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Intuitionistic Fuzzy *WI*-Ideals of Lattice Wajsberg Algebras

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Abstract. In the present paper, we introduce the notions of intuitionistic fuzzy *WI*-ideal and intuitionistic fuzzy lattice ideal of lattice Wajsberg algebras. We show that every intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of lattice Wajsberg algebra. Also, we discuss its converse part. Further, we obtain every intuitionistic fuzzy lattice ideal is an intuitionistic fuzzy *WI*-ideal in lattice *H*-Wajsberg algebra. Moreover, we discuss some characterizations of intuitionistic fuzzy *WI*-ideal.

Keywords: Wajsberg algebra; Lattice Wajsberg algebra; *WI*-ideal; Fuzzy *WI*-ideal; Intuitionistic fuzzy *WI*-ideal; Intuitionistic fuzzy lattice ideal.

AMS Mathematics Subject Classification (2010): 03B50, 03G10, 03G25, 03E72

1. Introduction

Non classical logic including many-valued logic and fuzzy logic takes the advantage of the classical logic to handle information with various facts of uncertainty, such as fuzziness and randomness. Therefore, nonclassical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. The theory of fuzzy set was introduced by Zadeh [13] in 1965 has several application in many fields. The idea of fuzzy set handles uncertainty and vagueness. In fuzzy set theory the membership of an element to a fuzzy set is a single value between zero and one. The generalization of fuzzy set was proposed by Atanassov [1, 2] as intuitionistic fuzzy set which incorporate the non-membership degree (that is, 1 minus sum of membership degree), and is quite interesting and has many useful applications in many areas. Fuzzy sets and intuitionistic fuzzy sets are two strong frameworks for uncertainty handling. The concept of Wajsberg algebra proposed by Mordchaj Wajsberg [12] in 1935. In [4], Chang introduced MV-algebras a kind of algebraic counterpart of the multivalued Lukasiewicz propositional Calculi [5]. Wajsberg algebras are formulated in terms of the operations "implication" and "quasi complement". Rose et al. [11] published the proof of Wajsberg algebra in 1958. In 1984, Font et al. [6] introduced lattice structure of Wajsberg algebra.

The authors [7] introduced the notion of Wajsberg implicative ideal (*WI*-ideal) of lattice Wajsberg algebra and discussed some related properties. Moreover, the authors [8] introduced the notions of fuzzy *WI*-ideal and normal fuzzy *WI*-ideal of lattice Wajsberg algebras, and investigated their properties with suitable illustrations.

In this paper, the concept of intuitionistic fuzzy set is applied to *WI*-ideal, that is we introduce the notions of intuitionistic fuzzy *WI*-ideal and intuitionistic fuzzy lattice ideal of lattice Wajsberg algebras. We show that every intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of lattice Wajsberg algebra. Also, we verify its converse part. Further, we discuss the relationship between intuitionistic fuzzy *WI*-ideal and intuitionistic fuzzy lattice ideal in lattice *H*-Wajsberg algebra. Also, we investigate some properties of intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebras. Finally, we show that collection of *WI*-ideals of lattice Wajsberg algebra is an intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebra.

2. Preliminaries

In this section, we recall some basic notions and their properties that are necessary to develop our main results.

Definition 2.1. [6] Let (A, $\rightarrow$, *****, 1) be an algebra with quasi complement "*" and a binary operation "$\rightarrow$" is called a Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in A$,

(i) $1 \rightarrow x = x$

(ii) $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$

(iii) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$

(iv) $(x^* \rightarrow y^*) \rightarrow (y \rightarrow x) = 1$.

