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Multicriteria Decision Making for Fuzzy Set Based on Strategic Games

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Abstract. A game is a decision-making situation with many players, each having objectives that conflict with each other. The players involved in the game usually make their decisions under conditions of risk or uncertainty. In this paper, a fuzzy approach is proposed to solve the strategic game problem in which the pure strategy set for each player is already defined. Based on the concepts of fuzzy set theory, this approach will use a multicriteria decision-making method to obtain the optimal strategy in the game, a method which shows more advantages than the classical game methods. Moreover, with this approach, some useful conclusions are reached concerning the famous ªprisoner's dilemmaº problem in game theory.

Keywords:Fuzzy set, game, membership function, prisoner's dilemma, strategic form.

AMS Mathematics Subject Classification (2010): 91A05

1. Introduction

A game *is* a description of a decision-making situation involving more than one decision maker The behavior of players in a game is assumed to be rational and influenced by other rational players' behaviors, which distinguishes a game from the general decision-making problem. The decision makers, in general decision-making problems, face only an anonymous nature, which is a set of relevant states. Certainly, this nature is not rational.

There are three ways to represent the interaction of players in a game. First, extensive form is the most complete description of the situation in a game. This form not only details the information available to and motivation of players, but it also shows the various stages of the interaction and conditions for a player to move. Second, strategic form is more abstract than extensive form; it only gives all possible strategies of each player, along with the payoffs that result from the strategy choices of players. Finally, characteristic function form describes the social interactions in which player scan make agreements with each other. This description is usually used for representing coalitions in cooperative games.

In order to analyze the behaviors of players and construct a method for each player to choose his action, a strategic game first defines each individual's alternative actions. The combination of all the players' strategies will determine a unique outcome to the game and the payoffs to all players.

A solution for a player in a game should allow that player to win or satisfy his objectives for the game. For example, he can maximize his own payoff and/or minimize his opponent's payoff.

Formally, the solution of a game is a situation in which each player plays a best response to the other players' actual strategy choices. This is the concept of equilibrium. There are several methods for obtaining the equilibria of some special kinds of games such as the dominant strategy for dominant strategy equilibrium or a mixed strategy for mixed strategy equilibrium. A linear programming method is used in matrix games. Most of these methods are based on the maximin principle for selecting optimal strategies.

In this paper, we introduce the concept of distribution in probability and statistics. The optimal solution to the game is found using this fuzzy model. At the same time, this concept allows us to obtain the solution for a player in a game. Compared with the models in classical game theory, this is a more appropriate method for representing the vagueness of knowledge in a game. At the same time, it does not introduce too many solutions. For example, in the Nash equilibrium solution method, we may have too many equilibria and, hence, no way for the player to decide on an action. Another advantage of conditional fuzzy sets is that the knowledge of a player's payoff value need not be a precise number. Hence, the fuzzy model is more appropriate to represent the real situation. This fuzzy approach combines the multiple goals of a player into one fuzzy model by using the multi-criteria method. Using the weight vector to represent the philosophical motives or moral characteristics of a player makes it more general than the maximin principle in classical game theory.

Relative fuzzy concepts will be given at the beginning of Section II, along with our fuzzy approach. In order to explain the approach clearly, we will give two examples. In Section III, the fuzzy approach is used for prisoner's dilemma problem. From the results, some useful conclusions can be drawn. Section IV ends the paper with some conclusions on the fuzzy approach, pointing out some open questions for future researchers.

2. Preliminaries

Fuzzy approach to strategic games

Fuzzy approach to strategy games means that. A complete plan for a player to decide how to play the game. Without detailed knowledge of the sequence of moves in a strategic game, we can still analyze the behaviors of players; all that is needed for a solution to II a game is to indicate what the player would do in the situation when he must make a move. So it is sufficient if we list each individual's strategies, the combination of which will determine a unique outcome (payoff to each player) o f a game. A game is in strategic form if only the set of players I, the set of strategies Si and the payoff functions $p_i(s)$ for each player i €I are given, where S= $(s_1, s_2 \ldots S_i)$ the concepts of game in strategic form and mixed strategies used in this paper. Some reason for this difficulty is that we do not consider other features of the players, such as moral or philosophical motives, etc. We assume that all possible strategies are equally possible to each player, which is not very appropriate in the classical game. Another disadvantage of the classical game method is the maximin principle. In classical game theory such as in a mixed strategy game, a mixed strategy of player i is a vector

