Intern. J. Fuzzy Mathematical Archive
Vol. 15, No. 1, 2018, 55-61
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 5 February 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v15n1a5

International Journal
Fuzzy Mathematical
Archive

Transportation Problem with Triangular Mixed Intutionistic Fuzzy Numbers Solved By BCM

***Priyanka A. Pathade*[1] and *Kirtiwant P. Ghadle*[2]**

Department of Mathematics
Dr. Babasaheb Ambedkar Marathwada University
Aurangabad-431004 (M.S.) India.
Email: [1]priyankapathade88@gmail.com; [2]drkp.ghadle@gmail.com
[1]Corresponding author

Received 29 December 2017; accepted 16 January 2018

Abstract. In this paper, we formulate a transportation problem in which sources, destinations and costs are different types of fuzzy numbers. We used real, fuzzy and intuitionistic fuzzy numbers are employed to get the optimal solution. Mixed intuitionistic fuzzy BCM is used to find the optimal solution in terms of triangular intuitionistic fuzzy numbers. The method is illustrated by a numerical examples.

Keywords: Fuzzytransportation problem, Intuionistic fuzzy number, Optimal solution.

AMS Mathematics Subject Classification (2010): 03E72, 03F55, 90B06

1. Introduction

The theory of fuzzy set introduced by Zadeh [6] in 1965 has achived good real life applications in many fields. Atanassov [3] proposed the concept of intuitionistic fuzzy sets in 1986. He found that it is useful in vague concept. Degree of membership (belongingness) and the degree of non membership (belongigness) of an element in the set has separated by intuitionistic fuzzy set.This is a first advantage of intuitionistic fuzzy set. Klir [2] has proved theory based applications in fuzzy environment.

Gani and Abbas [1] solved intutionistic fuzzy transportation problems using zero suffix method. Hussain and Kumar [8] solved intuitionistic fuzzy transportation problems using a newly defined ranking function. They allote a algorithmic approach to illustrate example. Shashi Aggarwal and Chavi Gupta [11] introduced a new ranking method for generalized trapezoidal intuitionistic fuzzy number and proposed a new method.This new method based on the new ranking method for solving generalized intuitionistic fuzzy transportation problems. Balanced transportation problems are necessary part for applying methods to solve a numerical examples. Ghadle and Pathade [4] solved hexagonal fuzzy numbers by balanced and unbalanced numerical examples and gives a comparative discussion. Kumar and Hussain [9] used mixed intutionistic fuzzy numbers and solve the example systematically. Biswas and Alam [10] developed a method to search for an intutionistic fuzzy shortest path from a source to destination. They used dijkstras algorithm to find out shortest path and used fuzzy numbers as a crisp values to improve the accuracy. Many researchers used fuzzy numbers but somewhere crisp values are important to verify the different types of number. Ranking process is different for different fuzzy numbers [12, 13,14]. Ghadle and Pathade [7] recently used octagonal fuzzy numbers.They solved the numerical example by BCM and find out the nearer optimal solution.

2. Preliminary

In this section, we collect some basic definitions that will be important to us in the sequel.

Definition 2.1. *A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse* X *to the unit interval [0,1] i.e* $A=\{(\mu_A(x); x\in X)\}$. *Here* $\mu_A(x): X \to [0,1]$ *is a mapping called the degree of membership value of* $x\, in X$ *in the fuzzy set* A. *These membership grades are often represented by real numbers ranking from* $[0,1]$.

