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Ranking of Students for Admission Process by Using Choquet Integral

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Abstract. Marks of the students in different subject are aggregated with respect to weights of the subjects. Weight of a subject indicates the relative importance of the subject. A λ −fuzzy measure is evaluated to obtain the index. Comparing indices we rank the students. The SCILAB programming is used to calculate fuzzy integral.

Keywords: Fuzzy measure, Choquet integral

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

The fuzzy set theory was introduced by Zadeh in 1965 in his paper 'Fuzzy Sets' [1]. In classical set theory crisp sets are defined by characteristic values either zero or one. But in Fuzzy set theory, these functions are replaced by membership functions [5]. Crisp set only give the quantitative information while the fuzzy set gives the qualitative information of data under consideration. Here the main focus is on the attribute of the data.

Fuzzy systems are found to be useful in dealing with the uncertainties and vague concepts. Sometimes decisions are to be made whenever there is insufficient or ambiguous information. In such situations fuzzy systems can be helpful to make good decisions [9-13].

The measure and integral w.r.t. measure are important concepts in Mathematics. The fuzzy measures which are non-additivehave been introduced by Sugeno in 1974 [8]. These non-additive measures and integrals generalize the traditional probability theory. Further it is observed that the fuzzy integral models derived from non-additive fuzzy measures have convincing advantage in decision theory as an aggregation operator [4].

The present paper gives the application of λ −fuzzy measure and Choquet integral to rank the student in admission process and also gives the comparison between Sugeno and Choquet integral. Here SCILAB programme is used to calculate λ −fuzzy measure and Choquet integral.

2. Basic terminology

Fuzzy Measure: A fuzzy measure μ on Θ is a function $\mu: 2^{\Theta} \longrightarrow [0,1]$ satisfying the following axioms

1. $\mu(\phi) = 0, \mu(\Theta) = 1$ (Boundary Condition)
2. $\theta_1 \subseteq \theta_2 \Longrightarrow \mu(\theta_1) \subseteq \mu(\theta_2)$ (Monotonicity)

The main characteristic of fuzzy measure is non-additivity, so fuzzy measures are also called as non-additive measure [2].

Sugeno's $\boldsymbol{\lambda}$ −fuzzy measure: Let $\lambda \in (-1, \infty)$. A normalized set function g_λ defined on 2^{Θ} is called as λ −fuzzy measure on Θ if for every pair of disjoint subsets $\theta_1$and θ_2 of Θ we have

$$g_\lambda(\theta_1 \cup \theta_2) = g_\lambda(\theta_1) + g_\lambda(\theta_2) + \lambda\, g_\lambda(\theta_1).\ g_\lambda(\theta_2)$$

Obviously if$\lambda = 0$, then a λ-fuzzy measure is a normalized additive measure, i.e. probability measure. A Dirac measure is a λ-fuzzy measure for all $\lambda > -1$. This is the monotone measure [3]. By following theorem the parameterλ is calculated.

Theorem 2.1.1. Let Θ be the finite set, $\Theta = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ and 2^{Θ} be the class of all subsets of Θ, the fuzzy measure $g_\lambda(\Theta) = g_\lambda(\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\})$ can be formulated as

$$g_\lambda(\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}) = \frac{1}{\lambda}\left[\prod_{i=1}^{n}[1 + \lambda\, g_\lambda(\{\varepsilon_i\})] - 1\right] \text{ where} \lambda \in (-1, \infty)$$

As $g_\lambda(\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}) = 1$ the formula becomes$\lambda + 1 = \prod_{i=1}^{n}[1 + \lambda\, g_\lambda(\{\varepsilon_i\})]$.
Here the value of λ has three cases.

i) If$\sum_{i=1}^{n} g_\lambda(\{\varepsilon_i\}) > g_\lambda(\Theta)$then $-1 < \lambda < 0$ ($\lambda -$ measure is subadditive).
ii) If $\sum_{i=1}^{n} g_\lambda(\{\varepsilon_i\}) = g_\lambda(\Theta)$then $\lambda = 0$ ($\lambda -$ measure is additive).
iii) If $\sum_{i=1}^{n} g_\lambda(\{\varepsilon_i\}) < g_\lambda(\Theta)$then $\lambda > 0$ ($\lambda -$ measure is superadditive)[3].

