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New Concepts on Mild Balanced Vague Graphs with Application

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Abstract. Recently, vague graph is a growing research topic as it is the generalization of fuzzy graphs. In this paper, we introduce intense subgraphs and feeble subgraphs based on their densities and discuss mild balanced vague graph and equally balanced vague subgraphs. The operations *sum* and *union* of subgraphs of vague graphs are analysed. Likewise, we investigated ϕ-complement of vague graph structure(VGS) and its isomorphic properties. Finally, an interesting application on vulnerability assessment of gas pipeline systems is given.

Keywords: Feeble subgraphs, mild balanced vague graphs, ϕ-complement of vague graph structure.

AMS Mathematics Subject Classification (2010): 05C72, 05C99

1. Introduction

Graph theory has found its importance in many real time problems. Recent applications in graph theory is quite interesting analysing any complex situations and moreover in engineering applications. It has got numerous applications on operations research, system analysis, network routing, transportation and many more. To analyse any complete information we make intensive use of graphs and its properties. For working on partial informations or incomplete informations or to handle the systems containing the elements of uncertainty we understand that fuzzy logic and its involvement in graph theory is applied. In 1975, Rosenfeld [17] discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman [12] in 1973. The fuzzy relation between fuzzy sets were also considered by Rosenfeld [20] who developed the structure of fuzzy graphs, obtaining various analagous results of several graph theoretical concepts. Bhattacharya [4] gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson [15]. Atanassov introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs [2, 3, 29, 30]. Talebi and Rashmanlou [33] studied the properties of isomorphism and complement of interval-valued fuzzy graphs. They defined isomorphism and some new operations on vague graphs [34, 35]. Borzooei and Rashmalou analysed new concepts of vague graphs [5], degree of vertices in vague graphs [6] , more results on vague graphs [7], semi global domination sets in vague graphs with application [8] and degree and total degree of edges in bipolar fuzzy graphs with application [9]. Rashmanlou et.al., defined the complete interval-valued fuzzy graphs [21]. Rashmanlou and Pal defined intuitionistic fuzzy graphs with categorical properties [23],some properties of highly irregular interval-valued fuzzy graphs [22], more results on highly irregular bipolar fuzzy graphs [24], balanced interval valued fuzzy graphs [23] and antipodal interval valued fuzzy graphs [19]. Samanta and Pal defined fuzzy k-competition and p-competition graphs in [28] . Also they introduced fuzzy tolerance graph [31], bipolar fuzzy hypergraphs [32] and investigated several properties on it. Pal and Rashmanlou [25] given lot of properties of irregular interval valued fuzzy graphs. Mishra and Pal [17] investigated about the concepts of magic labeling on interval-valued fuzzy graphs. Nivethana et al., [18] proposed the ideas of mild balanced intuitonistic fuzzy graphs. Kishore et al., [13, 14] analysed about new concepts on product vague graphs and the concept of Magic labeling on Interval-valued intuitionistic fuzzy graph. Vandana et al., [36] analyse the properties of ϕ-complement of intuitionistic fuzzy graph structure and investigated some properties of isomorphism on these structures. In this paper, we propose the ideas of density in vague graphs for degree of true and false membership values. We analyse the concepts of intense and feeble vague graphs and determine the knowledge of mild balanced vague graphs and strictly balanced vague graphs. Also, we discuss some properties of sum and union of vague graphs. For other notations and terminologies in this paper, the readers are referred to [1-6,11]

2 Preliminaries

In this section, we define some definitions which are prerequisites applied throughout this paper.

Definition 2.1. *A fuzzy graph G=(V,σ, μ) where V is the vertex set, σ is a fuzzy subset of V and μ is a membership value on σ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for every u,v $\in$ V. The underlying crisp graph of G is denoted by $G^* = (\sigma^*, \mu^*)$, where*
$\sigma^{\text{â}} = supp\rho(\sigma) = \{x \in V : \sigma(x) > 0\}$ *and*
$\mu^{\text{â}} = supp\rho(\mu) = \{(x, y) \in V \times V : \mu(x, y) > 0\}$. *$H = (\sigma;\mu)$ is a fuzzy subgraph of G if there exists X $\subseteq$ V such that, $\sigma : X \to [0,1]$ is a fuzzy subset and $\mu : X \times X \to [0,1]$ is a fuzzy relation on σ such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all x,y $\in$ X.*

Vague set is a generalization of fuzzy set. A vague set is characterized by two membership functions namely a truth membership function $t_v(i)$ and false membership function $f_v(i)$. $t_v(i)$ is the lower bound of the grade of membership function of i determined by the evidence of i and $f_v(i)$ is the negation of the grade of membership of i determined against the evidence of i. The difference $1-t_v(i)-f_v(i)$ is the uncertainty in the vague set. The uncertainty is determined based on the value of difference. If the difference value is small the knowledge is precisely relative and if it is large the knowledge is little. The boundedness of this vague set is represented by $t_v(i)\le\mu_v(i)\le 1-f_v(i)$ where $t_v(i)+f_v(i)\le 1$.

The example shows the vague set $X[t_B(x),1-f_B(x)]=[0.7,0.2]$. This indicates the degree of x belonging to the set B is 0.7 and the degree of x not belonging to the set B is 0.2. 0.1 is the degree representing the neutral position. This is a interval valued set on a vague relation.

