum divisor cordial graph.

3. Main theorems

Theorem 3.1. Switching of any vertex in cycle C_n admits face sum divisor cordial labeling for $n \geq 5$.

Proof: Let v_1, v_2, \dots, v_n be the successive vertices of C_n . G_v denotes the graph, which is obtained by switching of a vertex v of C_n . Without loss of generality let the switched vertex be v_1 . Let G be a graph G_{v_1} . Then v_1, v_2, \dots, v_n are vertices, $e_1, e_2, \dots, e_{2n-5}$ are edges and f_1, f_2, \dots, f_{n-4} are the interior faces of G . $e_i = v_1v_{i+2}$, for $1 \leq i \leq n-3$, $e_{n-3+i} = v_{i+1}v_{i+2}$, for $1 \leq i \leq n-2$ and $f_i = v_1v_{i+2}v_{i+3}v_1$ for $1 \leq i \leq n-4$. Then $|V(G)| = n$, $|E(G)| = 2n-5$ and $|F(G)| = n-4$.

Define $g : V(G) \rightarrow \{1, 2, 3, \dots, n\}$ as follows

Case 1 : $n = 5$.

$$g(v_1) = 1, \quad g(v_2) = 2, \quad g(v_3) = 4, \quad g(v_4) = 3 \text{ and } g(v_5) = 5.$$

Then induced edge labels are

$$g^*(e_1) = 0, \quad g^*(e_2) = 1, \quad g^*(e_3) = 1, \quad g^*(e_4) = 0 \text{ and } g^*(e_5) = 1.$$

Also the induced face label is

$$g^{**}(f_1) = 1.$$

In view of the above defined labeling pattern we have

$$e_f(0)+1 = e_f(1) = 3 \text{ and } f_g(0)+1 = f_g(1) = 1.$$

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Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n = 5$.

Case 2: $n = 6$.

$g(v_1) = 1, g(v_2) = 2, g(v_3) = 4, g(v_4) = 3$ and $g(v_{i+4}) = g(v_i) + 4$, for $1 \leq i \leq n - 4$.

Then induced edge labels are

$g^*(e_1) = 0, g^*(e_2) = 1, g^*(e_3) = 1, g^*(e_4) = 1, g^*(e_5) = 0, g^*(e_6) = 1$ and $g^*(e_7) = 0$.

Also the induced face labels are

$g^{**}(f_1) = 1$ and $g^{**}(f_2) = 0$.

In view of the above defined labeling pattern we have

$e_f(0) + 1 = e_f(1) = 4$ and $f_g(0) = f_g(1) = 1$.

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n = 6$.

Case 3: $n > 6$.

Sub Case 3.1: $n \equiv 0, 1, 2 \pmod{4}$

$g(v_1) = 1, g(v_2) = 2, g(v_3) = 4, g(v_4) = 3$ and $g(v_{i+4}) = g(v_i) + 4$, for $1 \leq i \leq n - 4$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0, g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \leq i \leq n - 7$.

$g^*(e_{2i+n-4}) = 1$ and $g^*(e_{2i+n-3}) = 0$, for $1 \leq i \leq \frac{n-2}{2}$, if $n \equiv 0, 2 \pmod{4}$.

$g^*(e_{2i+n-4}) = 1$, for $1 \leq i \leq \frac{n-1}{2}$ and $g^*(e_{2i+n-3}) = 0$, for $1 \leq i \leq \frac{n-3}{2}$, if $n \equiv 1 \pmod{4}$.

Also the induced face labels are

$g^{**}(f_{2i-1}) = 1$ and $g^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-4}{2}$, if $n \equiv 0, 2 \pmod{4}$.

$g^{**}(f_{2i-1}) = 1$, for $1 \leq i \leq \frac{n-3}{2}$ and $g^{**}(f_{2i}) = 0$, for $1 \leq i \leq \frac{n-5}{2}$, if $n \equiv 1 \pmod{4}$.

In view of the above defined labeling pattern we have

$e_f(0) + 1 = e_f(1) = n - 2$ and $f_g(0) + 1 = f_g(1) = \frac{n-3}{2}$, if $n \equiv 1 \pmod{4}$.

