

## Optimization of Fuzzy Inventory Model with Trended Deterioration and Salvage

N.K.Sahoo<sup>1</sup> and P.K.Tripathy<sup>2</sup>

<sup>1</sup>Department of Mathematics, S.S.S. Mahavidyalaya, Gaudakateni  
Dhenkanal-759001, India

<sup>2</sup>P.G. Department of Statistics, Utkal University, VaniVihar  
Bhubaneswar-751004, India. Email: [msccompsc@gmail.com](mailto:msccompsc@gmail.com)

<sup>1</sup>Corresponding author. Email: [narenmaths@yahoo.co.in](mailto:narenmaths@yahoo.co.in)

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**Abstract.** A fuzzy inventory model is developed with time dependent trended deterioration. Salvage value is incorporated to the cost of deteriorated items. The deterioration rate, holding cost, unit cost and salvage value are taken as trapezoidal fuzzy numbers. Both graded mean integration and signed distance method are used to defuzzify the total cost function. Mathematical model has been developed for determining the optimal order quantity, optimal cycle time and optimal total inventory cost. Numerical examples are given to validate the developed model. Sensitivity analysis is carried out to analyze the variability in the optimal solution with respect to change in various parameters.

**Keywords:** Deterioration, graded mean integration method, signed distance method.

**AMS Mathematics Subject Classification (2010):** 90B05

### 1. Introduction

It is a big challenge to develop robust inventory models for deteriorating items such as medicines, blood, electronic items, food stuffs and fashion cosmetics etc. The analysis of deteriorating inventory models began with Ghare and Schrader [4] who developed EOQ model with exponential rate of deterioration. Since then, many related research could be found in Jaggi, Aggarwal and Goel [7], Hariga [5], Behera and Tripathy [1]. On the other hand, the usual EOQ models do not consider the effect of salvage value, which cannot be neglected now due to piling of materials leading to its salvage value. Such a situation will motivate the procurement planner to earn revenue. So Mishra and Shah [10] has suggested a more realistic collaborative inventory model for deteriorating items with salvage value.

One of the weaknesses of present inventory model which is widely applied in commerce and business world is the inappropriate assumption of the various parameters. Bellan and Zadeh [2] first proposed the fuzzy set theory in decision making process. Zadeh ([19,20]) showed that for the new products and seasonal items it is better to use fuzzy numbers than probabilistic approaches. The uncertainties are shown to be captured by several articles by Chang, Yao and Lee [3] and Yao & Chiang [17]. There have been gradual progresses in the development of inventory models from crisp parameters to

fuzzy parameters. Park [12] and Vujosevic et al. [16] developed the inventory models in fuzzy sense in which ordering cost and holding cost are represented by Fuzzy numbers. Later Yao and Lee [18], Kao and Hsu [9], Hsieh [6], Jaggi et al. [8] have developed inventory models by taking major parameters as fuzzy for defuzzifying the fuzzy total inventory cost. In 2010, Sahoo et al. [14] worked on an inventory model for constant deteriorating items. Sahoo, Mohanty and Tripathy [13] represented a fuzzy inventory model with exponential demand and time-varying deterioration. Sahoo and Tripathy [15] discussed an EOQ model with three-parameter weibull deterioration, trended demand and time varying holding cost with salvage. Mohanty and Tripathy [11] formulated a fuzzy inventory model for deteriorating items with exponentially decreasing demand under fuzzified cost and partial backlogging.

In this paper it has presented an inventory model with time dependent trended deterioration and salvage value is incorporated to the cost of deteriorated items, in which the major costs are considered as fuzzy numbers. The trapezoidal type fuzzy number is used for representing fuzzy numbers and defuzzification is done using both Graded Mean Integration Method and Signed Distance Method. The optimal costs are discussed in crisp as well as fuzzy model. The results obtained by two methods are compared with discussing with numerical examples and sensitivity analysis.