Definition 2.2. *A vague relation B on a set V is a vague relation from V to V such that* $t_B(xy)\le min(t_A(x),t_A(y)),\ f_B(xy)\ge max(f_A(x),f_A(y))$ *where A is a vague set on a set V and for a vague relation B on A for all x,y*$\in$ *V.*

Definition 2.3. *Let* $G^*=(V,E)$ *be a crisp graph. A pair* $G=(A,B)$ *is called a vague graph on a crisp graph* $G^*=(V,E)$ *where* $A=(t_A,f_A)$ *is a vague set on V and* $B=(t_B,f_B)$ *is a vague set on* $E\subseteq V\times V$ *such that* $t_B(xy)\le min(t_A(x),t_A(y)),\ f_B(xy)\ge max(f_A(x),f_A(y))$ *for each edge x,y* $\in$ *E. Otherwise A is the vague set on V and B is a vague relation on V.*

Definition 2.4. *A vague graph is called complete vague graph if* $t_B(xy)=min(t_A(x),t_A(y)),\ f_B(xy)=max(f_A(x),f_A(y))$ *for each edge x,y* $\in$ *E.*

Remark 2.1. *The complete vague graph is also called strong vague graph.*

Definition 2.5. *An arc* (x,y) *of vague graph is said to be strong if both* $t_B(xy)=min(t_A(x),t_A(y)),\ f_B(xy)=max(f_A(x),f_A(y))$ *for each edge x,y* $\in$ *E.*

Definition 2.6. *The complement of an vague graph* $G=(A,B)$ *of graph* $G^*=(V,E)$ *is an vague graph* $\overline{G}=(A,B)$ *of* $G^*=(V,V\times V)$*, where* $\overline{A}=A=[t_A,f_A]$ *and* $\overline{B}=[\bar{t}_B,\bar{f}_B]$ *is defined by* $\bar{t}_B(xy)=min(t_A(x),t_A(y))\ -\ t_B(xy)$*, for all* $x,y\in V$, $\bar{f}_B(xy)=max(f_A(x),f_A(y))\ -\ f_B(xy)$ *for all* $x,y\in V$.

Definition 2.7. *Let* $H_1=(A_1,B_1)$ *and* $G=(A,B)$ *be two vague graphs whose underline graphs be* $H_1^*=(V_1,E_1)$ *and* $G^*=(V,E)$*. Then* H_1 *is said to be a subgraph* *of G if (a)* $V_1 \subseteq V$ *, where* $t_{A_1}(u_i) = t_A(u_i),\ f_{A_1}(u_i) = f_A(u_i)$ *for all* $u_i \in V_1, i = 1,2,3,...,n.$ *(b)* $E_1 \subseteq E$ *, where* $t_{B_1}(v_iv_j) = t_B(v_iv_j),\ f_{B_1}(v_iv_j) = f_B(v_iv_j)$ *for all* $v_iv_j \in E_1, i = 1,2,3,...,n$.

Definition 2.8. *A vague subgraph* $H_1 = (V_1, E_1)$ *is said to be connected vague subgraph if there exist at least one path between every pair of vertices in* V_1.

Definition 2.9. *Let* $G_1 : (V_1, E_1)$ *and* $G_2 : (V_2, E_2)$ *be two vague graphs with one or more vertices in common. Then the union of* G_1 *and* G_2 *is another vague graph* $G : (V, E) = (G_1 \cup G_2)$ *defined by*

$$(i) t_A(x) = \begin{cases} t_{1A}(x) & \forall x \in V_1 \\ t_{2A}(x) & \forall x \in V_2 \end{cases} \textit{ and } \quad f_B(x) = \begin{cases} f_{1A}(x) & \forall x \in V_1 \\ f_{2A}(x) & \forall x \in V_2 \end{cases}$$

$$(ii) t_B(xy) = \begin{cases} t_{1B}(xy) & \forall x \in E_1 \\ t_{2B}(xy) & \forall x \in E_2 \end{cases} \textit{and } \quad f_B(xy) = \begin{cases} f_{1B}(xy) & \forall x \in E_1 \\ f_{2B}(xy) & \forall x \in E_2 \end{cases}$$

Definition 2.10. *Let* $G_1 : (V_1, E_1)$ *and* $G_2 : (V_2, E_2)$ *be two vague graphs with one or more vertices in common. Then* $G_1 + G_2$ *is another vague graph on* $G : (V, E)$ *defined by*

$$(i) t_A(x) = \begin{cases} t_{1A}(x) & \forall x \in V_1 \\ t_{2A}(x) & \forall x \in V_2 \end{cases} \textit{ and } \quad f_B(x) = \begin{cases} f_{1A}(x) & \forall x \in V_1 \\ f_{2A}(x) & \forall x \in V_2 \end{cases}$$

$$(ii) t_B(xy) = \begin{cases} t_{1B}(xy) & \forall x \in E_1 \\ t_{2B}(xy) & \forall x \in E_2 \end{cases} \textit{and } \quad f_B(xy) = \begin{cases} f_{1B}(xy) & \forall x \in E_1 \\ f_{2B}(xy) & \forall x \in E_2 \end{cases}$$

(iii) There exists a strong edge between every pair of non-common vertices in G_1 and G_2

Definition 2.11. *The density of an vague graph* $G(V, E)$ *is* $D(G) = (D_t(G), D_f(G))$ *where* $D_t(G)$ *and* $D_f(G)$ *are defined by*

$$D_t(G)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V} t_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in V}(t_A(v_i)\wedge t_A(v_j))} \text{ and } D_f(G)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V} f_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in V}(f_A(v_i)\vee f_A(v_j))}$$

3. Mild balanced vague graphs

Definition 3.1. *A connected subgraph* H *of an vague graph* $G:(V,E)$ *is called Intense subgraph if (i)* $V(H)\subseteq V(G)$ *and* $E(H)\subseteq E(G)$,

(ii) $D_t(H)\le D_t(G)$ and $D_f(H)\le D_f(G)$.

Definition 3.2. *A connected subgraph* H *of an vague graph* $G:(V,E)$ *is called Feeble subgraph if (i)* $V(H)\subseteq V(G)$ *and* $E(H)\subseteq E(G)$,

(ii) $D_t(H) > D_t(G)$ and $D_f(H) > D_f(G)$.

