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Non-Linear Arithmetic Operations on Interval Valued Trapezoidal Fuzzy Numbers

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Abstract. Fuzzy numbers have been introduced by Zadeh in order to deal with imprecise numerical quantities in a practical way. In this paper, some types of interval valued trapezoidal fuzzy numbers (IVTrFNs) and some non-linear arithmetic operations on interval valued trapezoidal fuzzy numbers have been proposed. Also relevant numerical illustrations are included to justify the above said notions.

Keywords: Fuzzy number, Interval valued trapezoidal fuzzy number, Non-linear arithmetic operations.

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965[10]. A fuzzy set is characterized by a membership function, whose values are crisp number in [0,1]. The concept of fuzzy number is a special version of fuzzy set, which is defined on the set of real numbers. Kauffmann and Gupta [3] studied the concept on fuzzy arithmetic. Dubois and Prade [2] presented some operations on fuzzy numbers. Mizumoto and Tanaka [4] discussed some properties of fuzzy numbers. Rezvani [5-7] studied the concept of multiplication operation on trapezoidal fuzzy numbers and presented the graded mean representation with triangular fuzzy numbers. Also a new method for ranking of generalized trapezoidal fuzzy numbers using perimeters was presented by her.

Basal [1] presented some non-linear arithmetic operations on triangular fuzzy number and Vahidi and Rezvani [9] presented some non-linear arithmetic operations on trapezoidal fuzzy numbers. Dinagar and Abirami introduced the concept of more generalized interval valued fuzzy numbers and their arithmetic operations under the ranking function.

In this paper, we have studied the concept of interval valued trapezoidal fuzzy number (IVTrFNs), and some non-linear arithmetic operations on IVTrFNs.

The paper is organized as follows: Firstly in section 2 of this paper, we recall the definition of interval valued trapezoidal fuzzy number and some operations on Interval valued trapezoidal fuzzy numbers also we introduced some types of Interval valued trapezoidal fuzzy numbers (IVTrFNs). In section 3, we discuss some non-linear arithmetic operations on IVTrFNs. In section 4, relevant numerical illustrations are presented. Finally in section 5, conclusion is also included.

2. Interval valued trapezoidal fuzzy numbers

Definition 2.1. Fuzzy number

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is normal,

(ii) $\tilde{A}$ is a convex set,

(iii) The support of $\tilde{A}$ is closed and bounded.

Definition 2.2. Interval valued trapezoidal fuzzy numbers (IVTrFNs)

An interval valued trapezoidal fuzzy number (IVTrFN) $\tilde{A}$ on R is given by $\tilde{A} = \left\{x, \left(\mu_A^L(x), \mu_A^U(x)\right), x\epsilon R\right\}$ and $\mu_A^L(x) \leq \mu_A^U(x)$ for all $x\epsilon R$. And it is denoted by ${}_{IVTr}\widetilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right]$, where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$ and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$ are the trapezoidal fuzzy numbers. It is also noted that $a_1^U \leq a_1^L$, $a_2^U \leq a_2^L$, $a_3^L \leq a_3^U$, $a_3^L \leq a_3^U$, $a_4^L \leq a_4^U$.

The membership function of interval valued trapezoidal fuzzy number is defined by $\mu_{\tilde{A}}$: x → [0,1] has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1^L}{a_2^L - a_1^L}, & a_1^L \leq x \leq a_2^L \\ \dfrac{a_4^L - x}{a_4^L - a_3^L}, & a_3^L \leq x \leq a_4^L \\ 1, & a_2^U \leq x \leq a_3^U \; and \; a_2^L \leq x \leq a_3^L \\ \dfrac{x - a_1^U}{a_2^U - a_1^U}, & a_1^U \leq x \leq a_2^U \\ \dfrac{a_4^U - x}{a_4^U - a_3^U}, & a_3^U \leq x \leq a_4^U \\ 0, & otherwise \end{cases}$$

2.4. The proposed ranking function

An efficient for comparing the fuzzy number is by the use of ranking function defined as $\mathcal{R}$:F(R) → R, where F(R) is a set of fuzzy numbers defined on a set of real number where a natural order exists.