## 2. Assumptions and notations

- (i)  $R$  is the demand rate at any time  $t$  per unit time is deterministic and constant.
- (ii) The replenishment rate is infinite.
- (iii) The lead time is zero and shortages are not allowed.
- (iv)  $A$  is the ordering cost per order.
- (v)  $C$  is the purchase cost per unit.
- (vi)  $h$  is the holding cost per unit per unit time.
- (vii)  $T$  is the length of the cycle.
- (viii) Deterioration rate  $\theta(t) = a(1 + bt)$ , is assumed to be an increasing function of time i.e. where  $a$  and  $b$  are positive constants and  $a > 0, 0 < b < 1$ .
- (ix) The salvage value  $\beta C$  ( $0 \leq \beta < 1$ ) is associated to deteriorated units during the cycle time.
- (x) The deteriorating units cannot be repaired or replaced during the period under review.
- (xi)  $K(T)$  is the total inventory cost per unit time.
- (xii)  $\tilde{\theta}$  is the fuzzy deterioration rate.
- (xiii)  $\tilde{h}$  is the fuzzy holding cost per unit per unit time.
- (xiv)  $\tilde{C}$  is the fuzzy purchase cost per unit time.
- (xv)  $\tilde{\beta}\tilde{C}$  is the fuzzy salvage value.
- (xvi)  $K_{dG}(T)$  is the defuzzify value of  $\tilde{K}(T)$  by applying Graded Mean Integration Method.
- (xvii)  $K_{dS}(T)$  is the defuzzify value of  $\tilde{K}(T)$  by applying Signed Distance Method.

### 3. Mathematical model

Let  $Q(t)$  be the on hand inventory at any instant of time ( $0 \leq t \leq T$ ). The depletion of units in inventory is due to demand and deterioration. The instantaneous state of inventory  $Q(t)$  at any instant of time is governed by the following differential equation.

#### 3.1. Crisp model

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R, 0 \leq t \leq T \quad (3.1.1)$$

where initial condition  $Q(0) = Q$  and boundary condition  $Q(T) = 0$ .

Thus solution of differential equation using boundary condition  $Q(T) = 0$  is

$$Q(t) = R[(T-t) + \frac{1}{2}a(T^2 - 2Tt + t^2) + \frac{1}{6}ab(T^3 - 3Tt^2 + 2t^3)] \quad (3.1.2)$$

(By taking series expansion and neglecting powers of  $a$  and  $b$  higher than one)

Using  $Q(0) = Q$ , we get

$$Q = R[T + \frac{1}{2}aT^2 + \frac{1}{6}abT^3] \quad (3.1.3)$$

The total cost per time unit,  $K(T)$  comprise of following cost

(i)  $IHC = \text{Inventory holding cost}$

$$\begin{aligned} &= \int_0^T hQ(t) dt \\ &= hR[\frac{1}{2}T^2 + \frac{1}{6}aT^3 + \frac{1}{12}abT^4] \end{aligned}$$

(ii)  $OC = \text{Ordering Cost} = A$

(iii)  $CD = \text{Cost due to Deterioration}$

$$= C[Q - \int_0^T R(t)dt] = C[\frac{1}{2}aT^2 + \frac{1}{6}abT^3]$$

(iv)  $SV = \text{Salvage value of Deterioration items}$

$$\begin{aligned} &= \beta C[Q - \int_0^T R(t)dt] \\ &= \beta C[\frac{1}{2}aT^2 + \frac{1}{6}abT^3] \end{aligned}$$

Thus total cost per time unit is given by

$$\begin{aligned} K(T) &= \frac{1}{T}(IHC + OC + CD - SV) \\ K(T) &= \frac{1}{T}[hR(\frac{1}{2}T^2 + \frac{1}{6}aT^3 + \frac{1}{12}abT^4) + A + C(\frac{1}{2}aT^2 + \frac{1}{6}abT^3) - \beta C(\frac{1}{2}aT^2 + \frac{1}{6}abT^3)] \quad (3.1.4) \end{aligned}$$

#### 3.2. Fuzzy model

Let  $\tilde{a} = (a_1, a_2, a_3, a_4)$ ,  $\tilde{b} = (b_1, b_2, b_3, b_4)$ ,  $\tilde{h} = (h_1, h_2, h_3, h_4)$ ,  $\tilde{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$ ,  $\tilde{C} = (C_1, C_2, C_3, C_4)$  are as trapezoidal fuzzy number.