Definition 3.3. *A connected subgraph* H *of an vague graph* $G:(V,E)$ *is called partially Intense and Feeble subgraph if (i)* $V(H)\subseteq V(G)$ *and* $E(H)\subseteq E(G)$,

(ii) $D_t(H) < D_t(G)$ and $D_f(H) > D_f(G)$

or $D_t(H) > D_t(G)$ and $D_f(H) < D_f(G)$

Example 3.1. *Consider the vague graph* $G:(V,E)$ *with* $V=a,b,c,d,e$ *and* $E=ab,bc,cd,de,ea$ *We calculate the* t *density and* f *density of the below graph*

$$D_t(G)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V} t_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in V}(t_A(v_i)\wedge t_A(v_j))}=\frac{2[0.2+0.2+0.3+0.18]}{0.2+0.3+0.25+0.2}=1.853 \text{ and}$$

$$D_f(G)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V} f_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in V}(f_A(v_i)\vee f_A(v_j))}=\frac{2[0.7+0.75+0.7+0.7]}{0.6+0.65+0.65+0.6}=2.280$$

Subgraph	Vertices	Edges	$D_t(H)$	$D_f(H)$
H_1	{a,b}	{ab}	2	2.333
H_2	{b,c}	{bc}	1.333	2.308
H_3	{c,d}	{cd}	1.600	2.154
H_4	{d,a}	{da}	1.800	2.333
H_5	{a,b,c}	{ab, bc}	1.600	2.320

H_6	{b,c,d}	{bc ,cd}	1.455	2.231
H_7	{c,d,a}	{cd , da}	1.689	2.240
H_8	{a, b, c, d}	{ ab, bc, cd}	1.6	2.098
H_9	{b, c, d, a}	{ bc, cd, da}	0.750	2
H_{10}	{c, d, a, b}	{ cd, da, ab}	1.785	2.270
H_{11}	{d, a, b, c}	{ da, ab,bc}	3.086	2.324
H_{12}	{a,b,c,d,a}	{ab, bc, cd,da}	1.853	2.280

The above table shows the t-density and f-density of the subgraphs of the vague graph $G:(V,E)$. All the possible connected subgraphs of the above graph G have the values of their densities tabulated. It is observed from the above table that the subgraphs $\{H_3,H_6,H_7,H_8,H_9,H_{10},H_{12}\}$ are Intense subgraphs, $\{H_1,H_{11}\}$ are Feeble subgraphs and $\{H_2,H_4,H_5\}$ are partially intense and feeble subgraphs.

Definition 3.4. *A vague graph $G:(V,E)$ is mild balanced vague graph if all connected subgraphs of G are intense subgraphs.*

Definition 3.5. *Two intense vague connected subgraphs H_1 and H_2 of a vague graph $G:(V,E)$ are called equally balanced subgraphs if*

(i) $D_t(H_1) \le D_t(G)$ and $D_t(H_2) \le D_t(G)$

(ii) $D_f(H_1) \le D_f(G)$ and $D_f(H_2) \le D_f(G)$

(iii) $D_t(H_1) = D_t(H_2)$ and $D_f(H_1) = D_f(H_2)$.

Definition 3.6. *If $D_t(H_i) = D_t(H_i)$ and $D_f(H_i) = D_f(H_i)$ for all possible connected subgraphs H_i of G, then the graph $G:(V,E)$ is called a strictly balanced vague graph.*

Proposition 3.1. *For a strong vague graph, $D(G) = (2,2)$ and it is strictly balanced.*

Proof: Since all the edges of $G:(V,E)$ are strong

$t_B(v_iv_j) = t_A(v_i)\wedge t_A(v_j),\ f_B(v_iv_j) = f_A(v_i)\vee f_A(v_j)$ for each edge $v_i, v_j \in$ E.

By definition

$$D_t(G) = \frac{2\cdot \sum_{\forall v_i,v_j\in V} t_B(v_iv_j)}{\sum_{\forall v_i,v_j\in V}(t_A(v_i)\wedge t_A(v_j))} = \frac{2\cdot \sum t_A(v_i)\wedge \sum t_A(v_j)}{\sum t_A(v_i)\wedge \sum t_A(v_j)} = 2 \text{ and}$$

$$D_f(G)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V} f_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in V}(f_A(v_i)\vee f_A(v_j))}=\frac{2\cdot\sum f_A(v_i)\vee\sum f_A(v_j)}{\sum f_A(v_i)\vee\sum f_A(v_j)}=2$$

Hence $D(G)=(D_t(G),D_f(G))=(2,2)$. Also all the connected subgraphs of $G:(V,E)$ has strong edges and hence $D(H)=(2,2)$ for all subgraphs H of G. Hence $G:(V,E)$ is strictly balanced.

From the above graph we see that the density of the subgraphs of G and the vague graph G are same. ie., $D_t(H)=D_t(G)$ and $D_f(H)=D_f(G)$

Corollary 3.1. *A vague graph with few strong edges can never be a mild balanced vague graph.*

Proof: If a vague graph has one or a few strong edges (not all), then for the connected subgraph H which has only strong edges, $D_t(H)=2$ and $D_f(H)=2$.

Hence $D(H)=(2,2)>D(G)$. Hence it cannot be a mild balanced vague graph.

Proposition 3.2. *Union of two equally balanced connected vague subgraphs with one or more vertices in common is also equally balanced.*

Proof: Let H_1 and H_2 be two equally balanced connected vague subgraphs with at least one common vertex of a vague graph $G:(V,E)$

By definition,

$$D(H_1)=D(H_2)\le D(G).$$

$$D_t(H_1)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V(H_1)} t_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in E(H_1)}(t_A(v_i)\wedge t_A(v_j))}=\frac{2a}{b}\, and\, D_t(H_2)=\frac{2\cdot\sum\limits_{\forall v_i,v_j\in V(H_2)} t_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in E(H_2)}(t_A(v_i)\wedge t_A(v_j))}=\frac{2c}{d}$$

Since

$$D_t(H_1)=D_t(H_2)\frac{a}{b}=\frac{c}{d}\,\frac{c}{d}=\frac{k(a)}{k(b)}.\therefore D_t(H_1)=D_t(H_2)=\frac{2a}{b}$$

$$D_t(H_1\cup H_2)=\frac{2\cdot\sum\limits_{v_i,v_j\in V(H_1)} t_B(v_iv_j)+\sum\limits_{v_i,v_j\in V(H_2)} t_B(v_iv_j)}{\sum\limits_{\forall v_i,v_j\in E(H_1)}(t_A(v_i)\wedge t_A(v_j))+\sum\limits_{\forall v_i,v_j\in E(H_2)}(t_A(v_i)\wedge t_A(v_j))}$$

$$=\frac{2[a+c]}{b+d}=\frac{2(a+ka)}{b+kb}=\frac{2a(k+1)}{b(k+1)}\therefore D_t(H_1\cup H_2)=\frac{2a}{b}.$$