Proposition 2.2. [6] A Wajsberg algebra (A, $\rightarrow$, *****, 1) satisfies the following axioms for all $x, y, z \in A$,

(i) $x \rightarrow x = 1$

(ii) If $(x \rightarrow y) = (y \rightarrow x) = 1$ then $x = y$

(iii) $x \rightarrow 1 = 1$

(iv) $(x \rightarrow (y \rightarrow x)) = 1$

(v) If $(x \rightarrow y) = (y \rightarrow z) = 1$ then $x \rightarrow z = 1$

(vi) $(x \rightarrow y) \rightarrow ((z \rightarrow x) \rightarrow (z \rightarrow y)) = 1$

(vii) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$

(viii) $x \rightarrow 0 = x \rightarrow 1^* = x^*$

(ix) $(x^*)^* = x$

(x) $(x^* \rightarrow y^*) = y \rightarrow x$.

Definition 2.3. [6] A Wajsberg algebra A is called a lattice Wajsberg algebra if it satisfies the following conditions for all $x, y \in A$,

(i) The partial ordering "$\leq$" on a lattice Wajsberg algebra *A*, such that $x \leq y$ if and only if $x \to y = 1$

(ii) $(x \vee y) = (x \to y) \to y$

(iii) $(x \wedge y) = ((x^* \to y^*) \to y^*)^*$.

Note. From the definition 2.3 an algebra (*A*, $\vee$, $\wedge$, *****, 0, 1) is a lattice Wajsberg algebra with lower bound 0 and upper bound 1.

Proposition 2.4. [6] A lattice Wajsberg algebra (*A*, $\to$, *****, 1) satisfies the following axioms for all $x,\ y,\ z \in A$,

(i) If $x \leq y$ then $x \to z \geq y \to z$ and $z \to x \leq z \to y$

(ii) $x \leq y \to z$ if and only if $y \leq x \to z$

(iii) $(x \vee y)^* = (x^* \wedge y^*)$

(iv) $(x \wedge y)^* = (x^* \vee y^*)$

(v) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$

(vi) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$

(vii) $(x \to y) \vee (y \to x) = 1$

(viii) $x \to (y \vee z) = (x \to y) \vee (x \to z)$

(ix) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$

(x) $(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$

(xi) $(x \wedge y) \to z = (x \to y) \to (x \to z)$.

Definition 2.5. [7] The lattice Wajsberg algebra *A* is called a lattice *H*-Wajsberg algebra if $x \vee y \vee ((x \wedge y) \to z) = 1$ for all $x,\ y,\ z \in A$,
In a lattice *H*-Wajsberg algebra *A*, the following hold

(i) $x \to (x \to y) = (x \to y)$

(ii) $x \to (y \to z) = (x \to y) \to (x \to z)$.

Definition 2.6. [6] Let *L* be a lattice. An ideal *I* of *L* is a nonempty subset of *L* is called a lattice ideal if it satisfies the following axioms for all $x,\ y \in I$,

(i) $x \in I, y \in L$ and $y \leq x$ imply $y \in I$

(ii) $x, y \in I$ implies $x \vee y \in I$.

Definition 2.7. [7] Let *A* be a lattice Wajsberg algebra. Let *I* be a nonempty subset of *A*. Then *I* is called *WI*-ideal of lattice Wajsberg algebra *A* satisfies for all $x,\ y \in A$,

(i) $0 \in I$

(ii) $(x \to y)^* \in I$ and $y \in I$ imply $x \in I$.

Definition 2.8. [13] Let *X* be a set. A function $\mu : X \to [0, 1]$ is called a fuzzy subset on *X* for each $x \in X$, the value of $\mu(x)$ describes a degree of membership of *x* in μ.

Definition 2.9. [13] Let μ be a fuzzy subset of X then the complement of μ is denoted by μ^c and defined as $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$.

Definition 2.10. [8] Let A be a lattice Wajsberg algebra. A fuzzy subset μ of A is called a fuzzy *WI*-ideal of A if for any $x, y \in A$,

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq \min\{\mu((x \to y)^*), \mu(y)\}$.

Definition 2.11. [1] An intuitionistic fuzzy subset S in a non-empty set X is an object having the form $S = \{(x, \mu_S(x), \gamma_S(x)) / x \in X\} = (\mu_S, \gamma_S)$ where the functions $\mu_S(x): X \to [0, 1]$ and $\gamma_S(x): X \to [0, 1]$ denote the degree of membership and the degree of non-membership respectively, and $0 \leq \mu_S(x) + \gamma_S(x) \leq 1$ for any $x \in X$.