Definition 2.2. *A fuzzy number f in the real line R is a fuzzy set f: R*$\to$*[0,1] that satisfies the following properties.*

- *f is piecewise continuous.*
- *There exists an* x ε R *such that* $f(x)=1$.
- *f is convex* i.e if $x_1, x_2 \in$ *and* $a\in[0,1]$ *then* $f(\lambda x_1)+(1-\lambda x_2)\geq f(x_1)\wedge f(x_2)$

Definition 2.3. *A fuzzy number* A *is defined to be a traingular fuzzy number if its membership function* $\mu_A: R\to[0,1]$ *is equal to*

$$\mu_A(x)=\begin{cases}\dfrac{(x-a_1)}{(a_2-a_1)}, & if\, x\varepsilon[a_1,a_2]\\ \dfrac{(a_3-x)}{(a_3-a_2)}, & if\, x\varepsilon[a_2,a_3]\\ 0, & otherwise\end{cases} \tag{1}$$

where $a_1\leq a_2\leq a_3$.

This fuzzy number is denoted by $(a_1\leq a_2\leq a_3)$.

Definition 2.4. *A Triangular Intutionistic Fuzzy Number (* $\tilde{A}^I$ *is an intuitionistic fuzzy set in* R *with the following membership function* $(\mu_A(x) and\, \upsilon_A(x)$ *:)*

$$\mu_A(x)=\begin{cases}0, & for\;\; x<a_1\\ \dfrac{(x-a_1)}{(a_2-a_1)}, & for\;\; a_1\leq x\leq a_2\\ 1, & for\;\; x=a_2\\ \dfrac{(a_3-x)}{(a_3-a_2)} & for\;\; a_2\leq x\leq a_3\\ 0 & for\;\; x>a_3\end{cases} \tag{2}$$

and

$$\upsilon_A(x)=\begin{cases}1, & for\ x<a_1' \\ \frac{(a_2-x)}{(a_2-a_1')}, & for\ a_1'\leq x\leq a_2 \\ 0, & for\ x=a_2 \\ \frac{(x-a_2)}{(a_3'-a_2)} & for\ a_2\leq x\leq a_3' \\ 1 & for\ x>a_3'\end{cases} \quad (3)$$

where $a_1' \leq a_2 \leq a_3 \leq a_3'$ and $\mu_A(x),\ \ \upsilon_A(x) \leq 0.5$ for $\mu_A(x) = \upsilon_A(x)$,for all x $\in$ R. This TrIFN is denoted by $\tilde{A}^I$ =(a_1, a_2, a_3)(a_1', a_2, a_3').

Particular Cases: Let b $\tilde{A}^I$ =(a_1, a_2, a_3)(a_1', a_2, a_3') be a TrIFN. Then the following cases arise

Case 1: If $a_1' = a_1, a_3' = a_3$, then $\tilde{A}^I$ represent Triangular Fuzzy Number (TFN). It is denoted by $A = (a_1, a_2, a_3)$

Case 2: If $a_1' = a_1 = a_2 = a_3 = a_3' = m$, then $\tilde{A}^I$ represent a real number m.

Ranking of triangular intuitionistic fuzzy numbers

The ranking of a triagular intuitionistic fuzzy number

$$\tilde{A}^I \ = \frac{1}{3}[\frac{((a_3'-a_1')(a_2-2a_3'-2a_1')+(a_3-a_1)(a_1+a_2+a_3)+3(a_3'^2-a_1'^2)}{a_3'-a_1'+a_3-a_1}]$$

If $\upsilon_A(x)$=1$-\mu_a(x)$,then TrIFN $\tilde{A}^I$ = $(a_1, a_2, a_3)(a_1', a_2, a_3')$ will become the TrFN (a_1, a_2, a_3) .

Then $a_1' = a_1$ and $a_3' = a_3$ $R(\tilde{A}^I) = (a_1 + a_2 + a_3)/3$. The Rank of TrIFN $\tilde{A}$ = $(a_1, a_2, a_3)(a_1', a_2, a_3')$ is defined by $R(\tilde{A}^I)$ = $a_2 if a_2 - a_1 = a_3 - a_2$ and $a_2 - a'1 = a_3' - a_2$. Let $\tilde{A}^I$ =(8,10,12)(6,10,14) be a TrIFN, then its rank is defined by $R(\tilde{A}^I)$ =10.