Hence $D_t(H_1\cup H_2)=D_t(H_1)=D_t(H_2)$. Similarly, it can be shown that

$$D_f(H_1 \cup H_2) = D_f(H_1) = D_f(H_2). \therefore D(H_1 \cup H_2) = D(H_1) = D(H_2).$$

Corollary 3.2. *If all the possible connected subgraphs of a mild balanced vague graph are equally balanced then the graph in turn is strictly balanced vague graph.*
Proof: This can be proved by decomposing the graph into two connected subgraphs which are equally balanced. From the above proposition it follows that the union of two equally balanced connected vague subgraphs is equally balanced, the graph itself becomes a strictly balanced vague graph.

Proposition 3.3. *Two connected vague graphs G_1 and G_2 with at least one common vertex are intense subgraphs of vague graphs $G_1 + G_2$.*
Proof:

$$D_t(G_1) = \frac{2 \cdot \sum_{\forall v_i, v_j \in V_1} t_B(v_i v_j)}{\sum_{\forall v_i, v_j \in E_1)} (t_A(v_i) \wedge t_A(v_j))} = \frac{2a}{b} and D_t(G_2) = \frac{2 \cdot \sum_{\forall v_i, v_j \in V_2} t_B(v_i v_j)}{\sum_{\forall v_i, v_j \in E_2} (t_A(v_i) \wedge t_A(v_j))} = \frac{2c}{d}$$

$$D_t(G_1 + G_2) = \frac{2[\sum_{v_i, v_j \in V_1 V_2} t_B(v_i v_j) + \sum_{v_i, v_j \in V^*} t_B(v_i v_j)]}{\sum_{\forall v_i, v_j \in E_1 E_2} (t_A(v_i) \wedge t_A(v_j)) + \sum_{\forall v_i, v_j \in E^*} (t_A(v_i) \wedge t_A(v_j))}$$

where V^* and E^* are the set of vertices and strong edges between every pair of non-common vertices of G_1 and G_2. Obviously $t_b(v_i v_j) = t_A(v_i) \wedge t_A(v_j)$ for all $v_i v_j \in E^*$, since we add a strong edge between all pairs of non-common vertices of G_1 and G_2. ie., $\sum t_B(v_i v_j) = \sum t_A(v_i) \wedge t_A(v_j) \forall v_i v_j \in E^*$,

$$D_t(G_1 + G_2) = \frac{2(a + c + x)}{b + d + x} > \frac{2a}{d} > \frac{2c}{d}$$

$\therefore D_t(G_1 + G_2) < D_t(G_1) and D_t(G_1 + G_2) < D_t(G_2)$
Similarly it can be shown that
$D_f(G_1) > D_f(G_1 + G_2) and D_f(G_2) > D_f(G_1 + G_2)$. Hence G_1 and G_2 are intense subgraphs of $G_1 + G_2$. In particular, $D(G_1) = D(G_1 + G_2) = D(G_2)$ if all the graphs are strong vague graphs.
One can easily verify that both G_1 and G_2 cannot be intense subgraphs of $G_1 \cup G_2$.

Proposition 3.4. *Two connected vague graphs G_1 and G_2 with at least one common vertex are not intense subgraphs of their union.*

4. Φ -complement of vague graph structure

Definition 4.1. *Let $G = (V, R_1, R_2, R_3, ..., R_k)$ be a graph structure and $A, B_1, B_2, ... B_k$*

be vague subsets of $V, R_1, R_2, \ldots R_k$ *respectively such that*

$t_{B_i}(u,v) \le t_A(u) \wedge t_A(v)$ and $f_{B_i}(u,v) \ge f_A(u) \vee f_A(v) \forall u,v \in V$ and $i = 1,2,3,\ldots,k$

Then $\tilde{G} = (A, B_1, B_2, \ldots, B_k)$ is a vague graph structure of G.

Definition 4.2. *The complement of a fuzzy subgraph* $G = (\sigma, \mu)$ *is a fuzzy graph* $\overline{G} = (\overline{\sigma}, \overline{\mu})$ *where* $\overline{\sigma} = \sigma$ *and* $\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v) \forall u,v \in V$

Definition 4.3. *Consider the fuzzy graphs* $G_1 = (\sigma_1, \mu_1)$ *and* $G_2(\sigma_2, \mu_2)$ *with* $\sigma_1^* = V_1$ *and* $\sigma_2^* = V_2$. *An isomorphism between* $G_1 = (\sigma_1, \mu_1)$ *and* $G_2(\sigma_2, \mu_2)$ *is a one-to-one function* h *from* V_1 *onto* V_2 *that satisfies* $\sigma_1(u) = \sigma_2((h(u)))$ *and* $\mu_1(u,v) = \mu_2(h(u), h(v)), \forall u,v \in V$.

Definition 4.4. *Let* $\tilde{G} = (A, B_1, B_2, \ldots, B_k)$ *be a vague graph structure of graph structure* $G = (V, R_1, R_2, \ldots R_K)$. *Let* ϕ *denotes the permutation on the set* $\{R_1, R_2, \ldots R_k\}$ *and also the corresponding permutation on* $\{B_1, B_2, \ldots, B_k\}$ *ie.,* $\phi(B_i) = B_i^\phi = B_j$ *if and only if* $\phi(R_i) = R_j$, *then the* ϕ*-complement of* $\tilde{G}$ *is denoted* $\tilde{G}^\phi$ *and is given by* $\tilde{G}^\phi = (A, B_1^\phi, B_2^\phi, \ldots, B_k^\phi)$ *where for each* $i = 1,2,3,\ldots,k$, *we have*

$$t_{B_i}^\phi(uv) = t_A(u) \wedge t_A(v) - \sum_{j \ne i} (\phi t_{B_i})(uv) \quad \text{and} \quad f_{B_i}^\phi(uv) = \sum_{j \ne i} (\phi f_{B_i})(uv) - f_A(u) \vee f_A(v)$$