Choquet Integral: Let f be a nonnegative measurable function on $(\Theta, \mathfrak{B})$. The Choquet integral of f with respect to g_λ is denoted by $\mathfrak{C}_{g_\lambda}(f) = \sum_{i=1}^{n}(f(\varepsilon_i) - f(\varepsilon_{i-1}))\, g_\lambda(\theta_i)$ where
$\theta_i = \{\varepsilon_i, \varepsilon_{i+1}, \dots, \varepsilon_n\}, f(\varepsilon_0) = 0$ and $(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n)$ is a numbering of the elements of Θ satisfying the condition that $f(\varepsilon_1) \le f(\varepsilon_2) \dots \le f(\varepsilon_n)$.

Sugeno integral: Let g_λ be a normalized fuzzy measure on Θ and f be a function on $(\Theta, \mathfrak{B})$ with range $\{f(\varepsilon_1), f(\varepsilon_2) \dots, f(\varepsilon_n)\}$ where $0 \le f(\varepsilon_1) \le f(\varepsilon_2) \dots \le f(\varepsilon_n) \le 1$. The Sugeno integral $\mathfrak{S}_{g_\lambda}(f)$ with respect to measure g_λ is defined as $\mathfrak{S}_{g_\lambda}(f) = \bigvee_{i=1}^{n}[f(\varepsilon_i) \wedge g_\lambda(\theta_i)]$, where $\theta_i = \{\varepsilon_i, \varepsilon_{i+1}, \dots, \varepsilon_n\}$.

Aggregation of data by using fuzzy integral: Information fusion is a broad area that studies methods to combine data or information supplied by multiple sources. Aggregation is one of such process which is used in data analysis to obtain a single value from a set of values [3,4,6]. For this purpose the fuzzy integrals like Choquet integral, Sugeno integral etc. can be used as aggregation operators [9]. In decision theory we have to obtain aggregation of the preference values or satisfaction degrees.

Common aggregation operators like arithmetic mean, weighted mean, median, mode etc. have some drawbacks because they only express the quantitative approach. But to express the qualitative approach like relation between criteria, decision making etc. we need fuzzy integrals [7]. These integrals help in fusion of information and data mining effectively. Here we only consider the Choquet integral and Sugeno integral which are discussed in 2.3 and 2.4.

Let Θ be the finite set, $\Theta = \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n\}$ and 2^Θ be the class of all subsets of Θ.Consider $g_\lambda: 2^\Theta \longrightarrow [0,1]$ and $f: \Theta \longrightarrow \mathbb{R}$. Here g_λ indicates relative importance of the elements then $\mathfrak{C}_{g_\lambda}(f)$ and $\mathfrak{S}_{g_\lambda}(f)$ are the aggregation of functional values of f with respect to fuzzy measure g_λ.

3. SCILAB program to calculate Choquet and Sugeno integral

SCILAB is free software. It is helpful to solve any mathematical problem. Here we created a SCILAB program to calculate Choquet and Sugeno integral. It is helped in computing very complicated problems.

3.1. Algorithm to find g_λ measure

1. Start
2. Input the value of $g_\lambda(\{\varepsilon_i\})$.
3. Find the polynomial in λ by using the value of$g_\lambda(\{\varepsilon_i\})$.
4. Find the roots of the polynomial inλ.
5. If $\lambda \in (-1, \infty)$ then proceed else stop.
6. Let $\lambda \in (-1, \infty)$.If $\lambda = 0$ then print additive measure and stop else calculateg_λ for all various combinations.
7. Stop.