$e_f(0) + 1 = e_f(1) = n - 2$ and $f_g(0) = f_g(1) = \frac{n-4}{2}$, if $n \equiv 2 \pmod{4}$.

$e_f(0) = e_f(1) + 1 = n - 2$ and $f_g(0) = f_g(1) = \frac{n-4}{2}$, if $n \equiv 0 \pmod{4}$.

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$,

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n \equiv 0, 1, 2 \pmod{4}$.

Sub Case 3.2: $n \equiv 3 \pmod{4}$

$g(v_1) = 1, g(v_2) = 2, g(v_3) = 4, g(v_4) = 3, g(v_{i+4}) = g(v_i) + 4$, for $2 \leq i \leq n - 5$ and $g(v_n) = n$.

Then induced edge labels are

$g^*(e_1) = g^*(e_4) = 0, g^*(e_2) = g^*(e_3) = 1$ and $g^*(e_{i+4}) = g^*(e_i)$, for $1 \leq i \leq n - 7$.

$g^*(e_{2i+n-3}) = 1$ and $g^*(e_{2i+n-2}) = 0$, for $1 \leq i \leq \frac{n-3}{2}$. $g^*(e_{2n-5}) = 0$.

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n-3}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-5}{2}.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1)+1 = n-2$ and $f_g(0)+1 = f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n \equiv 3(\text{mod } 4)$.

Hence switching of any one vertex in cycle C_n is face sum divisor cordial graph for $n \geq 5$.

Example 3.1. Switching of a vertex v_1 in cycle C_8 and its face sum divisor cordial labeling is shown in figure 3.1.

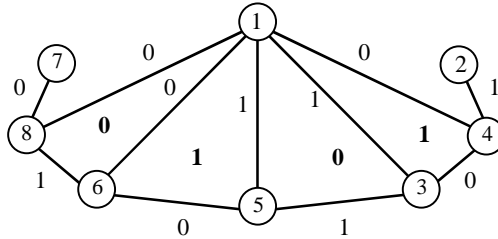


Figure 3.1:

Theorem 3.2. Switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \geq 4$.

Proof: Let v_1, v_2, \dots, v_n be the vertices of path P_n . v_1 and v_n are pendent vertex of path P_n . Without loss of generality, let the switched vertex be v_1 . The graph G is obtained by switching of a pendent vertex v_1 in path P_n .

The v_1, v_2, \dots, v_n are vertices, $e_1, e_2, \dots, e_{2n-4}$ are edges and f_1, f_2, \dots, f_{n-3} are the interior faces of G . $e_i = v_1 v_{i+2}$, for $1 \leq i \leq n-2$, $e_{n-2+i} = v_{i+1} v_{i+2}$, for $1 \leq i \leq n-2$ and $f_i = v_1 v_{i+2} v_{i+3} v_1$ for $1 \leq i \leq n-4$. Then $|V(G)| = n$, $|E(G)| = 2n-4$ and $|F(G)| = n-3$.

Define $g : V(G) \rightarrow \{1, 2, 3, \dots, n\}$ as follows

Case 1: $n = 4$.

$$g(v_1) = 1, \quad g(v_2) = 2, \quad g(v_3) = 4 \text{ and } g(v_4) = 3.$$

Then induced edge labels are

$$g^*(e_1) = 0, \quad g^*(e_2) = 1, \quad g^*(e_3) = 1 \text{ and } g^*(e_4) = 0.$$

Also the induced face label is

$$g^{**}(f_1) = 1.$$

In view of the above defined labeling pattern we have

$$e_f(0) = e_f(1) = 2 \text{ and } f_g(0)+1 = f_g(1) = 1.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n = 4$.

Case 2: $n = 5$.

$$g(v_1) = 5, \quad g(v_2) = 1, \quad g(v_3) = 3, \quad g(v_4) = 2 \text{ and } g(v_5) = 4.$$

Then induced edge labels are

$$g^*(e_1) = 1, \quad g^*(e_2) = 0, \quad g^*(e_3) = 0, \quad g^*(e_4) = 1, \quad g^*(e_5) = 0 \text{ and } g^*(e_6) = 1.$$

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Also the induced face labels are

$$g^{**}(f_1) = 1 \text{ and } g^{**}(f_2) = 0.$$

In view of the above defined labeling pattern we have

$$e_f(0) = e_f(1) = 3 \text{ and } f_g(0) = f_g(1) = 1.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n = 5$.