Example 4.1. *Consider the vague graph structure* $\tilde{G} = (A, B_1, B_2)$ *such that* $V = \{v_0, v_1, v_2, v_3, v_4, v_5\}$. *Let* $R_1 = \{(v_0, v_1)(v_0, v_2), (v_3, v_4)\}$ *and* $R_2 = \{(v_1, v_2)(v_4, v_5)\}$. *Let*

$A = \{< v_0, 0.2, 0.5 >, < v_1, 0.3, 0.4 >, < v_2, 0.4, 0.6 >, < v_3, 0.3, 0.5 >,$
$< v_4, 0.4, 0.6 >, < v_5, 0.3, 0.5 >\}, B_1 = \{< (v_0 v_1, (0.1, 0.6) >, < v_0 v_2, (0.1, 0.7) >, < v_3 v_4, (0.2, 0.65) >\},$
$B_2 = \{< (v_1 v_2, (0.2, 0.65) >, < v_4 v_5, (0.2, 0.7) >\}$

Let ϕ be the permutation on set $\{B_1, B_2\}$ defined by $\phi(B_1) = B_2$ then

$$t_{B_1}^\phi(uv) = t_A(u) \wedge t_A(v) - t_{B_1}(uv), \quad f_{B_1}^\phi(uv) = f_{B_1}(uv) - (f_A(u) \wedge f_A(v))$$

$$t_{B_2}^\phi(uv) = t_A(u) \wedge t_A(v) - t_{B_2}(uv)$$

$$f_{B_2}^\phi(uv) = f_{B_2}(uv) - (f_A(u) \wedge f_A(v))$$

$$t_{B_1}^\phi(v_0 v_1) = 0.1, \ f_{B_1}^\phi(v_0 v_1) = 0.1$$

$$t_{B_1}^\phi(v_0 v_2) = 0.1, \ f_{B_1}^\phi(v_0 v_2) = 0.1$$

$$t^{\phi}_{B_1}(v_3v_4) = 0.1,\ f^{\phi}_{B_1}(v_3v_4) = 0.05$$
$$t^{\phi}_{B_2}(v_1v_2) = 0.1,\ f^{\phi}_{B_2}(v_1v_2) = 0.05$$
$$t^{\phi}_{B_2}(v_4v_5) = 0.1,\ f^{\phi}_{B_2}(v_0v_1) = 0.1$$

Proposition 4.1. *If* ϕ *is a cyclic permutation on* $\{B_1, B_2, ...B_k\}$ *of order* $m(1 \le m \le k)$, *then* $\tilde{G}^{\phi^m} = \tilde{G}$.

Proof: Since, ϕ^m is an identity permutation, we have

$$\tilde{G}^{\phi^m} = (A, B_1^{\phi^m}, B_2^{\phi^m}, ..., B_k^{\phi^m}) = (A, B_1, B_2, ..., B_k) = \tilde{G}.$$

Proposition 4.2. *Let* $\tilde{G} = (A, B_1, B_2, ..., B_k)$ *be a vague graph structure of graph structure* $G = (V, R_1, R_2, R_3, ..., R_k)$ *and let* ϕ *and* ψ *be two permutations on* $\{B_1, B_2, ..., B_k\}$, *then*

$(\tilde{G}^{\phi}) = \tilde{G}^{(\phi\circ\psi)}$. In particular $\tilde{G}^{(\phi\circ\psi)} = \tilde{G}$ if and only if ϕ and ψ are inverse of each other.

Proof: Proof is obvious.

Definition 4.5. *Let* $\tilde{G} = (A, B_1, B_2, ..., B_k)$ *and* $\tilde{G}' = (A', B_1', B_2', ..., B_k')$ *be two vague graphs on graph structures* $G = (V, R_1, R_2, R_3, ..., R_k)$ *and* $G' = (V', R_1', R_2', R_3', ..., R_k')$ *respectively, then* $\tilde{G}$ *is isomorphic to* $\tilde{G}'$ *if there exists a bijective mapping* $f : V \to V^{p}rime$ *and a permutation* ϕ *on* $\{B_1, B_2, ..., B_k\}$ *such that* $\phi(B_i) = B_j'$ *and*

(i) $\forall u \in V, t_A(u) = t_{A'}(f(u))$ and $f_A(u) = f_{A'}(f(u))$

(ii) $\forall uv \in R_i, t_{B_i}(uv) = t_{B_j'}(f(u)f(v))$ and $f_{B_i}(uv) = f_{B_j'}(f(u)f(v))$.

In particular, if $V = V', A = A'$ and $B = B'$ for all $i = 1,2,3,...,k$, then the above two vague graphs $\tilde{G}$ and $\tilde{G}'$ are identical.

Remark 4.1. *Identical vague graphs are always isomorphic, but converse need not be true. In example above , vague graphs* $\tilde{G}$ *and* $\tilde{G}^{\phi}$ *are isomorphic but they are not identical.*

Example 4.2. *Consider the two VGS* $\tilde{G} = (A, B_1, B_2)$ *and* $\tilde{G}' = (A', B_1', B_2')$ *such that* $V = \{v_0, v_1, v_2, v_3, v_4, v_5\}$ *and* $V' = \{v_0', v_1', v_2', v_3', v_4', v_5'\}$ *Let*

$A = \{< v_0, 0.2, 0.5 >, < v_1, 0.3, 0.4 >, < v_2, 0.4, 0.6 >, < v_3, 0.3, 0.5 >,$
$< v_4, 0.4, 0.6 >, < v_5, 0.3, 0.5 >\}$

$B_1 = \{< (v_0 v_1, (0.1, 0.6) >, < v_0 v_2, (0.1, 0.7) >, < v_3 v_4, (0.2, 0.65) >\},$
$B_2 = \{< (v_1 v_2, (0.2, 0.65) >, < v_4 v_5, (0.2, 0.7) >\}$
be VGS on V.