Total cost of the system per unit time in fuzzy sense is given by

$$\tilde{K}(T) = \frac{1}{T} [A + \tilde{h}R(\frac{1}{2}T^2 + \frac{1}{6}\tilde{a}T^3 + \frac{1}{12}\tilde{a}\tilde{b}T^4) + \tilde{C}(\frac{1}{2}\tilde{a}T^2 + \frac{1}{6}\tilde{a}\tilde{b}T^3) - \tilde{\beta}\tilde{C}(\frac{1}{2}\tilde{a}T^2 + \frac{1}{6}\tilde{a}\tilde{b}T^3)] \quad (3.2.1)$$

We defuzzify the fuzzy total cost  $\tilde{K}(T)$  by Graded Mean Integration Method and Signed Distance Method

(i) By Graded Mean Integration Method, total cost is given by

$$K_{dG}(T) = \frac{1}{6} (K_{dG_1}(T) + 2K_{dG_2}(T) + 2K_{dG_3}(T) + K_{dG_4}(T))$$

$$= \frac{1}{6T} \left[ \begin{aligned} &6A + h_1R(\frac{1}{2}T^2 + \frac{1}{6}a_1T^3 + \frac{1}{12}a_1b_1T^4) + C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) - \beta_1C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) \\ &+ 2\{h_2R(\frac{1}{2}T^2 + \frac{1}{6}a_2T^3 + \frac{1}{12}a_2b_2T^4) + C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3) - \beta_2C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3)\} \\ &+ 2\{h_3R(\frac{1}{2}T^2 + \frac{1}{6}a_3T^3 + \frac{1}{12}a_3b_3T^4) + C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3) - \beta_3C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3)\} \\ &+ h_4R(\frac{1}{2}T^2 + \frac{1}{6}a_4T^3 + \frac{1}{12}a_4b_4T^4) + C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) - \beta_4C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) \end{aligned} \right] \quad (3.2.2)$$

The necessary condition for  $K_{dG}(T)$  to be minimum is

$$\frac{\partial K_{dG}(T)}{\partial T} = 0 \quad (3.2.3)$$

$$\Rightarrow \frac{1}{6} \left[ \frac{\partial K_{dG_1}(T)}{\partial T} + 2 \frac{\partial K_{dG_2}(T)}{\partial T} + 2 \frac{\partial K_{dG_3}(T)}{\partial T} + \frac{\partial K_{dG_4}(T)}{\partial T} \right] = 0$$

$$\Rightarrow \frac{1}{6T} \left[ \begin{aligned} &h_1R(T + \frac{1}{2}a_1T^2 + \frac{1}{3}a_1b_1T^3) + C_1(a_1T + \frac{1}{2}a_1b_1T^2) - \beta_1C_1(a_1T + \frac{1}{2}a_1b_1T^2) \\ &+ 2\{h_2R(T + \frac{1}{2}a_2T^2 + \frac{1}{3}a_2b_2T^3) + C_2(a_2T + \frac{1}{2}a_2b_2T^2) - \beta_2C_2(a_2T + \frac{1}{2}a_2b_2T^2)\} \\ &+ 2\{h_3R(T + \frac{1}{2}a_3T^2 + \frac{1}{3}a_3b_3T^3) + C_3(a_3T + \frac{1}{2}a_3b_3T^2) - \beta_3C_3(a_3T + \frac{1}{2}a_3b_3T^2)\} \\ &+ h_4R(T + \frac{1}{2}a_4T^2 + \frac{1}{3}a_4b_4T^3) + C_4(a_4T + \frac{1}{2}a_4b_4T^2) - \beta_4C_4(a_4T + \frac{1}{2}a_4b_4T^2) \end{aligned} \right]$$

$$= \frac{1}{6T^2} \left[ \begin{aligned} &6A + h_1R(\frac{1}{2}T^2 + \frac{1}{6}a_1T^3 + \frac{1}{12}a_1b_1T^4) + C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) - \beta_1C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) \\ &+ 2\{h_2R(\frac{1}{2}T^2 + \frac{1}{6}a_2T^3 + \frac{1}{12}a_2b_2T^4) + C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3) - \beta_2C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3)\} \\ &+ 2\{h_3R(\frac{1}{2}T^2 + \frac{1}{6}a_3T^3 + \frac{1}{12}a_3b_3T^4) + C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3) - \beta_3C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3)\} \\ &+ h_4R(\frac{1}{2}T^2 + \frac{1}{6}a_4T^3 + \frac{1}{12}a_4b_4T^4) + C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) - \beta_4C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) \end{aligned} \right] = 0 \quad (3.2.4)$$