Example: Let $\theta = \{\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4\}$, $g_\lambda(\{\varepsilon_1\}) = 0.11$, $g_\lambda(\{\varepsilon_2\}) = 0.07$, $g_\lambda(\{\varepsilon_3\}) = 0.03$, $g_\lambda(\{\varepsilon_4\}) = 0.05$. And the evaluation scores are $f(\varepsilon_1) = 0.1$, $f(\varepsilon_2) = 0.2$, $f(\varepsilon_3) = 0.2$, $f(\varepsilon_4) = 0.3$. Here we have to calculate Choquet integral and Sugeno integral.

For this we consider the SCILAB programing and its output:

```
-->// To find the lambda measure//
-->// here all A(1,j),1<=j<=4 are f(i)for 1<=i<=4//
-->A=[0.1 0.2 0.2 0.3 ;0.11 0.07 0.03 0.05]
A  =
   0.1     0.2     0.2     0.3
   0.11    0.07    0.03    0.05
-->c(1)=A(2,1);c(2)=A(2,2);c(3)=A(2,3);c(4)=A(2,4);//these are lambda measures which are given//
-->x=poly(0,'x')
x  =
x
-->p= (c(1)*x+1)*(c(2)*x+1)*(c(3)*x+1)*(c(4)*x+1)-x-1
p  =
 2          3           4
  - 0.74x + 0.0236x + 0.000886x + 0.0000116x
-->lambda=roots(p)
```

```
lambda  =
    0
    17.404142
  - 47.05705 + 38.300122i
  - 47.05705 - 38.300122i
-->l=17.404142 // here take lambda=l, we choose this value because this value lies in
between -1 to infinity//
l  =
    17.404142
-->n=4;
-->s=1;
-->// lambda measure is p for all criteria//
-->for i=1:1:n
-->      s=s*(1+l*c(i));
-->end
-->p=(1/l)*(s-1);
-->disp(p)
    1.0000000
-->// lambda measure only for two criteria//
-->for i=1:1:n-1
-->for j=i+1:1:n
-->            f=1;
-->      f=f*(1+l*c(i))*(1+l*c(j));
-->g(i,j)=(1/l)*(f-1);
-->disp(g(i,j))
-->end
-->end
    0.3140119
    0.1974337
    0.2557228
    0.1365487
    0.1809145
    0.1061062
-->// lambda measure only for three criteria//
-->for i=1:1:n-2
-->for j=i+1:1:n-1
-->for k=j+1:1:n
-->            f=1;
-->      f=f*(1+l*c(i))*(1+l*c(j))*(1+l*c(k));
-->g(i,j,k)=(1/l)*(f-1);
-->end
-->end
-->end

-->disp(g(1,2,3)),disp(g(1,2,4)),disp(g(1,3,4))
    0.5079651
```

```
   0.6372673
   0.4192418

-->disp(g(2,3,4))
   0.3053743
-->// Choquet integration is CI and Sugeno integration is SI//
-->CI=A(1,1)*p+(A(1,2)-A(1,1))*g(2,3,4)+(A(1,3)-A(1,2))*g(3,4)+(A(1,4)-A(1,3))*c(4)
CI  = 0.1355374

-->SI=max(min(A(1,1),p),min(A(1,2),g(2,3,4)),min(A(1,3),g(3,4)),min(A(1,4),c(4)))
SI  =0.2
```

Table 1: The Interdependencies measures which are obtained in SCILAB Programming

λ-measure	$g_\lambda(\{\varepsilon_1,\varepsilon_2\})$	$g_\lambda(\{\varepsilon_1,\varepsilon_3\})$	$g_\lambda(\{\varepsilon_1,\varepsilon_4\})$	$g_\lambda(\{\varepsilon_2,\varepsilon_3\})$	$g_\lambda(\{\varepsilon_2,\varepsilon_4\})$
Value	0.3140119	0.1974337	0.2557228	0.1365487	0.1809145
λ-measure	$g_\lambda(\{\varepsilon_1,\varepsilon_3\})$	$g_\lambda(\{\varepsilon_1,\varepsilon_2,\varepsilon_3\})$	$g_\lambda(\{\varepsilon_1,\varepsilon_2,\varepsilon_4\})$	$g_\lambda(\{\varepsilon_1,\varepsilon_3,\varepsilon_4\})$	$g_\lambda(\{\varepsilon_2,\varepsilon_3,\varepsilon_4\})$
Value	0.1061062	0.5079651	0.6372673	0.4192418	0.3053743

By SCILAB programming we get all theλ-measures in table 1 and the Choquet integral =0.1355374 and the Sugeno integral = 0.2.