Definition 4.6. *Then it can be easily verified that* $\tilde{G} = (A, B_1, B_2)$ *and* $\tilde{G}' = (A', B_1', B_2')$ *are VGSs. Let* ϕ *be a permutation on* $\{B_1, B_2\}$ *such that* $\phi(B_i) - B_i'$ *and* $h : V \to V'$ *be a map defined by*

$$h(v_k) = \begin{cases} v'_{k+1} & if k = 0,1,2,3,4 \\ v'_k & if k = 5 \end{cases} \tag{i}$$

$$t_A(v_k) = t'_A(h(v_k)),\ f_A(v_k) = f'_A(h(v_k)) \forall v_k \in V \tag{ii}$$

$t_{B_i}(uv) = t_{B'_i}(h(u)h(v)),\ f_{B_i}(uv) = f_{B'_i}(h(u)h(v)) \forall (u,v) \in V \times V$ and $i = 1,2$

Hence $\tilde{G} \cong \tilde{G}'$ Let $\tilde{G}$ be a vague graph of graph structure G and ϕ is a permutation on the set $\{B_1, B_2, ... B_k\}$ then $\tilde{G}$ is ϕ-complementary if $\tilde{G}$ is isomorphic to $\tilde{G}^\phi$ and $\tilde{G}$ is strong ϕ - self complementary if $\tilde{G}$ is identical to $\tilde{G}^\phi$.

Example 4.3. *Consider the VGS* $\tilde{G} = (A, B_1, B_2)$ *such that* $V = \{u_1, u_2, u_3, u_4\}$. *Let*
$A = \{< u_1, 0.3, 0.5 >, < u_2, 0.4, 0.6 >, < u_3, 0.4, 0.6 >, < u_4, 0.3, 0.5 >\}$ *and*
$B_1 = \{< (u_1 u_2), 0.2, 0.7 >, < (u_4 u_1), 0.2, 0.7 >\}, B_2 = \{< (u_2 u_3), 0.2, 0.7 >, < (u_4 u_3), 0.2, 0.7 >\}$

Let ϕ be the permutation on the set $\{B_1, B_2\}$ defined by $\phi(B_1, B_2)$ defined by $\phi(B_1) = B_2$ and $\phi(B_2) = B_1$, then
$t^\phi_{B_1}(u_1 u_2) = 0.2,\ f^\phi_{B_1}(u_1 u_2) = 0.7 \quad t^\phi_{B_1}(u_4 u_1) = 0.2,\ f^\phi_{B_1}(u_4 u_1) = 0.7$ and
$t^\phi_{B_2}(u_2 u_3) = 0.2,\ f^\phi_{B_1}(u_2 u_3) = 0.7 \quad t^\phi_{B_2}(u_4 u_3) = 0.2,\ f^\phi_{B_1}(u_4 u_3) = 0.7$
Let there exist a bijective mapping $h : V \to V$ defined by

$h(u_1) = u_3, h(u_2) = u_4, h(u_3) = u_1, h(u_4) = u_2$.
$t_A((h(u_1)) = t_A(u_3) = 0.3 = t_A(u_1)$ and $f_A((h(u_1)) = f_A(u_3) = 0.5 = f_A(u_1)$
$t_A((h(u_2)) = t_A(u_4) = 0.4 = t_A(u_2)$ and $f_A((h(u_2)) = f_A(u_4) = 0.6 = f_A(u_2)$
$h(u_1) = u_3, h(u_2) = u_4, h(u_3) = u_1, h(u_4) = u_2$.
$t_A((h(u_3)) = t_A(u_1) = 0.3 = t_A(u_3)$ and $f_A((h(u_3)) = f_A(u_1) = 0.5 = f_A(u_3)$
$t_A((h(u_4)) = t_A(u_2) = 0.4 = t_A(u_4)$ and $f_A((h(u_4)) = f_A(u_2) = 0.6 = f_A(u_4)$
$t^\phi_{B_1}(h(u_1)h(u_2)) = t_{B_2}(u_3 u_4) = 0.2 = t_{B_1}(u_1 u_2)$ and
$f^\phi_{B_1}(h(u_1)h(u_2)) = f_{B_2}(u_3 u_4) = 0.7 = f_{B_1}(u_1 u_2)$
$t^\phi_{B_1}(h(u_1)h(u_4)) = t_{B_2}(u_3 u_2) = 0.2 = t_{B_1}(u_1 u_4)$ and

$$f^{\phi}_{B_1}(h(u_1)h(u_4)) = f_{B_2}(u_3u_2) = 0.7 = f_{B_1}(u_1u_4)$$

$$t^{\phi}_{B_2}(h(u_1 2h(u_3)) = t_{B_1}(u_4u_1) = 0.2 = t_{B_2}(u_2u_3) \text{ and}$$

$$f^{\phi}_{B_2}(h(u_2)h(u_3)) = f_{B_1}(u_4u_1) = 0.7 = f_{B_2}(u_2u_3)$$

$$t^{\phi}_{B_2}(h(u_4)h(u_3)) = t_{B_1}(u_2u_1) = 0.2 = t_{B_1}(u_4u_3) \text{ and}$$

$$f^{\phi}_{B_2}(h(u_4)h(u_3)) = f_{B_1}(u_2u_1) = 0.7 = f_{B_2}(u_4u_3)$$

Definition 4.7. *Let* $\tilde{G}$ *be a vague graph of graph structure* G *. Then*

(i) $\tilde{G}$ is self complementary (SC) if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ for some permutation ϕ.

(ii) $\tilde{G}$ is strong self complementary (SSC) if $\tilde{G}$ is identical to $\tilde{G}^{\phi}$ for some permutation ϕ other than the identity permutation.

(iii) $\tilde{G}$ is totally self complementary (TSC) if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ for every permutation ϕ.