$K_{dG}(T)$  is minimum only if

$$\frac{\partial^2 K_{dG}(T)}{\partial T^2} > 0, \text{ for all } T > 0. \quad (3.2.5)$$

(ii) By Signed Distance Method, total cost is given by

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$$\begin{aligned}
 K_{ds}(T) &= \frac{1}{4}[K_{ds_1}(T) + K_{ds_2}(T) + K_{ds_3}(T) + K_{ds_4}(T)] \\
 &= \frac{1}{4T} \left[ \begin{aligned}
 &4A + h_1R(\frac{1}{2}T^2 + \frac{1}{6}a_1T^3 + \frac{1}{12}a_1b_1T^4) + C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) - \beta_1C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) \\
 &+ \{h_2R(\frac{1}{2}T^2 + \frac{1}{6}a_2T^3 + \frac{1}{12}a_2b_2T^4) + C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3) - \beta_2C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3)\} \\
 &+ \{h_3R(\frac{1}{2}T^2 + \frac{1}{6}a_3T^3 + \frac{1}{12}a_3b_3T^4) + C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3) - \beta_3C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3)\} \\
 &+ h_4R(\frac{1}{2}T^2 + \frac{1}{6}a_4T^3 + \frac{1}{12}a_4b_4T^4) + C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) - \beta_4C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3)
 \end{aligned} \right] \quad (3.2.6)
 \end{aligned}$$

The necessary condition for  $K_{ds}(T)$  to be minimum is

$$\frac{\partial K_{ds}(T)}{\partial T} = 0 \quad (3.2.7)$$

$$\begin{aligned}
 \Rightarrow \frac{1}{4} \left[ \frac{\partial K_{ds_1}(T)}{\partial T} + \frac{\partial K_{ds_2}(T)}{\partial T} + \frac{\partial K_{ds_3}(T)}{\partial T} + \frac{\partial K_{ds_4}(T)}{\partial T} \right] &= 0 \\
 \Rightarrow \frac{1}{4T} \left[ \begin{aligned}
 &h_1R(T + \frac{1}{2}a_1T^2 + \frac{1}{3}a_1b_1T^3) + C_1(a_1T + \frac{1}{2}a_1b_1T^2) - \beta_1C_1(a_1T + \frac{1}{2}a_1b_1T^2) \\
 &+ \{h_2R(T + \frac{1}{2}a_2T^2 + \frac{1}{3}a_2b_2T^3) + C_2(a_2T + \frac{1}{2}a_2b_2T^2) - \beta_2C_2(a_2T + \frac{1}{2}a_2b_2T^2)\} \\
 &+ \{h_3R(T + \frac{1}{2}a_3T^2 + \frac{1}{3}a_3b_3T^3) + C_3(a_3T + \frac{1}{2}a_3b_3T^2) - \beta_3C_3(a_3T + \frac{1}{2}a_3b_3T^2)\} \\
 &h_4R(T + \frac{1}{2}a_4T^2 + \frac{1}{3}a_4b_4T^3) + C_4(a_4T + \frac{1}{2}a_4b_4T^2) - \beta_4C_4(a_4T + \frac{1}{2}a_4b_4T^2)
 \end{aligned} \right] \\
 - \frac{1}{4T^2} \left[ \begin{aligned}
 &4A + h_1R(\frac{1}{2}T^2 + \frac{1}{6}a_1T^3 + \frac{1}{12}a_1b_1T^4) + C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) - \beta_1C_1(\frac{1}{2}a_1T^2 + \frac{1}{6}a_1b_1T^3) \\
 &+ \{h_2R(\frac{1}{2}T^2 + \frac{1}{6}a_2T^3 + \frac{1}{12}a_2b_2T^4) + C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3) - \beta_2C_2(\frac{1}{2}a_2T^2 + \frac{1}{6}a_2b_2T^3)\} \\
 &+ \{h_3R(\frac{1}{2}T^2 + \frac{1}{6}a_3T^3 + \frac{1}{12}a_3b_3T^4) + C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3) - \beta_3C_3(\frac{1}{2}a_3T^2 + \frac{1}{6}a_3b_3T^3)\} \\
 &+ h_4R(\frac{1}{2}T^2 + \frac{1}{6}a_4T^3 + \frac{1}{12}a_4b_4T^4) + C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3) - \beta_4C_4(\frac{1}{2}a_4T^2 + \frac{1}{6}a_4b_4T^3)
 \end{aligned} \right] = 0 \quad (3.2.8)
 \end{aligned}$$

