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Optimal Solution of a Mixed type Fuzzy Transportation Problem

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Abstract. In this paper, we solved transportation problem when the data are in mixed type. Transportation problem contains fuzzy numbers, intuitionistic fuzzy numbers and real numbers. The purpose of our paper is to find the least transportation cost in which cost, supply and demand are mixed fuzzy numbers. This procedure is illustrated with numerical example.

Keywords: Real numbers, hexagonal fuzzy numbers, trapezoidal fuzzy numbers, intuitionistic fuzzy numbers, ranking functions, optimal solution.

AMS Mathematics Subject Classification (2010): 90C08

1. Introduction

In today's highly competitive market, we have to find better way in transportation problem to minimize the transportation cost. Optimization methods and algorithms have lately become very valuable tools to optimize the problem. Due to uncertainty in real life we have introduced fuzzy numbers in transportation problem. There may be such a situation in which we have to take variety of fuzzy numbers in transportation problem.

The idea of fuzzy sets was introduced by Zadeh [8]. Klir [3] apply his logic in fuzzy sets and fuzzy logic. Ahmed et al. [5] develop a new algorithm for finding an initial basic feasible solution of transportation problem when transportation matix contains fuzzy numbers and real numbers. Prabha et al. [7] worked on mixed intuitionistic fuzzy transportation problem and solved by best candidate method.

In section 2 basic definitions are discussed. In section 3 we discussed about algorithm. In section 4 we solved some numerical example using different ranking techniques. In section 5 conclusion is discussed.

2. Preliminaries

Definition 2.1. Fuzzy set. The characteristic function μ_A of a crisp set $A \subset X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$: X→ [0,1]. The assigned value indicate the membership function and the set A= $\left\{\left(x, \mu_{\tilde{A}}(x)\right); x \in X\right\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called fuzzy set.

Definition 2.2. Fuzzy number. A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number is a convex normalized fuzzy set on the real line R such that:

1) There exist at least one $x \in R\ with\ \mu_{\tilde{A}}(x) = 1$.
2) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.3. Trapezoidal fuzzy number. A fuzzy number $\tilde{A}$ = (a,b,c,d) is said to be trapezoidal fuzzy number if its membership function is given the following expression,

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0 & otherwise \end{cases}$$

Definition 2.4. Hexagonal fuzzy number. A fuzzy number A is said to be a hexagonal fuzzy number denoted by $A = (a_1, a_2, a_3, a_4, a_5, a_6)$, where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and it's membership function μ_A is given by:

$$\mu_A(x) = \begin{cases} \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)}, & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\frac{(x-a_4)}{(a_5-a_4)}, & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2}\frac{(a_6-x)}{(a_6-a_5)}, & \text{for } a_5 \leq x \leq a_6 \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.5. Intuitionistic fuzzy set. Let X denote universe of discover, then an intuitionistic fuzzy set $\bar{A}^I$ in X is given by $\bar{A}^I$=$\left\{x, \mu_{\bar{A}^I}(x), v_{\bar{A}^I}(x)/x\epsilon X\right\}$ where $\mu_{\bar{A}^I}(x)$, $v_{\bar{A}^I}(x)$: X→[0,1] are functions such that 0 ≤ $\mu_{\bar{A}^I}(x)$ + $v_{\bar{A}^I}(x)$ ≤1 for all $x\epsilon X$. For each x the membership function $\mu_{\bar{A}^I}(x)$ and $v_{\bar{A}^I}(x)$ represent the degree of membership and non-membership of each element $x\epsilon X$ to

A ⊂ X respectively.

Definition 2.6. Intuitionistic fuzzy number. An intuitionistic fuzzy subset $\bar{A}^I=\{x, \mu_{\bar{A}^I}(x), v_{\bar{A}^I}(x)/x\epsilon X\}$ of the real line R is called an intuitionistic fuzzy number if the following holds:

i) There exist m ∈ R, $\mu_{\bar{A}^I}(m)$ = 1 and $v_{\bar{A}^I}(m)$ = 0, m is called the mean value of $\bar{A}^I$.

ii) $\mu_{\bar{A}^I}$ is a continuous mapping from R to the closed interval [0,1] and for all $x\epsilon R$, the relation $0 \le \mu_{\bar{A}^I}(x) + v_{\bar{A}^I}(x) \le 1$ holds.

The membership and non-membership function of $\bar{A}^I$ is of the following form

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0 & -\infty < x \le m-\alpha \\ f_1(\mathrm{x}) & x\epsilon\ [m-\alpha, m] \\ 1 & x = \mathrm{m} \\ h_1(\mathrm{x}) & x\ \epsilon\ [m, m+\beta] \\ 0 & \mathrm{m}+\beta \le x \le \infty \end{cases}$$

$$v_{\bar{A}^I}(x) = \begin{cases} 1 & -\infty < x \le m-\alpha' \\ f_1(\mathrm{x}) & x\epsilon[m-\alpha', m],\ 0 \le f_1(\mathrm{x}) + f_2(\mathrm{x}) \le 1 \\ 0 & x = \mathrm{m} \\ h_1(\mathrm{x}) & x\ \epsilon[m, m+\beta'],\ 0 \le h_1(\mathrm{x}) + h_2(\mathrm{x}) \le 1 \\ 0 & \mathrm{m}+\beta' \le x \le \infty \end{cases}$$