Definition 2.5. *Let* $\tilde{A}^I$ *and* $\tilde{B}^I$ *be two TrIFNs. The ranking of* $\tilde{A}^I$ *and* $\tilde{B}^I$ *by the R(.) on E, the set of TrIFN is defined as follows: (i)* $\tilde{A}^I > \tilde{B}^I$ *iff* $A^I \succ B^I$

(ii) $\tilde{A}^I < \tilde{B}^I$ iff $A^I \prec B^I$

(iii $\tilde{A}^I = \tilde{B}^I$ iff $A^I \approx B^I$

(iv) $R(\tilde{A}^I + \tilde{B}^I) = R(\tilde{A}^I) + R(\tilde{B}^I)$

(v) $R(\tilde{A}^I - \tilde{B}^I) = R(\tilde{A}^I) - R(\tilde{B}^I)$

(vi) $R(\tilde{A}^I \otimes \tilde{B}^I) = R(\tilde{A}^I) \otimes R(\tilde{B}^I)$

Definition 2.6. *The ordering* $\tilde{A}^I \approx \tilde{B}^I \geq$ *and* $\leq$ *between any two TrIFNs* $\tilde{A}^I and \tilde{B}^I$ *are defined as follows(i)* $\tilde{A}^I \geq \tilde{B}^I$ *iff* $\tilde{A}^I \succ \tilde{B}^I$ *or* $\tilde{A}^I \approx \tilde{B}^I$, *(ii)* $\tilde{A}^I \leq \tilde{B}^I$ *iff* $\tilde{A}^I \prec \tilde{B}^I$ *or* $\tilde{A}^I \approx \tilde{B}^I$.

Arithmetic operation:

Let $\tilde{A}^I = (a_1, a_2, a_3)\ (a_1', a_2, a_3')$ and $\tilde{B}^I = (b_1, b_2, b_3)\ (b_1', b_2, b_3')$ be any two TrIFN then the arithmetic operation as follows,

Addition: $\tilde{A}^I \bigoplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3)\ (a_1' + b_1', a_2 + b_2, a_3' + b_3')$.

Subtraction: $\tilde{A}^I\ !\tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1)\ (a_1' - b_3', a_2 - b_2, a_3' - b_1')$.

Multiplication: If $R(\tilde{A}^I)$, $R(\tilde{B}^I) \geq 0$, then

$$\tilde{A}^I \otimes \tilde{B}^I = [(\frac{a_1(b_1 + b_2 + b_3)}{3}), (\frac{a_2(b_1 + b_2 + b_3)'}{3}), (\frac{a_3(b_1 + b_2 + b_3)}{3})$$
$$(\frac{a_1'(b_1' + b_2 + b_3')}{3}), (\frac{a_2(b_1' + b_2 + b_3')'}{3}), (\frac{a_3(b_1' + b_2 + b_3')}{3})].$$

If $R(\tilde{A}^I)$, $R(\tilde{B}^I)$ <0, then

$$\tilde{A}^I \otimes \tilde{B}^I = [(\frac{a_3(b_1 + b_2 + b_3)}{3}), (\frac{a_2(b_1 + b_2 + b_3)'}{3}), (\frac{a_1(b_1 + b_2 + b_3)}{3})$$
$$(\frac{a_3'(b_1' + b_2 + b_3')}{3}), (\frac{a_2(b_1' + b_2 + b_3')'}{3}), (\frac{a_1(b_1' + b_2 + b_3')}{3})].$$

Scalar Addition: $R(K + \tilde{A}^I) = K + R(\tilde{A}^I)$.