For ${}_{IVTr}\widetilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right] \in$ F(R), then the ranking function $\mathcal{R}$:F(R) → R is defined as $\underline{\mathcal{R}}({}_{IVTr}\widetilde{A}) = (a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U)/8$.

Also we define orders on F(R) by

$\mathcal{R}({}_{IVTr}\widetilde{A}) \geq \mathcal{R}({}_{IVTr}\widetilde{B})$ if and only if ${}_{IVTr}\widetilde{A} \underset{\mathcal{R}}{\geq} {}_{IVTr}\widetilde{B}$,

$\mathcal{R}({}_{IVTr}\widetilde{A}) = \mathcal{R}({}_{IVTr}\widetilde{B})$ if and only if ${}_{IVTr}\widetilde{A} \underset{\mathcal{R}}{=} {}_{IVTr}\widetilde{B}$

and $\mathcal{R}({}_{IVTr}\widetilde{A}) \leq \mathcal{R}({}_{IVTr}\widetilde{B})$ if and only if ${}_{IVTr}\widetilde{A} \underset{\mathcal{R}}{\leq} {}_{IVTr}\widetilde{B}$.

Pictorial representation of IVTrFN ${}_{IVTr}\widetilde{A}$ is provided in fig.2.1.

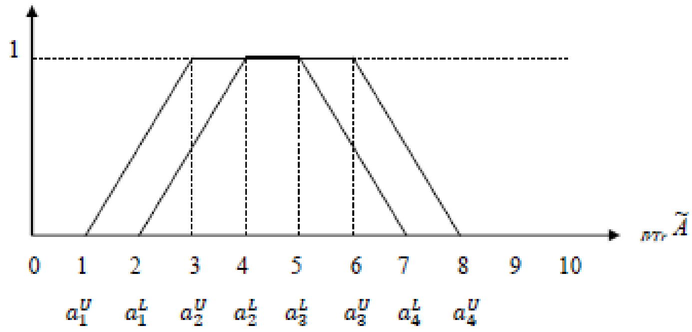

Figure 2.1: IVTrFN ${}_{IVTr}\widetilde{A}$

2.6. Arithmetic operations on interval valued trapezoidal fuzzy numbers (IVTrFNs)

Let ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ and

${}_{IVTr}\widetilde{B} = [(b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U)]$

(i) Addition for IVTrFNs:

${}_{IVTr}\widetilde{A} + {}_{IVTr}\widetilde{B} = [(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U)]$

(ii) Subtraction for IVTrFNs:

${}_{IVTr}\widetilde{A} - {}_{IVTr}\widetilde{B} = [(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L), (a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U)]$

(iii) Scalar multiplication for IVTrFNs:

If $k \geq 0$ and $k \in R$ then $k({}_{IVTr}\widetilde{A}) = [(ka_1^L, ka_2^L, ka_3^L, ka_4^L), (ka_1^U, ka_2^U, ka_3^U, ka_4^U)]$ and
if $k < 0$ and $k \in R$ then $k({}_{IVTr}\widetilde{A}) = [(ka_4^L, ka_3^L, ka_2^L, ka_1^L), (ka_4^U, ka_3^U, ka_2^U, ka_1^U)]$

(iv) Multiplication for IVTrFNs:

If $\mathcal{R}({}_{IVTr}\widetilde{B}) = (b_1^L + b_2^L + b_3^L + b_4^L + b_1^U + b_2^U + b_3^U + b_4^U)/8$.

${}_{IVTr}\widetilde{A} \times {}_{IVTr}\widetilde{B} = [(a_1^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_2^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_3^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_4^L \mathcal{R}({}_{IVTr}\widetilde{B})),$
$(a_1^U \mathcal{R}({}_{IVTr}\widetilde{B}), a_2^U \mathcal{R}({}_{IVTr}\widetilde{B}), a_3^U, \mathcal{R}({}_{IVTr}\widetilde{B}), a_4^U \mathcal{R}({}_{IVTr}\widetilde{B}))]$, when $\mathcal{R}({}_{IVTr}\widetilde{B}) \geq 0$