Definition 2.7. Pentagonal intuitionistic fuzzy number. A pentagonal intuitionistic fuzzy number A^I of an intuitionistic fuzzy set is defined as,
A^I={$(a_1, b_1, c_1, d_1, e_1)$ $(a_2, b_2, c_2, d_2, e_2)$}, where all $a_1, b_1, c_1, d_1, e_1, a_2, b_2, c_2, d_2, e_2$ are real numbers and it's membership function $\mu_{\bar{A}^I}(x)$ and non-membership $v_{\bar{A}^I}(x)$ function is defined as,

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{(x-a_1)}{(b_1-a_1)} & \text{for } a_1 \le x \le b_1 \\ \frac{(x-b_1)}{(c_1-b_1)} & \text{for } b_1 \le x \le c_1 \\ 1 & \text{for } x = c_1 \\ \frac{(d_1-x)}{(d_1-c_1)} & \text{for } c_1 \le x \le d_1 \\ \frac{(e_1-x)}{(e_1-d_1)} & \text{for } d_1 \le x \le e_1 \\ 0 & \text{for } x > e_1 \end{cases}$$

$$v_{\bar{A}^I}(x) = \begin{cases} 1 & \text{for } x < a_1 \\ \frac{(b_2-x)}{(b_2-a_2)} & \text{for } a_2 \le x \le b_2 \\ \frac{(c_2-x)}{(c_2-b_2)} & \text{for } b_2 \le x \le c_2 \\ 0 & \text{for } x = c_1 \\ \frac{(x-c_2)}{(d_2-c_2)} & \text{for } c_2 \le x \le d_2 \\ \frac{(x-d_2)}{(e_2-d_2)} & \text{for } d_1 \le x \le e_1 \end{cases}$$

Ranking techniques

Definition 2.8. Ranking of trapezoidal fuzzy numbers [4]
The Ranking of a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is defined by,

$$R(\tilde{A}) = \frac{a_1+a_2+a_3+a_4}{4}$$

Definition 2.9. Ranking of hexagonal fuzzy numbers [2]
The Ranking of a hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ is defined by,

$$R(\tilde{A}) = \frac{a_1+a_2+a_3+a_4+a_5+a_6}{6}$$

Definition 2.10. Ranking of pentagonal Intuitionistic fuzzy numbers [1]
The Ranking of a pentagonal Intuitionistic fuzzy number
$\tilde{A} = (a_1, a_2, a_3, a_4, a_5)\ (a'_1, a'_2, a_3, a'_4, a'_5)$ is defined by,

$$R(\tilde{A}) = \frac{(a_1+a'_1)+(a_2+a'_2)+(a_3+a'_3)+(a_4+a'_4)+(a_5+a'_5)}{5}$$

3. Algorithm [6]

Step 1: Construct the mixed type fuzzy transportation table for a given fuzzy transportation problem.

Step 2: Using ranking function convert all fuzzy numbers into the crisp numbers.

Step 3: Check whether the transportation table is balanced or not, if not, make it balanced.

Step 4: After defuzzify the quantities of the problem, if any of values are not integers, round off into integers.

Step 5: Select the minimum odd cost from all cost in the matrix. Suppose all costs are even, multiply each column by 1/2.

Step 6: Subtract selected least odd cost only from odd cost in the matrix. Now there will be at least one zero and remaining all cost become even.

Step 7: Allocate minimum of supply/ demand at the place of zero.

Step 8: After the allotment, multiply each column by 1/2.

Step 9: Again select minimum odd cost in the remaining column except zeros in that column.

Step 10: Go to step 6 and repeat step 7and 8 till optimal solution are obtained.

Step 11: Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply /demand.

4. Numerical example

Consider a mixed type fuzzy transportation problem,

	D_1	D_2	D_3	Supply
O_1	4	(2,4,6,8,10,12)	(1,3,5,7,9; 0,2,5,8,10)	10
O_2	(1,1,1,1,1,1)	7	(2,7,9,10,15; 1,6,9,11,20)	(2,4,6,8,10,12)
O_3	5	(1,3,3,5)	(0,1,2,5)	(2,6,9,11,15; 1,4,9,13,20)
Demand	(2,3,4,7,9; 1,2,4,8,10)	(4,8,12,16,32,48)	5	

By defuzzifying the quantities we get,

	D_1	D_2	D_3	Supply
O_1	4	7	10	10
O_2	1	7	18	7
O_3	5	3	2	18
Demand	10	20	5	

Since the minimum odd cost in the odd matrix is 1, subtract 1 from all the odd costs and allocate minimum of supply or demand to the cell where there is zero cost then delete the row or column.

	D_1	D_2	D_3	Supply
O_1	4	6	10	10
O_2	**7** 0	6	18	7
O_3	4	2	2	18
Demand	10	20	5	

Now all the cost is even, hence multiply all the cost by ½ and subtract the minimum of odd cost from all the odd cost.

	D_1	D_2	D_3	Supply
O_1	2	3	5	10
O_3	2	1	1	18
Demand	3	20	5	

	D_1	D_2	D_3	Supply
O_1	2	2	4	10
O_3	2	0	**5** 0	18
Demand	3	20	5	

Proceeding like this, we get

	D_1	D_2	D_3	Supply
O_1	4	**10** 7	10	10
O_2	**7** 1	7	18	7
O_3	**3** 5	**10** 3	**5** 2	18
Demand	10	20	5	

The optimum cost is given by,
$10\times7+7\times1+3\times5+10\times3+5\times2 = 132$.

5. Conclusion

In this paper, cost, supply and demand of fuzzy transportation problem are considered as mixed fuzzy numbers, i.e hexagonal fuzzy numbers, trapezoidal fuzzy numbers, real numbers and intuitionistic pentagonal fuzzy numbers. By using different ranking functions the quantities are converted to crisp values and optimal solution is obtained by the given methodology. This method is very easy to solved.

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