3. Intuitionistic fuzzy Wajsberg implicative ideal (Intuitionistic fuzzy *WI*-ideal)

In this section, we introduce the concept of an intuitionistic fuzzy *WI*-ideal and an intuitionistic fuzzy lattice ideal of lattice Wajsberg algebras. Also, we obtain some properties of an intuitionistic fuzzy *WI*-ideal.

Definition 3.1. Let A be a lattice Wajsberg algebra. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of A is called an intuitionistic fuzzy *WI*-ideal of A if it satisfies the following inequalities for any $x, y \in A$,

(i) $\mu_S(0) \geq \mu_S(x)$ and $\gamma_S(0) \leq \gamma_S(x)$

(ii) $\mu_S(x) \geq \min\{\mu_S((x \to y)^*), \mu_S(y)\}$

(iii) $\gamma_S(x) \leq \max\{\gamma_S((x \to y)^*), \gamma_S(y)\}$.

Example 3.2. Let $A = \{0, a, b, c, d, r, s, t, 1\}$ be a set with Figure (1) as a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on A as in Table (1) and Table (2).

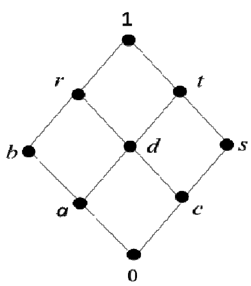


Figure 1:

x	x^*
0	1
a	t
b	b
c	r
d	d
r	c
s	b
t	a
1	0

Table 1:

$\to$	0	a	b	c	d	r	s	t	1
0	1	1	1	1	1	1	1	1	1
a	t	1	1	t	1	1	t	1	1
b	b	t	1	s	t	1	s	t	1
c	r	r	r	1	1	1	1	1	1
d	d	r	r	t	1	1	t	1	1
r	c	d	r	s	t	1	s	t	1
s	b	b	b	r	r	r	1	1	1
t	a	b	b	d	r	r	t	1	1
1	0	a	b	c	d	r	s	t	1

Table 2:

Define $\vee$ and $\wedge$ operations on *A* as follows,

$(x \vee y) = (x \to y) \to y$,

$(x \wedge y) = ((x^* \to y^*) \to y^*)^*$ for all $x, y \in A$.

Then (*A*, $\vee$, $\wedge$, *****, 0, 1) is a lattice Wajsberg algebra.

Consider an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on *A* as,

$$\mu_S(x) = \begin{cases} 1 & if \ \ x \in \{0, b\} \quad for\ all\ x \in A \\ 0.6 & otherwise \quad for\ all\ x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & if \ \ x \in \{0, b\} \quad for\ all\ x \in A \\ 0.4 & otherwise \quad for\ all\ x \in A \end{cases}$$

Then *S* is an intuitionistic fuzzy *WI*-ideal of *A*.

In the same Example 3.2, let us consider an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on *A* as,

$$\mu_S(x) = \begin{cases} 1 & if \ \ x \in \{a, b\} \quad for\ all\ x \in A \\ 0.32 & otherwise \quad for\ all\ x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & if \ \ x \in \{a, b\} \quad for\ all\ x \in A \\ 0.56 & otherwise \quad for\ all\ x \in A \end{cases}$$

Then *S* is not an intuitionistic fuzzy *WI*-ideal of *A* for $\mu_S(t) < \min\{\mu_S((t \to b)^*), \mu_S(b)\}$, and $\gamma_S(t) > \max\{\gamma_S((t \to b)^*), \gamma_S(b)\}$.