$K_{ds}(T)$  is minimum only if

$$\frac{\partial^2 K_{ds}(T)}{\partial T^2} > 0, \text{ for all } T > 0. \quad (3.2.9)$$

#### 4. Numerical examples

Consider an inventory model with the following parametric values with proper unit

##### Crisp model

$A = Rs200/\text{order}$ ,  $R = 10000\text{units}/\text{year}$ ,  $C = Rs 20/\text{unit}$ ,  $h = Rs 5/\text{units}/\text{year}$ ,  
 $a = 150\text{units}/\text{year}$ ,  $b = 0.1\text{units}/\text{year}$ ,  $\beta = 0.5\text{units}/\text{year}$

The solution of crisp model using Mathematica 5.1 we get

$$K(T) = Rs 8033.98, T = 0.0398814 \text{ year}$$

##### Fuzzy model

$$\tilde{a} = (120, 140, 160, 180) \quad , \quad \tilde{b} = (0.07, 0.09, 0.11, 0.13) \quad , \quad \tilde{C} = (17, 19, 21, 23) \quad ,$$

$$\tilde{\beta} = (0.2, 0.4, 0.6, 0.8) \quad , \quad \tilde{h} = (2, 4, 6, 8)$$

The solution of fuzzy model using Mathematica 5.1 can be determined by following two methods.

By **Graded Mean Integration Method**, we have

- (i) When  $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\beta}, \tilde{h}$  are trapezoidal fuzzy numbers  
 $K_{dG}(T) = Rs\ 8128.25, T = 0.0393445\ year$
- (ii) When  $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\beta}$  are trapezoidal fuzzy numbers  
 $K_{dG}(T) = Rs\ 8031.88, T = 0.0398858\ year$
- (iii) When  $\tilde{a}, \tilde{b}, \tilde{\beta}$  are trapezoidal fuzzy numbers  
 $K_{dG}(T) = Rs\ 8032.61, T = 0.0398837\ year$
- (iv) When  $\tilde{a}, \tilde{b}$  are trapezoidal fuzzy numbers  
 $K_{dG}(T) = Rs\ 8034.07, T = 0.0398795\ year$

By **Signed Distance Method**, we have

- (i) When  $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\beta}, \tilde{h}$  are trapezoidal fuzzy numbers  
 $K_{dS}(T) = Rs\ 8161.9, T = 0.039157\ year$
- (ii) When  $\tilde{a}, \tilde{b}, \tilde{C}, \tilde{\beta}$  are trapezoidal fuzzy numbers  
 $K_{dS}(T) = Rs\ 8031.11, T = 0.0398878\ year$
- (iii) When  $\tilde{a}, \tilde{b}, \tilde{\beta}$  are trapezoidal fuzzy numbers  
 $K_{dS}(T) = Rs\ 8032.11, T = 0.0398849\ year$
- (iv) When  $\tilde{a}, \tilde{b}$  are trapezoidal fuzzy numbers  
 $K_{dS}(T) = Rs\ 8034.11, T = 0.0398792\ year$