${}_{IVTr}\widetilde{A} \times {}_{IVTr}\widetilde{B} = [(a_4^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_3^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_2^L \mathcal{R}({}_{IVTr}\widetilde{B}), a_1^L \mathcal{R}({}_{IVTr}\widetilde{B})),$
$(a_4^U \mathcal{R}({}_{IVTr}\widetilde{B}), a_3^U \mathcal{R}({}_{IVTr}\widetilde{B}), a_2^U \mathcal{R}({}_{IVTr}\widetilde{B}), a_1^U \mathcal{R}({}_{IVTr}\widetilde{B}))]$, when $\mathcal{R}({}_{IVTr}\widetilde{B}) < 0$

(v) Division for IVTrFNs:

Whenever $\mathcal{R}({}_{IVTr}\widetilde{B}) \neq 0$, we define division as follows:

If $\mathcal{R}\,({}_{IVTr}\widetilde{B})>0$, then

$$\frac{{}_{IVTr}\widetilde{A}}{{}_{IVTr}\widetilde{B}} = [(a_1^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_2^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_3^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_4^L/\mathcal{R}\,({}_{IVTr}\widetilde{B})),\ (a_1^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_2^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_3^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_4^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}))]$$

If $\mathcal{R}\,({}_{IVTr}\widetilde{B})<0$, then

$$\frac{{}_{IVTr}\widetilde{A}}{{}_{IVTr}\widetilde{B}} = [(a_4^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_3^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_2^L/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_1^L/\mathcal{R}\,({}_{IVTr}\widetilde{B})),\ (a_4^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_3^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_2^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}),\ a_1^U/\mathcal{R}\,({}_{IVTr}\widetilde{B}))]$$

Definition 2.7. Zero interval valued trapezoidal fuzzy number
If ${}_{IVTr}\widetilde{A}=$ [(0,0,0,0), (0,0,0,0)], then ${}_{IVTr}\widetilde{A}$ is said to be a zero interval valued trapezoidal fuzzy number. It is denoted by ${}_{IVTr}0$.

Definition 2.8. Zero-equivalent interval valued trapezoidal fuzzy number
An interval valued trapezoidal fuzzy number is said to be a zero equivalent interval valued trapezoidal fuzzy number if $\mathcal{R}\,({}_{IVTr}\widetilde{A}) = 0$. It is denoted by ${}_{IVTr}\widetilde{0}$.

Definition 2.9. Unit interval valued trapezoidal fuzzy number
If ${}_{IVTr}\widetilde{A}=$ [(1,1,1,1), (1,1,1,1)], then ${}_{IVTr}\widetilde{A}$ is said to be an unit interval valued trapezoidal fuzzy number. It is denoted by ${}_{IVTr}1$.

Definition 2.10. Unit-equivalent interval valued trapezoidal fuzzy number
An interval valued trapezoidal fuzzy number is said to be an unit-equivalent interval valued trapezoidal fuzzy number if $\mathcal{R}\,({}_{IVTr}\widetilde{A}) = 1$. It is denoted by ${}_{IVTr}\widetilde{1}$.

Definition 2.11. Inverse interval valued trapezoidal fuzzy number
If ${}_{IVTr}\widetilde{A}$ be an interval valued trapezoidal fuzzy number and ${}_{IVTr}\widetilde{A} \neq {}_{IVTr}\widetilde{0}$. Then its inverse interval valued trapezoidal fuzzy number is defined by ${}_{IVTr}\widetilde{A}^{-1} = \dfrac{{}_{IVTr}\widetilde{1}}{{}_{IVTr}\widetilde{A}}$.

Definition 2.12. Positive interval valued trapezoidal fuzzy number
An interval valued trapezoidal fuzzy number is said to be a positive interval valued trapezoidal fuzzy number if $\mathcal{R}\,({}_{IVTr}\widetilde{A}) > 0$.

Definition 2.13. Negative interval valued trapezoidal fuzzy number

An interval valued trapezoidal fuzzy number is said to be a negative interval valued trapezoidal fuzzy number if $\mathcal{R}\,({}_{IVTr}\widetilde{A}) < 0$.