Example 3.3. Let *A* be a lattice Wajsberg algebra defined in example 3.2, define an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of *A* as follows,

(i) $\mu_S(0) = \mu_S(c) = 1$

(ii) $\mu_S(x) = m$ for any $x \in \{a, b, c, d, r, s, t, 1\}$

(iii) $\gamma_S(0) = \gamma_S(c) = 0$

(iv) $\gamma_S(x) = n$ for any $x \in \{a, b, c, d, r, s, t, 1\}$.

where $m, n \in [0, 1]$ and $m + n \leq 1$. Then $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A*.

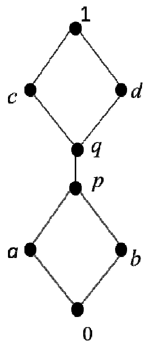
Example 3.4. Let $A = \{0, a, b, p, q, c, d, 1\}$ be a set with Figure (2) as a partial ordering. Define a quasi complement "$*$" and a binary operation "$\to$" on *A* as in Table (3) and Table (4).

Define $\vee$ and $\wedge$ operations on *A* as follows,

$(x \vee y) = (x \to y) \to y$,

$(x \wedge y) = ((x^* \to y^*) \to y^*)^*$ for all $x, y \in A$.

Then (*A*, $\vee$, $\wedge$, *****, 0, 1) is a lattice Wajsberg algebra.



x	x^*
0	1
a	b
b	a
p	0
q	0
c	0
d	0
1	0

$\rightarrow$	0	a	b	p	q	c	d	1
0	1	1	1	1	1	1	1	1
a	b	1	b	1	1	1	1	1
b	a	a	1	1	1	1	1	1
p	0	a	b	1	1	1	1	1
q	0	a	b	p	1	1	1	1
c	0	a	b	p	d	1	d	1
d	0	a	b	p	c	c	1	1
1	0	a	b	p	q	c	d	1

Figure 2: **Table 3:** **Table 4:**

Consider an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on A as,

$$\mu_S(x) = \begin{cases} 1 & if \;\; x \in \{0, \; p\} \quad for \; all \; x \in A \\ 0.56 & otherwise \quad for \; all \; x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & if \;\; x \in \{0, \; p\} \quad for \; all \; x \in A \\ 0.38 & otherwise \quad for \; all \; x \in A \end{cases}$$

Then S is an intuitionistic fuzzy *WI*-ideal of A.

In the same Example 3.4, let us consider an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ on A as,

$$\mu_S(x) = \begin{cases} 1 & if \;\; x \in \{0, p, b\} \;\; for \; all \; x \in A \\ 0.45 & otherwise \quad for \; all \; x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & if \;\; x \in \{0, p, b\} \;\; for \; all \; x \in A \\ 0.32 & otherwise \quad for \; all \; x \in A \end{cases}$$

Then S is not an intuitionistic fuzzy *WI*-ideal of A for $\mu_S(d) < \min\{\mu_S((d \rightarrow p)^*), \mu_S(p)\}$, and $\gamma_S(d) > \max\{\gamma_S((d \rightarrow p)^*), \gamma_S(p)\}$.

Proposition 3.5. Every intuitionistic fuzzy *WI*-ideal $S = (\mu_S, \gamma_S)$ of lattice Wajsberg algebra A is an intuitionistic monotonic, that is, if $x \leq y$, then $\mu_S(x) \geq \mu_S(y)$ and $\gamma_S(x) \leq \gamma_S(y)$.

Proof: Let $x, y \in A$, $x \leq y$. Then $(x \rightarrow y)^* = 1^* = 0$

$$\begin{aligned} \mu_S(x) &\geq \min\{\mu_S((x \rightarrow y)^*), \mu_S(y)\} \\ &= \min\{\mu_S(0), \mu_S(y)\} \\ &= \mu_S(y) \end{aligned}$$

Therefore $\mu_S(x) \geq \mu_S(y)$

Now, $\gamma_S(x) \leq \max\{\gamma_S((x \rightarrow y)^*), \gamma_S(y)\}$

$= \max\{\gamma_S(0), \gamma_S(y)\}$
$= \gamma_S(y)$

Therefore $\gamma_S(x) \le \gamma_S(y)$. ■

Proposition 3.6. Let an intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebra *A*. For any $x, y, z \in A$ which satisfies $x \le y^* \to z$ then $\mu_S(x) \ge \min\{\mu_S(y), \mu_S(z)\}$ and $\gamma_S(x) \le \max\{\gamma_S(y), \gamma_S(z)\}$.