(iv) $\tilde{G}$ is totally strong self complementary (TSSC) if $\tilde{G}$ is isomorphic to $\tilde{G}^{\phi}$ for every permutation ϕ, where ϕ is a permutation on the set $\{B_1, B_2, ..., B_k\}$

Remark 4.2. *Totally self complementary* $\Rightarrow$ *self complementary and totally strong self complementary* $\Rightarrow$ *strong self complementary, but converse is not true.*

Theorem 4.1. *Let* $\tilde{G}$ *be self complementary vague graph structure, for some permutation* ϕ *on the set* $\{B_1, B_2, ..., B_k\}$ *then for each* $i = 1,2,3,..k$ *we have*

$$\sum_{u \neq v} t_{B_i}(uv) + \sum_{u \neq v}\sum_{j \neq i}(\phi t_{B_j})(uv) = \sum_{u \neq v}(t_A(u) \wedge t_A(v)) \text{ and}$$

$$\sum_{u \neq v} f_{B_i}(uv) + \sum_{u \neq v}(f_A(u) \wedge f_A(v)) = \sum_{u \neq v}\sum_{j \neq i}(\phi f_{B_j})(uv)$$

Proof: Given $\tilde{G} = (A, B_1, B_2, ..., B_k)$ is ϕ - self complementary vague graph structure. There exists a bijective mapping $h : V \to V$ such that

$t_A(h(u)) = t_A(u)$ and $f_A(h(u)) = f_A(u)$, where

$t^{\phi}_{B_j}(h(u)h(v)) = t_{B_j}(uv)$ and $f^{\phi}_{B_j}(h(u)h(v)) = f_{B_j}(uv) \forall u, v \in V$ and $j = 1,2,..k$

By definition of ϕ-complement of VGS, we have

$\tilde{G}^{\phi} = (A, B_1^{\phi}, B_2^{\phi}, ..., B_k^{\phi})$ where for each $i = 1,2,3,...,k$, we have

$$t^{\phi}_{B_i}(h(u)h(v)) = t_A(h(u)) \wedge t_A(h(v)) - \sum_{j \neq i}(\phi t_{B_i})(h(u)h(v)) \text{ and}$$

$$f^{\phi}_{B_i}(h(u)h(v)) = \sum_{j \neq i}(\phi f_{B_i})(h(u)h(v)) - f_A(h(u)) \vee f_A(h(v))$$

$$t^{\phi}_{B_i}(uv) = t_A(u) \wedge t_A(v) - \sum_{j \neq i}(\phi t_{B_i})(h(u)h(v)) \text{ and}$$

$$f^{\phi}_{B_i}(h(u)h(v)) = \sum_{j\neq i}(\phi f_{B_i})(h(u)h(v)) - f_A(u)\vee f_A(v)$$

Now, $\sum_{u\neq v} t^{\phi}_{B_i}(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi t_{B_i})(h(u)h(v))$ and

$$\sum_{u\neq v} f^{\phi}_{B_i}(h(u)h(v)) = \sum_{u\neq v}\sum_{j\neq i}(\phi f_{B_i})(h(u)h(v)) - \sum_{u\neq v}(f_A(u)\vee f_A(v))$$

$\Rightarrow \sum_{u\neq v} t^{\phi}_{B_i}(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v)) - \sum_{u\neq v}\sum_{j\neq i}(\phi t_{B_i})(uv)$ and

$$\sum_{u\neq v} f^{\phi}_{B_i}(uv) = \sum_{u\neq v}\sum_{j\neq i}(\phi f_{B_i})(uv) - \sum_{u\neq v}(f_A(u)\vee f_A(v))$$

$\Rightarrow \sum_{u\neq v} t^{\phi}_{B_i}(uv) + \sum_{u\neq v}\sum_{j\neq i}(\phi t_{B_i})(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v))$ and

$$\sum_{u\neq v} f^{\phi}_{B_i}(uv) + \sum_{u\neq v}(f_A(u)\vee f_A(v)) = \sum_{u\neq v}\sum_{j\neq i}(\phi f_{B_i})(uv)$$

Remark 4.3. *The above theorem holds good for a strong self complementary VGS* $\tilde{G}$ *by using the identity mapping as the isomorphism.*

Corollary 4.1. *If an VGS* $\tilde{G}$ *is totally self complementary, then*

$\sum_{u\neq v}\sum_{j}(\phi t_{B_i})(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v))$ and

$$\sum_{u\neq v}\sum_{j}(\phi f_{B_i})(uv) = \sum_{u\neq v}(f_A(u)\vee f_A(v))$$

Proof: By theorem, $\sum_{u\neq v} t_{B_i}(uv) + \sum_{u\neq v}\sum_{j}(t_{B_i})(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v))$ and $\sum_{u\neq v} f_{B_i}(uv) + \sum_{u\neq v}(f_A(u)\vee f_A(v)) = \sum_{u\neq v}\sum_{j\neq i}(\phi f_{B_j})(uv)$ hold for every permutation ϕ.

Using the identity permutation ϕ, we have

$\sum_{u\neq v}\sum_{j}(\phi t_{B_i})(uv) = \sum_{u\neq v}(t_A(u)\wedge t_A(v))$ and

$\sum_{u\neq v}\sum_{j}(\phi f_{B_i})(uv) = \sum_{u\neq v}(f_A(u)\vee f_A(v))$ ie.,

The sum of membership(non-membership) of all B_i edges $i = 1,2,3,...,k$ is equal to the sum of the minumum(maximum) of the membership(non-membership) of the corresponding vertices.

Remark 4.4. *The above result holds if VGS* $\tilde{G}$ *is totally strong self complementary.*

Theorem 4.2. *In an VGS* $\tilde{G}$*, if for all* $u, v\in V$ *and*

$t_{B_i}(uv) + \sum_{j\neq i}(\phi t_{B_i})(uv) = t_A(u)\wedge t_A(v)$ *and* $f_{B_i}(uv) + f_A(u)\vee f_A(v) = \sum_{j\neq i}(\phi f_{B_i})(uv)$,

then $\tilde{G}$ *is self complementary for a permutation* ϕ *on the set* $\{B_1, B_2, ..., B_k\}$