Scalar Multiplication:

i. k $\tilde{A}^I = (ka_1, ka_2, ka_3)(ka_1', ka_2, ka_3')$, for $k \geq 0$

ii. k $\tilde{A}^I = (ka_3, ka_2, ka_1')$, for $<\ 0$

3. Intuitionistic fuzzy transportation problem

Consider a transportation with m origins (rows) and n destinations (columns). Let $\tilde{c}_{ij}{}^I$ be the cost of transporting one unit of the product from i^{th} origin to j^{th} destination. Let $\tilde{a}_i{}^I$ be the quality of commodity available at origin i. Let $\tilde{b}_j{}^I$ be the quantity of commodity needed at destination j and $\tilde{\tilde{x}}_i j^I$ is the quantity transported from i^{th} origin to

j^{th} destination, so as to minimize the Intuitionistic Fuzzy Transportation Cost [IFTC].

(IFTP) Minimize $\tilde{Z}^I = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^{\ I} \otimes \tilde{x}_{ij}^{\ I}$ Subject to,

$\sum_{j=1}^{n} \tilde{x}_{ij}^I \approx \tilde{a}_i^I$ for i=1,2,...,m

$\sum_{j=1}^{m} \tilde{x}_{ij}^I \approx \tilde{b}_i^I$ for i=1,2,...,n

$x_{ij}^I \approx \tilde{0}^I$, for i=1,2,...,m and j=1,2,...n

where m= the number of supply points, n= the number of demand points

$\tilde{c}_{ij}^I = (c_{ij}^1, c_{ij}^2, c_{ij}^3)\ \ (c_{ij}^{1'}, c_{ij}^2, c_{ij}^{3'}), \tilde{a}_i^{\ I} = (a_i^1, a_i^2, a_i)^3 (a_i^{1'}, a_i^2, a_i^3),)$

$\tilde{b}_j^I = (b_j^1, b_j^2, b_j^3)\ \ (b_j^{1'}, b_j^2, b_j^{3'}), \tilde{x}_i\, j^I = (x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'})$

4. Numerical example

Consider the 3×3 MIFTP

	D_1	D_2	D_1	**supply**
O_1	(8,10,12)(6,10,14)	4	(10,15,20)	(4,6,8)(8,6,9)
O_2	3	(6,12,18)	(4,6,8)(2,6,10)	8
O_1	(4,8,12)	(3,4,5)(2,4,6)	6	(2,5,8)
demand	(3,4,5)	(2,6,10)(1,6,11)	9	

The corresponding balanced intuitionistic fuzzy transportation table (BIFTT) is

!

	D_1	D_2	D_1	**supply**
O_1	(8,10,12)(6,10,14)	(4,4,4)(4,4,4)	(10,15,20)(10,15,20)	(4,6,8)(8,6,9)
O_2	(3,3,3)(3,3,3)	(6,12,18)(6,12,18)	(4,6,8)(2,6,10)	(8,8,8)(8,8,8)
O_1	(4,8,12)(4,8,12)	(3,4,5)(2,4,6)	(6,6,6)(6,6,6)	(2,5,8)(2,5,8)
demand	(3,4,5)(3,4,5)	(2,6,10)(1,6,11)	(9,9,9)(9,9,9)	(14,19,24)(13,19,25)

Since $\sum_{i=1}^{m} \tilde{a}_j^{\,I} = \sum_{i=1}^{n} \tilde{b}_j^{\,I}$ =(14,19,24)(13,19,24), the problem is BIFTP. Now, using the Best Candidate Method[5]

!

	D_1	D_2	D_1	**supply**
O_1	10	4	15	(4,6,8)(8,6,9)
O_2	3	12	6	(8,8,8)(8,8,8)
O_1	8	4	6	(2,5,8)(2,5,8)
demand	(3,4,5)(3,4,5)	(2,6,10)(1,6,11)	(9,9,9)(9,9,9)	(14,19,24)(13,19,25)

	D_1	D_2	D_1	**supply**
O_1	10	(-2,6,14)(-3,6,15)	15	(4,6,8)(8,6,9)
O_2	(3,4,5)(3,4,5)	12	(3,4,5)(3,4,5)	(8,8,8)(8,8,8)
O_1	8	(-4,0,4)(-4,0,4)	(4,5,6)(4,5,6)	(2,5,8)(2,5,8)
demand	(3,4,5)(3,4,5)	(2,6,10)(1,6,11)	(9,9,9)(9,9,9)	(14,19,24)(13,19,25)