$m_i =$(mi1,mi2……mini) with $\sum_{k=1}^{n} \mathrm{mk} = 1$ and $m_{ik} \geq 0$

The meaning of this mixed strategy is a representation of the probability of choosing each pure strategy. So it can be noted that all elements of m_i are equally probable to the player i, which, in the real world, is not always true. A player's moral, aesthetic, or philosophical motives should also be considered in deciding the optimal strategy. This may result in some strategy being more or less likely to be chosen by a player. Probability and statistics now are insufficient to represent the ambiguity and vagueness in such models, especially for linguistically represented imprecision. Fortunately, we have another utility to measure the uncertainty: the fuzzy set theory. Indeed, imprecision is used in the sense of vagueness rather than vague conceptual phenomena that can be precisely and rigorously studied.

A classical (crisp) set is defined as a collection of elements or objects $x \in A$. The number of elements in the set can be finite, countable, or uncountable. We note that each single element can either belong to or not belong to a set $A\, A \subseteq X$. Such a classical set can be described by using the characteristic function. For instance, by stating a condition for membership such as $A = [x \in X/x \leq .5]$ we define a membership (characteristic) function $A(X)= 1$ means membership function of *x* in *A*. *A (x)* indicates non membership. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set *X*

For a fuzzy set A in terms of a relevant universal set there is a membership function

$$A : X \rightarrow [0,1],\ 0 \leq A(x) \leq 1.$$

$A(x)$ represents the degree to which x to A. Com-pared with a crisp subset of X , the range of the membership function of fuzzy set is a continuous interval [0,1] rather than the two element set [0, 1]. *A(x)* is the membership function that maps to *x* to the membership space S and the value of *A (x)* is the game of membership. *A (x)* is also the degree of compatibility or degree of truth of *x* in A. when S contains only two points 0 and 1, *A* is nonfuzzy and *A (x)* is to the characteristic function of a non fuzzy set. In other words, a crisp set (a non fuzzy set) is a special case of fuzzy sets.

A game contains conflict. There are at least two *rational* players whose decisions influence each other's payoffs. Usually their objective functions (payoff functions) conflict with each other. The players are regarded as rational if they try to maximize their objective functions based on their belief about their environment. This rationality, as well as moral and philosophical motives, usually affect the player's decisions. For example, if the player is cooperative, he may want to maximize his own objective functions as well as maximize other players' payoffs. If he is non cooperative, even *malicious*, he would not only like to maximize his own payoff, but at the same time minimize his opponents' payoffs. The attitude of players cannot be precisely represented in classical game theory. Here fuzzy set theory is used to construct a model that will overcome this shortcoming in the classical game model.

Let $\Omega_i = \{p_i \mid p_{ik} \geq 0\, \Sigma_{n_i}\, p_{ik} = 1\}$ be the strategy set of player i.

Assume that the game situation considered is *symmetrical* with respect to the uncertainty about the other players' attitude about strategies, so we need concern ourselves with one players view. , for instance player one's view. The others' actions can be similarly analyzed.

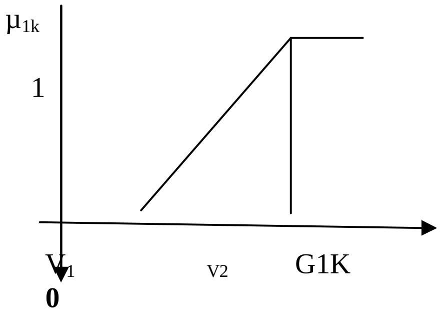


Figure 1: Membership function of F_{1k}

In order to represent more than one motive for the player, the multicriteria decision-making method is used. For example, the first goal of player 1 is to maximize the expected value of his own payoff. The second goal is to maximize his opponent's expected payoff, etc. This kind of model can represent the cooperative game in classical game theory. If the second goal is not to maximize his opponent's payoff, but to minimize his opponent's outcome, the model corresponds to the noncooperative game. G_{ik} is used to represent the kth goal function of player i, which is a function that measures the attainment of the kth goal for the player. Each goal function is assumed to be a function of $(p_1 \ldots p_k \ldots p_I)$ where I is the set of players. Obviously, each other player's reaction can affect the players payoff

Now the player 1 constructs a conditional fuzzy set Fi_k in Ω_k, which corresponds to the goal G_{1k} on the condition that th strategies of other players are μ_k, when the other player play the strategy combination ($p_1, p_2, \ldots p_k \ldots p_I$),player 1 can react to a change in his goal structure. The membership function F_{ik} is obtained as

$$\mu F_{ik} = \mu_{1k}(G_{1K}(P_1, P_{-1}) / P_{-1})$$

This membership function represents the degree of player 1 maximizing his kth goal $G_{1k}()$ and usually we choose the function shown in Fig. 1 to define this membership function.