**5. Sensitivity analysis**

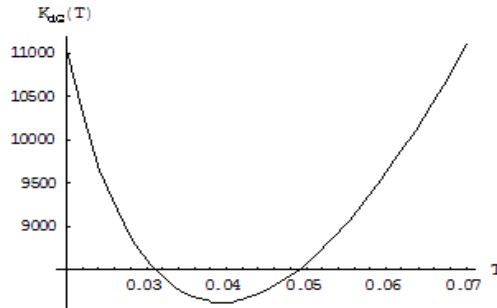
**Table 1:**

$a$	$T$	$K_{dG}(T)$		$b$	$T$	$K_{dG}(T)$
120	0.0424801	7603.35		0.070	0.0393576	8126.67
135	0.0410957	7824.99		0.085	0.0393526	8127.27
150	0.0398810	8033.17		0.100	0.0393476	8127.87
165	0.0388022	8229.80		0.115	0.0393426	8128.48
180	0.0378345	8416.42		0.130	0.0393376	8129.08

Table-1 indicates that as the value of  $a$  increases, fuzzy cost  $K_{dG}(T)$  increases regularly while  $T$  decreases gradually similarly as the value of  $b$  increases, fuzzy cost  $K_{dG}(T)$  increases slightly while  $T$  decreases slowly.

If we plot the total cost function  $K_{dG}(T)$  with time values  $T$ , then we get the convex graph of total cost function as given below

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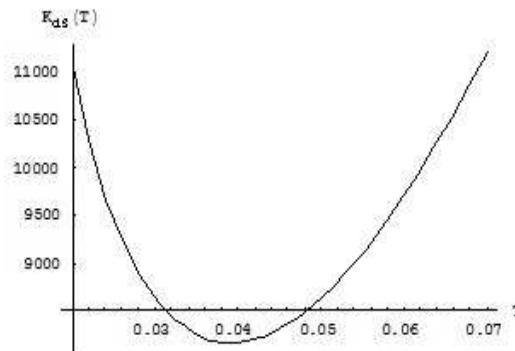
**Figure 1:** Total fuzzy cost  $K_{dG}(T)$  vs  $T$

**Table 2:**

$a$	$T$	$K_{ds}(T)$		$b$	$T$	$K_{ds}(T)$
120	0.0424801	7603.35		0.070	0.0393576	8126.67
135	0.0410957	7824.99		0.085	0.0393526	8127.27
150	0.039881	8033.17		0.100	0.0393476	8127.87
165	0.0388022	8229.80		0.115	0.0393426	8128.48
180	0.0378345	8416.42		0.130	0.0393376	8129.08

Analysing Table-2, it indicates that as the value of  $a$  increases, fuzzy cost  $K_{ds}(T)$  increases regularly while  $T$  decreases gradually similarly as the value of  $b$  increases, fuzzy cost  $K_{ds}(T)$  increases slightly while  $T$  decreases slowly.

If we plot the total cost function  $K_{ds}(T)$  with time values  $T$ , then we get the convex graph of total cost function as given below



**Figure 2:** Total fuzzy cost  $K_{ds}(T)$  vs  $T$

### 6. Conclusion

In this paper, we have developed a time dependent trended fuzzy inventory model for deteriorating items with salvage. To capture the real life situation we have considered that the major parameters are uncertain and it is possible to describe it by trapezoidal fuzzy numbers. For defuzzification Graded Mean Integration and Signed Distance Method are employed to evaluate the optimal time period of total cycle length  $T$  which minimize the

total cost. Numerically we ventured to compare the crisp model with fuzzy model and conclude that if the uncertainties are accounted for in an appropriate manner, the time would decrease. Sensitivity analysis is carried out to see how far the output of the model is affected by changes in its input parameters. The preliminary results indicate that the total inventory cost increases when we increase the parameters  $a$  and  $b$ . With the increased value of these parameters, it will subsequently increase the fuzzy cost, but decrease the time period. Similarly with the decreased value of these parameters it will subsequently decrease the fuzzy cost but increase the time period.

Here we use different examples to illustrate both the crisp as well as fuzzy model which demonstrate the effect of fuzziness of the parameters on the optimal solution. In comparison with the crisp model, the fuzzy model is giving the relatively better optimal solution.

### Future suggestions

In the future study it is hoped to incorporate the proposed model into more realistic assumptions, such as stochastic demand, credit policy and partial backlogging. The model presented in this study can be extended in various ways. For instance, the model can be extended for non-instantaneous receipt of orders as well as shortages.

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