Proof: Let $x, y, z \in A$, $x \le y^* \to z$

Then, we have $1 = x \to (y^* \to z)$
$= z^* \to (x \to y)$
$= (x \to y)^* \to z$ [from (x) of proposition 2.2]

and so, $((x \to y)^* \to z)^*) = 0$.

It follows from definition 3.1 that,

$$\mu_S(x) \ge \min\{\mu_S((x \to y)^*), \mu_S(y)\}$$
$$\ge \min\{\min\{\mu_S((x \to y)^* \to z)^*), \mu_S(z)\}, \mu_S(y)\}$$
$$= \min\{\min\{\mu_S(0), \mu_S(z)\}, \mu_S(y)\}$$
$$= \min\{\mu_S(y), \mu_S(z)\}$$

Hence $\mu_S(x) \ge \min\{\mu_S(y), \mu_S(z)\}$

$$\gamma_S(x) \le \max\{\gamma_S((x \to y)^*), \gamma_S(y)\}$$
$$\le \max\{\max\{\gamma_S((x \to y)^* \to z)^*), \gamma_S(z)\}, \gamma_S(y)\}$$
$$= \max\{\max\{\gamma_S((0), \gamma_S(z)\}, \gamma_S(y)\}$$
$$= \max\{\ \gamma_S(y), \gamma_S(z)\}$$

Hence $\gamma_S(x) \le \max\{\ \gamma_S(y), \gamma_S(z)\}$. ■

Definition 3.7. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ of lattice *A* is called an intuitionistic fuzzy lattice ideal of *A* if it satisfies the following for all $x, y \in A$,

(i) $S = (\mu_S, \gamma_S)$ is intuitionistic monotonic

(ii) $\mu_S(x \vee y) \ge \min\{\mu_S(x), \mu_S(y)\}$

(iii) $\gamma_S(x \vee y) \le \max\{\gamma_S(x), \gamma_S(y)\}$ for all $x, y \in A$.

Remark 3.8. In the definition 3.7 (ii) and (iii) can be equivalently replaced by $\mu_S(x \vee y) = \min\{\mu_S(x), \mu_S(y)\}$ and $\gamma_S(x \vee y) = \max\{\gamma_S(x), \gamma_S(y)\}$ respectively by γ.

Example 3.9. Let *A* be a lattice Wajsberg algebra defined in the Example 3.2 and $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy subset of *A* defined by

$$\mu_S(x) = \begin{cases} 1 & \text{if } x \in \{0, d\} \quad \text{for all } x \in A \\ m & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & \text{if } x \in \{0, d\} \quad \text{for all } x \in A \\ n & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

where $m, n \in [0, 1]$ and $m + n \leq 1$. Then $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy lattice ideal of *A*.

Proposition 3.10. Let *A* be a lattice Wajsberg algebra. Every intuitionistic fuzzy *WI*-ideal of *A* is an intuitionistic fuzzy lattice ideal of *A*.

Proof: Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy lattice ideal of *A*. Then, from the proposition 3.5 shows that $S = (\mu_S, \gamma_S)$ is intuitionistic monotonic.

Now $((x \vee y) \to y)^* = (((x \to y) \to y)) \to y)^* = (x \to y)^* \leq (x^*)^* = x$ for any $x, y \in A$, it follows from definition 3.1and definition 3.7 that,

$$\mu_S(x \vee y) \geq \min\{\mu_S((x \vee y) \to y)^*), \mu_S(y)\}$$
$$\geq \min\{\mu_S(x), \mu_S(y)\}$$
$$\gamma_S(x) \leq \max\{\gamma_S((x \vee y) \to y)^*), \gamma_S(y)\}$$
$$\leq \max\{\gamma_S(x), \gamma_S(y)\}$$

Hence, we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy lattice ideal of *A*. ■

The following example shows that the converse of proposition 3.10 is not true.