So $\mu_{1k}(G_{1k}(p_1 p_{-1}) | p_{-1})$ is

$$\mu_{1k}(G_{1K}(P_1, /P_{-1}) = \frac{G1K(P1,P-1)-V1}{V2-V1}$$

For example, player one's first goal is to maximize his payoff. We simply assume the membership function to be

$$\mu_{1k}(G_{1K}(P_1, P_{-1}) / P_{-1}) = \frac{G1K(P1,P-1)-\min}{\max-\min} \quad (1)$$

where min and max are the minimum and maximum value of the objective function. If one of them is infinite, we will use a sufficiently large number to represent it. This is reasonable since in reality the objective function usually has a finite range.

So the membership function F_{1K} for goal $G_{1K}$is liner. At the maximum value of the objective function, it equals 1; at the minimum if G_{1k} will be 0 (sec Fig.2).
If his golal is to minimize a value, the membership function can be

$$\mu_{1k}(G_{1K}(P_1, P_{-1}) / P_{-1}) = \frac{G1K(P1,P-1)-\max}{\min-\max} \quad (2)$$

when $G_{1k}(p_1) = \max(\mu_{1k}(G_{1K}(P_1, P_{-1}) / P_{-1}) = 0$,if $G_{1K}(P_1, P_{-1}) = \min$, $\mu_{1k}(G(P_1, P_{-1}) / P_{-1}) = 1$.

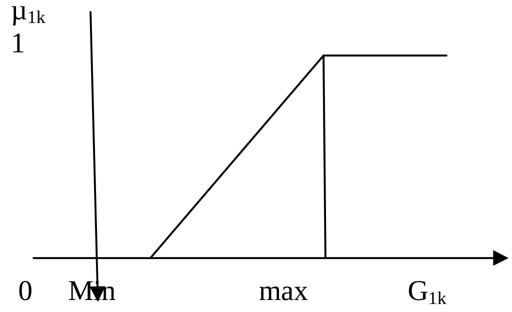


Figure 2: Membership function of F_{1k}

Note that player 1 is still uncertain about the strategies that the other players employ. We need another fuzzy set F_1 in $\Omega_1 x \Omega_2 x \ldots x \Omega_I$ to represent the possibility that strategy combination p_{-1} is chosen. The membership function of fuzzy set F_1 is denoted as μ_1. In fact, player 1 can use his experience or knowledge of the raw score outcomes of the game to construct the membership function F_1(Fig 3). Generally, if the player 1 does not have any knowledge of what the other players will choose or if he has little experience, he will assume the average value has mostly happened. For example if there only two players in the game and player 2 only has two pure strategy set S_2 the membership function for fuzzy set F1 is assumed to be

$$\mu_1(p_2) = \begin{cases} 2p2, if 0 \leq p2 \leq 0.5 \\ -2p2 + 2 \; if 0.5 \leq p2 \leq 1 \end{cases}$$

player 1 has no bias toward any particular strategy which player 2 will choose because he does not have enough experience or does not wish to do so.

In order to obtain the optimal strategy for player1 we first need to construct an unconditional fuzzy set F_{ik}' is

$$\mu'_{1k}(p_1) = \max(\mu_{1k}(G_{1K}(P_1,P_{-1})/ P_{-1}) . \mu_1(P_{-1})) \quad (3)$$

The multiplication $\mu_{1k}.\mu_1$ accounts for the passivity of the two events happening at the same time. The maximum operator on strategy combination P-will choose the most possible situation for the player 1 on all strategies of the other players. Thus based on all $\mu_{1k}'(p_1)$ that correspond to the kth goal of player 1,weight vector w= $(w_1,w_2 \ldots\ldots w_{n1})^T$ $\sum_{k=1}^{n1} \mathrm{WK} = 1$ is introduced to represent player 1's attitude about the structure of his goals. Then all fuzzy set are aggregated by the weight vector w to their membership functions. The result will be

$$\mu'_{1k}(p_1) = \max(\mu_{1k}(G_{1K}(P_1,P_{-1})/ P_{-1}) . \mu_1(P_{-1})). \quad (4)$$

The optimal strategy p_1* for player 1 is eventually determined as

$$\mu(p_1^*) = \max\left(\sum_{k=1}^{n1} wk.\mu 1k(p1)\right) \quad (5)$$

The optimal strategies for the other players are similarly obtained.