3. Non-linear arithmetic operations on interval valued trapezoidal fuzzy numbers

3.1. Modulus of an IVTrFN

Let ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L)\,,(a_1^U, a_2^U, a_3^U, a_4^U)\,]$ be an IVTrFN. Then its modulus is given by $\left|{}_{IVTr}\widetilde{A}\right| = \left|[(a_1^L, a_2^L, a_3^L, a_4^L)\,,(a_1^U, a_2^U, a_3^U, a_4^U)\,]\right|$

$$= \begin{cases} \left[\left(a_1^L, a_2^L, a_3^L, a_4^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U\right)\right], & \text{if } \mathcal{R}\left({}_{IVTr}\widetilde{A}\right) \geq 0, \\ \left[\left(-a_4^L, -a_3^L, -a_2^L, -a_1^L\right), \left(-a_4^U, -a_3^U, -a_2^U, -a_1^U\right)\right], & \text{if } \mathcal{R}\left({}_{IVTr}\widetilde{A}\right) < 0. \end{cases}$$

3.2. Square root of an IVTrFN

Whenever $\mathcal{R}\,({}_{IVTr}\widetilde{A}) > 0$, the square root of an IVTrFN ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L)\,,(a_1^U, a_2^U, a_3^U, a_4^U)\,]$ is defined by

$$\sqrt{{}_{IVTr}\widetilde{A}} = \left[\begin{array}{l} \left(\frac{a_1^L}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_2^L}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_3^L}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_4^L}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}\right) \\ , \left(\frac{a_1^U}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_2^U}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_3^U}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}, \frac{a_4^U}{\sqrt{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}}\right) \end{array}\right]$$

3.3. Exponentiation of a positive IVTrFN

By using the multiplication of two IVTrFNs, we define the exponentiation of an IVTrFN.
Let ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L)\,,(a_1^U, a_2^U, a_3^U, a_4^U)\,]$ be a positive IVTrFN.

When n ≥ 0,

$$\left({}_{IVTr}\widetilde{A}\right)^n = \Bigg[\left(a_1^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_4^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right),$$
$$\left(a_1^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_4^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right)\Bigg].$$

When n < 0,

$$\left({}_{IVTr}\widetilde{A}\right)^n = \Bigg[\left(\frac{a_1^L}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_2^L}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_3^L}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_4^L}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}\right),$$

$$\left(\frac{a_1^U}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_2^U}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_3^U}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}, \frac{a_4^U}{\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{1-n}}\right)\Bigg].$$

3.4. Exponentiation of a negative IVTrFN

Let ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ be a negative IVTrFN.

When n (≥ 1) is odd,

$$\left({}_{IVTr}\widetilde{A}\right)^n =$$
$$\Bigg[\left(a_1^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_4^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right),$$
$$\left(a_1^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_4^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right)\Bigg].$$

When n (≥ 1) is even,

$$\left({}_{IVTr}\widetilde{A}\right)^n =$$
$$\Bigg[\left(a_4^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_1^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right),$$
$$\left(a_4^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_3^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_2^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}, a_1^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{n-1}\right)\Bigg].$$

3.5. Exponential of a positive IVTrFN

We intend to formulate the exponential of an IVTrFN using the Taylor series expansion method. Since the Taylor series expansion of exponential of a real number is given by,

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots, -\infty < x < \infty.$$

For $1 = {}_{IVTr}1$ and $x = {}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ be a positive IVTrFN, we have

$$e^{{}_{IVTr}\widetilde{A}} = \Bigg[\Bigg(1 + \frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)} - 1\right), 1 + \frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)} - 1\right), 1$$
$$+ \frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)} - 1\right), 1 + \frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)} - 1\right)\Bigg),$$

$$\left(1+\frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),\right.$$

$$\left.1+\frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\right)\Bigg].$$

3.6. Exponential of a negative IVTrFN

Replacing 1 by ${}_{IVTr}1$ and x by ${}_{IVTr}\widetilde{A}=[(a_1^L,a_2^L,a_3^L,a_4^L),(a_1^U,a_2^U,a_3^U,a_4^U)]$ be a negative IVTrFN in above equation, we have

$$e^{-{}_{IVTr}\widetilde{A}}=\left[\left(1+\frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1\right.\right.$$

$$+\frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1$$

$$\left.+\frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\right)\left(1\right.$$

$$+\frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),$$

$$\left.\left.1+\frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\right)\right].$$