Example 3.11. Let *A* be a lattice Wajsberg algebra defined in the Example 3.3 and $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy subset of *A* defined by,

$$\mu_S(x) = \begin{cases} 1 & \text{if } x \in \{0, b, d\} \quad \text{for all } x \in A \\ 0.7 & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

$$\gamma_S(x) = \begin{cases} 0 & \text{if } x \in \{0, b, d\} \quad \text{for all } x \in A \\ 0.3 & \text{otherwise} \quad \text{for all } x \in A \end{cases}$$

Then, we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy lattice ideal of *A*, but not an intuitionistic fuzzy *WI*-ideal of *A* for $\mu_S(p) < \min\{\mu_S((p \to d)^*), \mu_S(d)\}$ and

$$\gamma_S(p) > \max\{\gamma_S((p \to d)^*), \gamma_S(d)\}.$$

Proposition 3.12. In a lattice *H*-Wajsberg algebra *A*, every intuitionistic fuzzy lattice ideal of *A* is an intuitionistic fuzzy *WI*-ideal of *A*.

Proof: Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy lattice ideal of *A*. Clearly $\mu_S(0) \geq \mu_S(x)$ and $\gamma_S(0) \leq \gamma_S(x)$ for any $x \in A$. Now, $x \leq x \vee y$ for all $x, y \in A$

It follows from definition 3.7 that,

$$\mu_S(x) \geq \mu_S(x \vee y)$$
$$= \mu_S(y \vee (x^* \vee y)^*)$$
$$= \mu_S(y \vee (x \to y)^*)$$
$$\mu_S(x) \geq \min\{\mu_S(y), \mu_S(x \to y)^*\},$$

and $\gamma_S(x) \le \gamma_S(x \vee y)$

$= \gamma_S(y \vee (x^* \vee y)^*)$

$= \gamma_S(y \vee (x \to y)^*)$

$\gamma_S(x) \le \max\{\gamma_S(y), \gamma_S(x \to y)^*)\}$.

Thus $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A*. ■

Proposition 3.13. Let *A* be a lattice Wajsberg algebra. An intuitionistic fuzzy subset $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A* if and only if the fuzzy subsets μ_S and γ_S^c are fuzzy *WI*-ideals of *A*, where $\gamma_S^c(x) = 1 - \gamma_S(x)$ for any $x \in A$.

Proof: Let $S = (\mu_S, \gamma_S)$ be an intuitionistic fuzzy *WI*-ideal of *A*. Clearly, μ_S is a fuzzy *WI*-ideal of *A*. For any $x, y \in A$, we have

$\gamma_S^c(0) = 1 - \gamma_S(0)$

$\ge 1 - \gamma_S(x)$

$\gamma_S^c(0) = \gamma_S^c(x)$

and $\gamma_S^c(x) = 1 - \gamma_S(x)$

$\ge 1 - \max\{\gamma_S((x \to y)^*), \gamma_S(y)\}$

$= \min\{1 - \gamma_S((x \to y)^*), 1 - \gamma_S(y)\}$

$\gamma_S^c(x) = \min\{\gamma_S^c((x \to y)^*), \gamma_S^c(y)\}$

Hence, we have γ_S^c is a fuzzy *WI*-ideal of *A*.

Conversely, assume that μ_S and γ_S^c are fuzzy *WI*-ideals of *A*. For any $x, y \in A$, we get

$\mu_S(0) \ge \mu_S(x)$ and $1 - \gamma_S(0) = \gamma_S^c(0) \ge \gamma_S^c(x)$

$= 1 - \gamma_S(x)$

$\gamma_S(0) \le \gamma_S(x)$

$\mu_S(x) \ge \min\{\mu_S(x \to y)^*, \mu_S(y)\}$,

and $1 - \mu_S(x) = \mu_S^c(x)$

$\ge \min\{\mu_S^c((x \to y)^*), \mu_S^c(y)\}$

$= \min\{1 - \mu_S((x \to y)^*), 1 - \mu_S(y)\}$

$= 1 - \max\{\mu_S((x \to y)^*), \mu_S(y)\}$

$\gamma_S(x) \le \max\{\gamma_S((x \to y)^*), \gamma_S(y)\}$

Hence, we have $S = (\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A*. ■

Proposition 3.14. Let A be a lattice Wajsberg algebra and $S=(\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A*. Then $S=(\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A* if and only if (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy *WI*-ideals of *A*.