Optimal Solution of an Intuitionistic Fuzzy Transportation Problem

Min $\tilde{Z}$ =4(-2,6,4)(-3,6,15)+3(3,4,5)(3,4,5)+6(3,4,5)(3,4,5)+4(-4,0,4)(-4,0,4) + 6(4,5,6)(4,5,6)

= (-8,24,16)(-12,24,60)+(9,12,15)(9,12,15)+(18,24,30)(18,24,30)+(-16,0,16)(-16,0,16) + (24,30,36)(24,30,36)

=(-8,24,16;-12,24,60)+(9,12,15;9,12,15)+(18,24,30;18,24,30)+(-16,0,16;-16,0,16) + (24,30,30;24,30,36)

Hence, the minimum total intutionistic fuzzy transportation cost is Min $\tilde{Z}$ =(27,90,107;23,90,157)

5. Conclusion

Intutionistic fuzzy transportation problem and procedure for finding an intutionistic fuzzy optimal solution of BIFTP are discussed with numerical example. In this paper we used mixed Intutionistic fuzzy numbers solved by new BCM method.This is a new concept to find a nearer optimal solution. New concepts are helpful to solve the upcoming transportation problems in the real world.

REFERENCES

1. A.N.Gani and S.Abbas, A new method for solving intutionistic fuzzy transportation problem, *Applied Mathematical Sciences*, 7(28) (2013) 1357-1365.
2. G.J.Klir, Fuzzy set theory foundations and applications, Prentice hall, Inc, 1997.

3. K.T.Attansov, Intutionistic fuzzy sets, *Fuzzy Sets and Systems,* 20(1) (1983) 87-96.
4. K.P.Ghadle and P.A.Pathade, Optimal solution of balanced and unbalanced fuzzy transportation problem using hexagonal fuzzy numbers, *International Journal of Mathematical Research (Pakinsight),* 5(2) (2016) 131-137.
5. T.Pathinathan and K.Ponnivalam, Pentagonal fuzzy number, *International Journal of Computing Algorithm*, 3 (2014) 1003-1005.
6. L.A.Zadeh, Fuzzy sets, *Information and Control,* 8 (1965) 338-353.
7. P.A.Pathade and K.P.Ghadle, Optimal solution of balanced and unbalanced fuzzy transportation problem by using octagonal fuzzy numbers, *International Journal of Pure and Applied Mathematics*, Accepted.
8. R.J.Hussain and P.S.Kumar, Algorithmic approach for solving intutionistic fuzzy transportation problem, *Applied Mathematical Sciences,* 6(80) (2012) 3981-3989.
9. P.S.Kumar and R.J.Hussain, A systematic approach for solving mixed intuitionistic fuzzy transportation problems, *International Journal of Pure and Applied Mathematics,* 92(2) (2013) 181-190.
10. S.S.Biswas and B.Alam, An algorithm for extracting intuitionistic fuzzy shortest path in a graph, *Applied Computational Intelligence and Soft Computing,* 2013, Article ID 970197, 5 pages, doi:org/10.1155/2013/970197.
11. S.Aggarwal and C.Gupta, A novel algorithm for solving intutionistic fuzzy transportation problem via new ranking method, *Annals of Fuzzy Mathematics and Informatics,* 8(5) (2014) 753-768.
12. S.Roseline and H.Amirthraj, New approaches to find the solution for the intuitionistic fuzzy transportation problem with ranking of intuitionistic fuzzy numbers, *International Journal of Innovative Research in Sciences Engineering and Technology,* 4(10) (2015) 10222-10230.
13. V.Dhanalakhsmi and F.C.Kennedy, Some ranking methods for octagonal fuzzy numbers, *International Journal of Mathematical Archive,* 5(6) (2014) 177-188.