3.7. Inverse exponential of a positive IVTrFN

Using the definition of exponential of an positive IVTrFN, we have

$$e^{-{}_{IVTr}\widetilde{A}}=\left[\left(1+\frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-\right.\right.\right.$$

$$\left.1\right),\ 1+\frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\Bigg)\left(1+\right.$$

$$\frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+$$
$$\frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\Bigg)\Bigg].$$

3.8. Inverse exponential of a negative IVTrFN

Using the definition of exponential of a positive IVTrFN, we have

$$\begin{aligned} e^{-{}_{IVTr}\widetilde{A}} = \Bigg[\Bigg(&1+\frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1 \\ &+\frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1 \\ &+\frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\Bigg),\Bigg(1 \\ &+\frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1+\frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right), \\ &1+\frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right),1 \\ &+\frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{-\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}-1\right)\Bigg)\Bigg]. \end{aligned}$$

3.9. Properties on exponential of an IVTrFN

(i) $e^{{}_{IVTr}\widetilde{A}} \times e^{{}_{IVTr}\widetilde{B}} \underset{\mathcal{R}}{=} e^{{}_{IVTr}\widetilde{A}+{}_{IVTr}\widetilde{B}}$.

(ii) $\dfrac{e^{{}_{IVTr}\widetilde{A}}}{e^{{}_{IVTr}\widetilde{B}}} \underset{\mathcal{R}}{=} e^{{}_{IVTr}\widetilde{A}-{}_{IVTr}\widetilde{B}}$.

(iii) $\left(e^{{}_{IVTr}\widetilde{A}}\right)^{a} \underset{\mathcal{R}}{=} e^{a\,{}_{IVTr}\widetilde{A}}$.

(iv) $\left(e^{{}_{IVTr}\widetilde{A}}\right)^{e^{{}_{IVTr}\widetilde{B}}} \underset{\mathfrak{R}}{=} e^{{}_{IVTr}\widetilde{A} \times {}_{IVTr}\widetilde{B}}$.

3.10. Logarithm of a positive IVTrFN

We intend to formulate the logarithm of an IVTrFN using the Taylor series expansion method.

Since the Taylor series expansion of logarithm of a real number is given by,

$$log_e x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots, x > 0. \quad \ldots\ldots\ldots\ldots(2)$$

For $1 = {}_{IVTr}1$ and $x = {}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ be a positive IVTrFN, we have

$$log_e\left({}_{IVTr}\widetilde{A}\right) =$$

$$\left[\begin{array}{l}\left((a_1^L - 1)\frac{log_e\left({}_{IVTr}\widetilde{A}\right)}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_2^L - 1)\frac{log_e\left({}_{IVTr}\widetilde{A}\right)}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_3^L - 1)\frac{log_e\left({}_{IVTr}\widetilde{A}\right)}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_4^L -)\frac{log_e\left({}_{IVTr}\widetilde{A}\right)}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}\right), \\ \left((a_1^U - 1)\frac{log_e({}_{IVTr}\widetilde{A})}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_2^U - 1)\frac{log_e({}_{IVTr}\widetilde{A})}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_3^U - 1)\frac{log_e({}_{IVTr}\widetilde{A})}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}, (a_4^U - 1)\frac{log_e({}_{IVTr}\widetilde{A})}{\mathfrak{R}\left({}_{IVTr}\widetilde{A}\right)_{-1}}\right)\end{array}\right]$$

3.12. Properties on logarithm of an IVTrFN

(i) $log_e\left({}_{IVTr}\widetilde{A} \times {}_{IVTr}\widetilde{B}\right) \underset{\mathfrak{R}}{=} log_e\left({}_{IVTr}\widetilde{A}\right) + log_e\left({}_{IVTr}\widetilde{B}\right)$.

(ii) $\frac{log_e\left({}_{IVTr}\widetilde{A}\right)}{log_e\left({}_{IVTr}\widetilde{B}\right)} \underset{\mathfrak{R}}{=} log_e\left({}_{IVTr}\widetilde{A}\right) - log_e\left({}_{IVTr}\widetilde{B}\right)$.

(iii) $\boldsymbol{e}^{log_e\left({}_{IVTr}\widetilde{A}\right)} \underset{\mathfrak{R}}{=} {}_{IVTr}\widetilde{A}$.