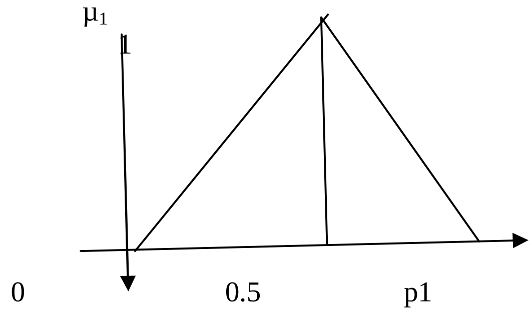


Figure 3: Membership function of F_1

3. Main results

Fuzzy model of the prisoner's dilemma using graphical method

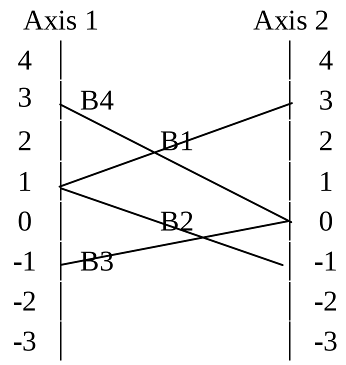
		P$_2$'s Strategies	
		B1	B2
P$_2$'s Strategies	A1	(1, 1)	(-1, 3)
	A2	(3, -1)	(0, 0)

Consider the problem in matrix form

P2's strategies

$$\text{P1'strategies}\quad \begin{matrix} & B_1 & B_2 & B_3 & B_4 \\ A_1 & 1 & 1 & -1 & 3 \\ A_2 & 3 & -1 & 0 & 0 \end{matrix}$$

Consider the two axes say axis I, axis II vertically at unit distance apart



Lower envelope

Maxmin Lines B_2,B_3

	B2	B3
A1	1	-1

A2	-1	0

Now the matrix be $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

where $\lambda = a_{11} + a_{22} - (a_{21} + a_{12})$

$= 1 - (-1 - 1) = 3$

$P_1 = a_{22} - a_{12} / \lambda$

$= 0 - (-1) / 3 = 1/3$

$P_2 = (1 - P_1) = (1 - 1/3) = 2/3$

The optimal strategy for A = (1/3, 2/3)

$q_1 = a_{22} - a_{21} / \lambda = 0 - (-1)/3 = 1/3$

$q_2 = (1 - q_1) = (1 - 1/3) = 2/3$

The optimal strategy for B = (0,1/3, 2/3,0)

Value of the game $V = a_{22}\, a_{11} - a_{12}\, a_{21} / \lambda$

Value of the game V = 1/3.

From this special case of the prisoner' dilemma, we see that the fuzzy model can easily represent the game situation. Moreover, it is possible to describe the prisoner's subjective attitudes or moral characteristics that are uncertain in the game and cannot be described by probability and statistics in classical game, although they are certain to affect the strategy chosen by players. This is only a special case of prisoner's dilemma, but we hope this model is general enough to represent all prisoner's dilemma problems, whatever the payoff matrix might be. The relationship between the payoff matrix and the weight vector is worthy of further study to help the prisoner decide his optimal strategy.

4. Conclusion

In this paper we recommend a fuzzy approach to strategies games. Using this method, a fuzzy model is constructed to represent the conflict situation in strategic games. This model uses the fuzzy set to represent the philosophical motives or moral characteristics of players in the game, which are usually vague and uncertain in practice and not easy to represent in the models of classical game theory. Moreover, the approach combines multiple goals of one player in the game into one model. This makes the model simpler. Compared with prisoner's dilemma problems in the classical game theory and given the weight vector information that represents the prisoner's subjective characteristics, the optimal strategy can be easily determined. This model is also more general since the maximin pinciple is a special case of it. Finally, we use the prisoner's dilemma using graphical method, we get the optimal strategy. It will be easier for the decision maker to choose his action based upon the above knowledge. From the examples, we see the solution will be difficult to compute for n-person games n>2, although this approach can be extended to n- person games.

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