4. Case study

In admission process to any stream it is difficult to rank the student because the seats are limited. Normally the admissions are given on the basis of performance of student in previous examination or on the basis of entrance test. But it could not give proper justice to student because intelligence quotient, subject linking, responsibility etc. varies student to student and subject to subject. Fuzzy measures and integrals are appropriate tools to collect the information.

4.1. Ranking according to fuzzy integral

To decide the rank of student for admission to M.Sc. in Mathematics, we use Choquet integral. Here, the students of Third year B.Sc. are evaluated according to their marks in Algebra, Analysis, Differential Equation, Complex Analysis, Numerical Analysis and practical examination.

Here, the departmental committee gives the equal importance to algebra and analysis and less importance to all other subjects. Consider the grades of importance i.e. λ- measure of the different subjects.

$$g_\lambda(x_1) = g_\lambda(\{\text{Algebra}\}) = 0.8$$

$g_\lambda(x_2) = g_\lambda(\{\text{Analysis}\}) = 0.8$
$g_\lambda(x_3) = g_\lambda(\{\text{Differential Equation}\}) = 0.5$
$g_\lambda(x_4) = g_\lambda(\{\text{Complex Analysis}\}) = 0.7$
$g_\lambda(x_5) = g_\lambda(\{\text{Numerical Analysis}\}) = 0.5$
$g_\lambda(x_6) = g_\lambda(\{\text{Practical}\}) = 0.4$

Let $\{J_1, J_2, \dots, J_{10}\}$ be the set of 10 students. The marks of the different subjectsof each student in a scale 0 to 50 are given in table 2.

Table 2: Subject wise students marks

Subjects ⇒ **Students** ⇓	x_1	x_2	x_3	x_4	x_5	x_6
	Algebra	Analysis	Differential Equation	Complex Analysis	Numerical Analysis	Practical
J_1	45	40	48	45	30	40
J_2	49	40	41	47	49	45
J_3	48	48	49	50	46	49
J_4	42	44	46	48	43	48
J_5	35	37	38	40	49	49
J_6	48	48	48	48	49	49
J_7	48	50	35	40	43	43
J_8	38	38	40	48	48	49
J_9	45	46	30	33	30	45
J_{10}	43	44	41	34	42	47

By using SCILAB programming Sugeno's λ-measure is computed. Here we get sixth degree equation as

$$2.7\lambda + 5.63\lambda^2 + 4.507\lambda^3 + 2.0012\lambda^4 + 0.4672\lambda^5 + 0.0448\lambda^6 = 0 \qquad (1)$$

Solving this equation we get six roots as {0 ,- 0.9981565 ,- 1.2942895 + 1.7393075i, - 1.2942895 - 1.7393075i ,- 3.420918 + 1.0690765i , - 3.420918 - 1.0690765i }. Among these four roots are complex, we reject these roots. Thus the roots 0 and - 0.9981565 are only under consideration. If $\lambda = 0$ then the measure is additive measure. Hence we only take$\lambda = -0.9981565$. As there are six subjects, it is necessary to define $2^6 = 64$ subsets of subjects.We have λ-measure for six sets as mentioned earlier. Again, λ-measure for empty set is zero and λ-measure for whole set is 1.

By using SCILAB programming we calculate the other values and also calculate the Choquet integral for each student to rank them. All possible λ-measure are given in the table 3 and aggregate values by using Choquet integration are mentioned in table 4.