Case 3: $n > 5$.

Sub Case 3.1: $n \equiv 0(\text{mod } 4)$

$$g(v_1) = 1, g(v_2) = 2, g(v_3) = 4, g(v_4) = 3 \text{ and } g(v_{i+4}) = g(v_i)+4, \text{ for } 1 \leq i \leq n-4.$$

Then induced edge labels are

$$g^*(e_1) = g^*(e_4) = 0, g^*(e_2) = g^*(e_3) = 1 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-6.$$

$$g^*(e_{2i+n-3}) = 1 \text{ and } g^*(e_{2i+n-2}) = 0, \text{ for } 1 \leq i \leq \frac{n-2}{2}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n-2}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-4}{2}.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n-2$ and $f_g(0)+1 = f_g(1) = \frac{n-2}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 0(\text{mod } 4)$.

Sub Case 3.2: $n \equiv 1(\text{mod } 4)$

$$g(v_1) = n, g(v_2) = 1, g(v_3) = 3, g(v_4) = 2, g(v_5) = 4 \text{ and } g(v_{i+4}) = g(v_i)+4, \text{ for } 2 \leq i \leq n-4.$$

Then induced edge labels are

$$g^*(e_1) = g^*(e_4) = 1, g^*(e_2) = g^*(e_3) = 0 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-6.$$

$$g^*(e_{2i+n-3}) = 1, \text{ for } 1 \leq i \leq \frac{n-1}{2} \text{ and } g^*(e_{2i+n-2}) = 0, \text{ for } 1 \leq i \leq \frac{n-3}{2}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n-3}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-3}{2}.$$

In view of the above defined labeling pattern we have $e_f(0) = e_f(1) = n-2$ and $f_g(0) = f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 1(\text{mod } 4)$.

Sub Case 3.3: $n \equiv 2(\text{mod } 4)$

$$g(v_1) = n, g(v_2) = 1, g(v_3) = 2, g(v_4) = 4, g(v_5) = 3, g(v_6) = 5 \text{ and } g(v_{i+4}) = g(v_i)+4, \text{ for } 3 \leq i \leq n-4.$$

Then induced edge labels are

$$g^*(e_1) = g^*(e_2) = 1, g^*(e_3) = g^*(e_4) = 0 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-6.$$

$$g^*(e_{2i+n-3}) = 0, \text{ for } 1 \leq i \leq \frac{n-2}{2} \text{ and } g^*(e_{2i+n-2}) = 1, \text{ for } 1 \leq i \leq \frac{n-2}{2}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n-2}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-4}{2}.$$

In view of the above defined labeling pattern we have $e_t(0) = e_t(1) = n-2$ and $f_g(0)+1 = f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 2(\pmod 4)$.

Sub Case 3.4: $n \equiv 3(\pmod 4)$

$g(v_1) = n-1, g(v_2) = 1, g(v_3) = 3, g(v_4) = 2, g(v_5) = 4, g(v_6) = 5, g(v_7) = 7$ and $g(v_{i+4}) = g(v_i)+4$, for $4 \leq i \leq n-4$.

Then induced edge labels are

$$g^*(e_1) = g^*(e_4) = 0, g^*(e_2) = g^*(e_3) = 1 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-6.$$

$$g^*(e_{2i+n-3}) = 1, \text{ for } 1 \leq i \leq \frac{n-1}{2} \text{ and } g^*(e_{2i+n-2}) = 0, \text{ for } 1 \leq i \leq \frac{n-3}{2}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 0, \text{ for } 1 \leq i \leq \frac{n-3}{2} \text{ and } g^{**}(f_{2i}) = 1, \text{ for } 1 \leq i \leq \frac{n-3}{2}.$$

In view of the above defined labeling pattern we have $e_t(0) = e_t(1) = n-2$ and $f_g(0) = f_g(1) = \frac{n-3}{2}$. Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus switching of a pendent vertex in path P_n is face sum divisor cordial graph for $n \equiv 3(\pmod 4)$.