Proof: Let $S=(\mu_S, \gamma_S)$ be an intuitionistic fuzzy *WI*-ideal of *A*, then μ_S and γ_S^c are fuzzy *WI*-ideals of *A* from proposition 3.13. Hence, we have (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy *WI*-ideals of *A*.

Conversely, if (μ_S, μ_S^c) and (γ_S^c, γ_S) are intuitionistic fuzzy *WI*-ideals of *A*, then the fuzzy subsets μ_S and γ_S^c are fuzzy *WI*-ideals of *A*, hence $S=(\mu_S, \gamma_S)$ is an intuitionistic fuzzy *WI*-ideal of *A*. ■

Proposition 3.15. Let *A* be a lattice Wajsberg algebra, *V* a non-empty subset of [0, 1] and $\{I_t / t\in V\}$ a collection of *WI*-ideals of *A* such that

(i) $A=\bigcup\limits_{t\in v} I_t$

(ii) $r>t$ if and only if $I_r\subseteq I_t$ for any $r,t\in V$ then the intuitionistic fuzzy subset $S=(\mu_S, \gamma_S)$ of A defined by $\mu_S = Sup\{t\in V / x\in I_t\}$ and $\gamma_S = Inf\{t\in V / x\in I_t\}$ for any $x\in A$ is an intuitionistic fuzzy *WI*-ideal of *A*.

Proof: According to proposition 3.13, it is sufficient to show that μ_S and γ_S^c are fuzzy *WI*-ideals of *A* for any $x\in A$.

$$\mu_S(0) = Sup\{t\in V / 0\in I_t\} = SupV \geq \mu_S(x)$$

If there exist $x, y\in A$ such that $\mu_S(x)<\min\{\mu_S((x\to y)^*), \mu_S(y)\}$.

Then there exist t_1 such that $\mu_S(x)<t_1<\min\{\mu_S((x\to y)^*), \mu_S(y)\}$.

It follows that $t_1<\mu_S((x\to y)^*)$, $t_1<\mu_S(y)$,

and hence there exist $t_2, t_3\in V$, $t_2>t_1$, $t_3>t_1$, $(x\to y)^*\in I_{t_2}$ and $y\in I_{t_3}$.

It follows that $(x\to y)^*\in I_{t_2\wedge t_3}$ and $y\in I_{t_2\wedge t_3}$, hence $x\in I_{t_2\wedge t_3}$.

That is, $\mu_S(x) = Sup\{t\in V / x\in I_t\} \geq t_2\wedge t_3 > t_1$

Therefore, $\mu_S(x) > t_1$

This is a contradiction. Hence, we have μ_S is a fuzzy *WI*-ideal of *A*. γ_S^c is a fuzzy *WI*-ideal, which can be proved by similar method. ■

4. Conclusion

In this paper, we have introduced the definitions of intuitionistic fuzzy *WI*-ideal and intuitionistic fuzzy lattice ideal of lattice Wajsberg algebra. We have discussed some of their properties with illustrations. Also, we have shown that every intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebra is an intuitionistic fuzzy lattice ideal of lattice Wajsberg algebra. But, the converse part is true only in the lattice *H*-Wajsberg algebras. Finally, we have shown that collection of *WI*-ideals of lattice Wajsberg algebras is an intuitionistic fuzzy *WI*-ideal of lattice Wajsberg algebras. We hope that more links of intuitionistic fuzzy subsets and logics emerge by the stipulating of this work.

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