Proof: Let $h:V \to V$ be the identity map. Then we have,
$t_A(h(u)) = t_A(u)$ and $f_A(h(u)) = f_A(u)$
By definition of ϕ - complement of VGS, we have

$$t_{B_i}^{\phi}(h(u)h(v)) = t_A(h(u) \wedge t_A(h(v)) - \sum_{j \neq i}(\phi t_{B_i}(h(u)h(v)) = t_A(u) \wedge t_A(v) - \sum_{j \neq i}(\phi t_{B_i})(uv) = t_{B_i}(uv)$$
$$+\sum_{j \neq i}(\phi t_{B_i})(uv) - \sum_{j \neq i}(\phi t_{B_i})(uv) = t_{B_i}(uv)$$

And

$$f_{B_i}^{\phi}(h(u)h(v)) = \sum_{j \neq i}(\phi f_{B_i})(h(u)h(v)) - f_A(h(u)) \vee f_A(h(v)) = \sum_{j \neq i}(\phi f_{B_i}(uv) - (f_A(u) \vee f_A(v))$$
$$= \sum_{j \neq i}(\phi f_{B_i})(uv) - (\sum_{j \neq i}(\phi f_{B_i})(uv) - f_{B_i}(uv)) = f_{B_i}(uv)$$

$\therefore \tilde{G}$ is ϕ - complementary. Hence $\tilde{G}$ is self complementary for some permutation ϕ.

Remark 4.5. *In an VGS* $\tilde{G}$ *, if* $\forall u, v \in V$, *we have*

$$t_{B_i}(uv) + \sum_{j \neq i}(\phi t_{B_i})(uv) = t_A(u) \wedge t_A(v) \quad \textit{and} \quad f_{B_i}(uv) + f_A(u) \vee f_A(v) = \sum_{j \neq i}(\phi f_{B_i})(uv)$$

then $\tilde{G}$ *is self complementary for a permutation* ϕ *on the set* $\{B_1, B_2, ..., B_k\}$, *then* $\tilde{G}$ *is totally self complementary.*

5. Vague digraph in vulnerability assessment of gas pipeline networks

Vulnerability assessment of gas network can be categorized into structural components reliability, connectivity reliability, flow performance reliability,and interdependent reliability. These reliabilities depended on the type of pipe and fittings used, their again, and the connection between fitting and pipe. In most cases, we do not know the exact age and condition of connectivity. We can present these factors as a vague set. Any gas network can be represented as a vague digraph $G(F,P)$, where F is the vague set of pipe fittings, presenting their ages and connectivity conditions as degrees of membership $t_F(x)$ and non-membership $f_F(x)$, and P is a vague set of pipelines between fittings. In graph theoretic terms, P is a set of edges (ie., pipelines)between two vertices (ie., fittings). The degrees of membership $t_P(x,y)$ and non-membership $f_P(x,y)$ are calculated as $t_P(x,y) \leq min(t_F(x), t_F(y))$, and $f_P(x,y) \geq max(f_F(x), f_F(y))$. Consider the vague set of pipe fittings:

	C_1 C_2 C_3 C_4 C_5 C_6
$t_F(x)$	0.1 0.3 0.3 0.2 0.4 0.3
$f_F(x)$	0.7 0.5 0.6 0.7 0.5 0.5

The vague digraph $G = (F, P)$ of the gas pipeline network shown in Fig. 7 is represented by the following adjacency matrix as follows

$$G = \begin{bmatrix} (0.0,1.0) & (0.1,0.8) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.2,0.7) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.3,0.6) & (0.2,0.7) \\ (0.1,0.8) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.2,0.8) & (0.1,0.7) \\ (0.1,0.7) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \\ (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) & (0.0,1.0) \end{bmatrix}$$

The final weighted digraph WG that can be used for different kind of vulnerabilities can be calculated by finding the ranks of edges $S_i = f_{P(i)} - t_{P(i)} * \pi_{P(i)}$. The final adjacency matrix and weighted digraph, shown in Fig 6 are developed based on these weights.

$$WG = \begin{bmatrix} 0 & 0.73 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.51 & 0.6 \\ 0.73 & 0 & 0 & 0 & 0.68 & 0.64 \\ 0.64 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The overall algorithm is explained in Algorithm 1. It takes a vague set of pipeline fittings as an input. Lines 3-6 calculate the degrees of membership and non-membership for edges, and Line 7 assigns them to vague set of edges and adjacency matrix is prepared in Line 8. Finally, a weighted adjacency matrix is calculated in Lines 9-12 using rank techniques based on the degrees of membership and non-membership. This weighted matrix is printed in Line 13 and is used for calculating vulnerability in Line 14.

Algorithm 1:

```
      void fuzzy Pipeline Vulnerability()
        F = Vague set of Pipeline fitting;   Count Fitt = count(F);   P = Empty Vague set;
          for(int x = 0; x  <  Count Fitt ; x++){
          for(int y = 0; y  <  Count Fitt ; y++){
             if ( $F(x)$ is adjacent to $F(y)$){
                  $t_{P(xy)} = min(t_{F(x)}, t_{F(y)})$;    $f_{P(xy)} = max(f_{F(x)}, f_{F(y)})$;
              }
           }
        }
     $P$ = Vague set of edges;   $G$ = Vague relation (Adjacency matrix of $F \times F$);
   $WG$ = Weighted relation (Adjacency matrix of $F \times F$);   no. of Edges = Count(P);
for(int i=0 ; i  <  no. of Edges ; i++){
```

```
        $S_i = f_{P(i)} - t_{P(i)} * \pi_{P(i)}$
        x = Adjacent from Node of $P_i$;
        y = Adjacent from Node of $P_i$;   $WG_{xy} = S_i$;
        }
        print $WG$   Calculated vulnerability using $WG$
    }
```

6. Conclusion

Vague graph was very much useful to analyse the partial information on the boundaries and hence get the knowledge on that information to calculate the approximations. Most of the actions in real life situations are time dependent and also ambigous in partial information, symbolic models in expert system are more effective than traditional methods to identify bounds of the true and false membership values. In this paper, we introduced the concept of intense and feeble vague graphs based on the density of true and false degree of membership. We also define the mild balanced vague graph and strictly balanced vague graphs. Also understand some of the properties of sum and union of vague graphs which are mild balanced in nature. An application of vague graph is also discussed showing the vulnerability assessment of gas pipeline networks on vague graph.

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