Therefore switching of a pendent vertex in path P_n is face sum divisor graph for $n \geq 4$.

Example 3.2. Switching of a pendent vertex of path P_6 and its face sum divisor cordial labeling is shown in figure 3.2.

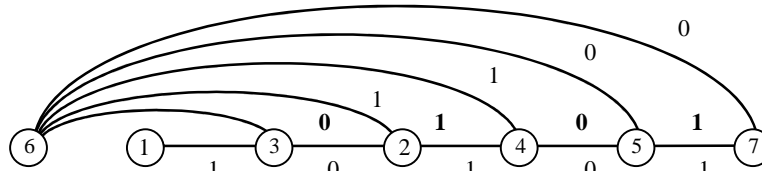


Figure 3.2:

Theorem 3.3. The graph $S'(K_{1,n})$ is face sum divisor cordial graph for $n \geq 2$.

Proof: Let v, v_1, \dots, v_n be the vertices of $K_{1,n}$. Let $G = S'(K_{1,n})$. Then $v, v_1, \dots, v_n, v', v'_1, \dots, v'_n$ are the vertices, e_1, e_2, \dots, e_{3n} are the edges and f_1, f_2, \dots, f_{n-1} are the interior faces of G , where $e_i = v'v_i, e_{n+i} = v_i v$ and $e_{2n+i} = v v'_i$ for $1 \leq i \leq n$ and $f_i = v'v_i v v_{i+1} v'$ for $1 \leq i \leq n-1$. Then $|V(G)| = 2n+2, |E(G)| = 3n$ and $|F(G)| = n-3$.

Case 1: $n = 2$.

$$g(v') = 1, g(v) = 2, g(v_1) = 3, g(v_2) = 4, g(v'_1) = 5 \text{ and } g(v'_2) = 6.$$

Then induced edge labels are

$$g^*(e_1) = g^*(e_4) = g^*(e_6) = 1 \text{ and } g^*(e_2) = g^*(e_3) = g^*(e_5) = 0.$$

Also the induced face label is

$$g^{**}(f_1) = 1.$$

In view of the above defined labeling pattern we have

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$$e_f(0) = e_f(1) = 3 \text{ and } f_g(0)+1 = f_g(1) = 1.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n = 2$.

Case 2: $n = 3$.

$$g(v') = 1, g(v) = 2, g(v_1) = 3, g(v_2) = 5, g(v_3) = 4 \text{ and } g(v'_i) = n+2+i, \text{ for } 1 \leq i \leq 3.$$

Then induced edge labels are

$$g^*(e_1) = g^*(e_2) = g^*(e_6) = g^*(e_7) = g^*(e_9) = 1 \text{ and } g^*(e_3) = g^*(e_4) = g^*(e_5) = g^*(e_8) = 0.$$

Also the induced face labels are

$$g^{**}(f_1) = 0 \text{ and } g^{**}(f_2) = 1.$$

In view of the above defined labeling pattern we have

$$e_f(0) + 1 = e_f(1) = 5 \text{ and } f_g(0) = f_g(1) = 1.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n = 3$.

Case 3: $n \geq 4$.

Sub Case 3.1: $n \equiv 0, 1, 2 \pmod{4}$

$g(v') = 1, g(v) = 2, g(v_1) = 3, g(v_2) = 4, g(v_3) = 6, g(v_4) = 5, g(v_{i+4}) = g(v_i) + 4$, for $1 \leq i \leq n-4$ and $g(v'_i) = n+2+i$, for $1 \leq i \leq n$.