Table 3: The Interdependencies measures among two or more subjects

Between two subjects	**λ-measure**	**Among three subjects**	**λ-measure**
x_1,x_2	0.9611798	x_2,x_4,x_5	0.9713836
x_1,x_3	0.9007374	x_2,x_4,x_6	0.9653133
x_1,x_4	0.9410324	x_2,x_5,x_6	0.9411066
x_1,x_5	0.9007374	x_3,x_4,x_5	0.9261067
x_1,x_6	0.8805899	x_3,x_4,x_6	0.9110144
x_2,x_3	0.9007374	x_3,x_5,x_6	0.8508299
x_2,x_4	0.9410324	x_4,x_5,x_6	0.9110144
x_2,x_5	0.9007374	**Among four subjects**	**λ-measure**
x_2,x_6	0.8805899	x_1,x_2,x_3,x_4	0.9957093
x_3,x_4	0.8506452	x_1,x_2,x_3,x_5	0.9916426
x_3,x_5	0.7504609	x_1,x_2,x_3,x_6	0.9896093
x_3,x_6	0.7003687	x_1,x_2,x_4,x_5	0.9957093
x_4,x_5	0.8506452	x_1,x_2,x_4,x_6	0.9944863
x_4,x_6	0.8205162	x_1,x_2,x_5,x_6	0.9896093
x_5,x_6	0.7003687	x_1,x_3,x_4,x_5	0.9865872
Among three subjects	**λ-measure**	x_1,x_3,x_4,x_6	0.9835464
x_1,x_2,x_3	0.9814759	x_1,x_3,x_5,x_6	0.9714208
x_1,x_2,x_4	0.9895943	x_1,x_4,x_5,x_6	0.9835464
x_1,x_2,x_5	0.9814759	x_2,x_3,x_4,x_5	0.9865872
x_1,x_2,x_6	0.9774167	x_2,x_3,x_4,x_6	0.9835464
x_1,x_3,x_4	0.9713836	x_2,x_3,x_5,x_6	0.9714208
x_1,x_3,x_5	0.9511990	x_2,x_4,x_5,x_6	0.9835464
x_1,x_3,x_6	0.9411066	x_3,x_4,x_5,x_6	0.9563469
x_1,x_4,x_5	0.9713836	**Among five subjects**	**λ-measure**
x_1,x_4,x_6	0.9653133	x_1,x_2,x_3,x_4,x_5	0.9987725
x_1,x_5,x_6	0.9411066	x_1,x_2,x_3,x_4,x_6	0.9981598
x_2,x_3,x_4	0.9713836	x_1,x_2,x_3,x_5,x_6	0.9957168
x_2,x_3,x_5	0.9511990	x_1,x_2,x_4,x_5,x_6	0.9981598
x_2,x_3,x_6	0.9411066	x_1,x_3,x_4,x_5,x_6	0.9926798
		x_2,x_3,x_4,x_5,x_6	0.9926798

Table 4: Calculated Choquet integral (C.I.) for each student.

Student	J_1	J_2	J_3	J_4	J_5
C.I.	46.338515	46.869631	49.607332	47.439286	46.067052
Student	J_6	J_7	J_8	J_9	J_{10}
C.I.	48.700367	49.365525	47.600808	45.512458	45.917632

Here, by sorting Choquet integral values we get ranking as
$\mathbf{J_9 \prec J_{10} \prec J_5 \prec J_1 \prec J_2 \prec J_4 \prec J_8 \prec J_6 \prec J_7 \prec J_3}$.
It is observed that the Choquet integral is useful for calculating indices for each student than that of Sugeno integral because marks of the students are in between 0 and 50. In case of Sugeno integral, the functional values are in between 0 and 1.

5. Conclusion

This paper presents the calculation of λ-fuzzy measure, Choquet integral and Sugeno integral by SCILAB programming. The case study shows that the marks of students are aggregated with respect to weight of the subject. Here relative indices by using Choquet integral are obtained to rank the students in admission process. As weight of the subject indicates the relative importance of the subject, our ranking shows that the student who has the more index value is good at that subject and should be admitted firstly for the M.Sc. Course.

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