Then induced edge labels are

$$g^*(e_1) = g^*(e_4) = 0, g^*(e_2) = g^*(e_3) = 1 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-4.$$

$$g^*(e_{n+1}) = g^*(e_{n+4}) = 1, g^*(e_{n+2}) = g^*(e_{n+3}) = 0 \text{ and } g^*(e_{n+4+i}) = g^*(e_{n+i}), \text{ for } 1 \leq i \leq n-4.$$

$$g^*(e_{2n+2i-1}) = 0, \text{ for } 1 \leq i \leq \frac{n}{2} \text{ and } g^*(e_{2n+2i}) = 1, \text{ for } 1 \leq i \leq \frac{n}{2}, \text{ if } n \equiv 0, 2 \pmod{4}.$$

$$g^*(e_{2n+2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n+1}{2} \text{ and } g^*(e_{2n+2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-1}{2}, \text{ if } n \equiv 1 \pmod{4}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-2}{2}, \text{ if } n \equiv 0, 2 \pmod{4}.$$

$$g^{**}(f_{2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n-1}{2} \text{ and } g^{**}(f_{2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-1}{2}, \text{ if } n \equiv 1 \pmod{4}.$$

In view of the above defined labeling pattern we have

$$e_f(0) = e_f(1) = \frac{3n}{2} \text{ and } f_g(0)+1 = f_g(1) = \frac{n}{2}, \text{ if } n \equiv 0, 2 \pmod{4}.$$

$$e_f(0)+1 = e_f(1) = \frac{3n+1}{2} \text{ and } f_g(0) = f_g(1) = \frac{n-1}{2}, \text{ if } n \equiv 1 \pmod{4}.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n \equiv 0, 1, 2 \pmod{4}$.

Sub Case 3.2: $n \equiv 3 \pmod{4}$

$g(v') = 1, g(v) = 2, g(v_1) = 3, g(v_2) = 5, g(v_3) = 4, g(v_4) = 6, g(v_{i+4}) = g(v_i) + 4$, for $1 \leq i \leq n-4$ and $g(v'_i) = n+2+i$, for $1 \leq i \leq n$.

Then induced edge labels are

$$g^*(e_1) = g^*(e_2) = 1, g^*(e_3) = g^*(e_4) = 0 \text{ and } g^*(e_{i+4}) = g^*(e_i), \text{ for } 1 \leq i \leq n-4.$$

$$g^*(e_{n+1}) = g^*(e_{n+2}) = 0, g^*(e_{n+3}) = g^*(e_{n+4}) = 1 \text{ and } g^*(e_{n+4+i}) = g^*(e_{n+i}), \text{ for } 1 \leq i \leq n-4.$$

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$$g^*(e_{2n+2i-1}) = 1, \text{ for } 1 \leq i \leq \frac{n+1}{2} \text{ and } g^*(e_{2n+2i}) = 0, \text{ for } 1 \leq i \leq \frac{n-1}{2}.$$

Also the induced face labels are

$$g^{**}(f_{2i-1}) = 0 \text{ and } g^{**}(f_{2i}) = 1, \text{ for } 1 \leq i \leq \frac{n-1}{2}.$$

In view of the above defined labeling pattern we have

$$e_f(0)+1 = e_f(1) = \frac{3n+1}{2} \text{ and } f_g(0) = f_g(1) = \frac{n-1}{2}.$$

Then $|e_g(0) - e_g(1)| \leq 1$ and $|f_g(0) - f_g(1)| \leq 1$.

Thus $S'(K_{1,n})$ is face sum divisor cordial graph for $n \equiv 3 \pmod{4}$.

Hence the graph $S'(K_{1,n})$ is face integer edge cordial graph for $n \geq 2$.

Example 3.3. The graph $S'(K_{1,4})$ and its face sum divisor cordial labeling is shown in figure 3.3.

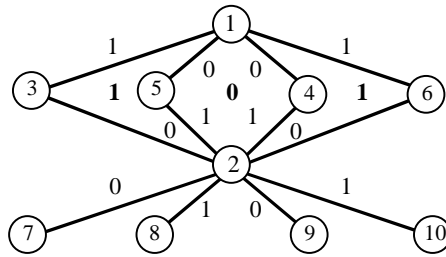


Figure 3.3:

4. Conclusions

In this paper, we presented the face sum divisor cordial labeling of switching of any vertex in cycle C_n , switching of a pendent vertex in path P_n and $S'(K_{1,n})$.

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