3.13. Positive solution of n^th root of IVTrFN

If ${}_{IVTr}\widetilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ be a positive IVTrFN. Then the n^{th} ($n > 0$) root of ${}_{IVTr}\widetilde{A}$ is defined by,

$$\sqrt[n]{{}_{IVTr}\widetilde{A}} =$$

$$\left[\left(a_1^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_2^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_3^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_4^L\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}\right),\right.$$
$$\left.\left(a_1^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_2^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_3^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}, a_4^U\left(\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\right)^{\frac{1-n}{n}}\right)\right].$$

3.14. Formula for $a^{{}_{IVTr}\widetilde{A}}$, where a ≥ 1

Let $a^{{}_{IVTr}\widetilde{A}} = e^{\log_e a^{{}_{IVTr}\widetilde{A}}} = e^{{}_{IVTr}\widetilde{A}\log_e a}$

$$= e^{[(a_1^L \log_e a, a_2^L \log_e a, a_3^L \log_e a, a_4^L \log_e a),(a_1^U \log_e a, a_2^U \log_e a, a_3^U \log_e a, a_4^U \log_e a)]}$$

When ${}_{IVTr}\widetilde{A}$ is a positive IVTrFN, since from the definition of exponential of a positive IVTrFN, we get

$$a^{{}_{IVTr}\widetilde{A}} = \left[\left(1 + \frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right)\right)\right.$$
$$\left.\left(1 + \frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right), 1 + \frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)\log_e a} - 1\right)\right)\right].$$

When ${}_{IVTr}\widetilde{A}$ is a negative IVTrFN, since from the definition of exponential of a negative IVTrFN, we get

$$a^{{}_{IVTr}\widetilde{A}} = \Bigg[\Bigg(1+\frac{a_4^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_3^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_2^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_1^L}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right)\Bigg)$$

$$\Bigg(1+\frac{a_4^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_3^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_2^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right),1+\frac{a_1^U}{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\widetilde{A}\right)log_e a}-1\right)\Bigg)\Bigg].$$

3.15. Formula for ${}_{IVTr}\widetilde{A}^{{}_{IVTr}\widetilde{B}}$, where ${}_{IVTr}\widetilde{A}$ be a positive IVTrFN

If ${}_{IVTr}\widetilde{B}$ be a positive IVTrFN, then

$$
{}_{IVTr}\tilde{A}^{\;{}_{IVTr}\tilde{B}}
$$

$$
=\left[\begin{pmatrix}\left(1+\frac{b_1^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_2^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),\right.\\ \left.1+\frac{b_3^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_4^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right)\right)\end{pmatrix},\right.
$$

$$
\left.\begin{pmatrix}1+\frac{b_1^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_2^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),\\ 1+\frac{b_3^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_4^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right)\end{pmatrix}\right].
$$

If ${}_{IVTr}\tilde{B}$ be a negative IVTrFN, then

$$
{}_{IVTr}\tilde{A}^{\;{}_{IVTr}\tilde{B}}
$$

$$
=\left[\begin{pmatrix}\left(1+\frac{b_4^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_3^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),\right.\\ \left.1+\frac{b_2^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_1^L}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right)\right)\end{pmatrix},\right.
$$

$$
\left.\begin{pmatrix}1+\frac{b_4^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_3^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),\\ 1+\frac{b_2^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right),1+\frac{b_1^U}{\mathcal{R}\left({}_{IVTr}\tilde{B}\right)}\left(e^{\mathcal{R}\left({}_{IVTr}\tilde{A}\right)\mathcal{R}\left(\log_{e\,IVTr}\tilde{A}\right)}-1\right)\end{pmatrix}\right].
$$

3.16. Properties

(i) ${}_{IVTr}\tilde{A}^{\;{}_{IVTr}\tilde{1}} \underset{\mathcal{R}}{=} {}_{IVTr}\tilde{A}$.

(ii) ${}_{IVTr}\tilde{A}^{\;{}_{IVTr}\tilde{0}} \underset{\mathcal{R}}{=} {}_{IVTr}\tilde{1}$.

(iii) ${}_{IVTr}\tilde{A}^{\;{}_{IVTr}\tilde{B}} \underset{\mathcal{R}}{=} \left(\dfrac{{}_{IVTr}\tilde{1}}{{}_{IVTr}\tilde{A}}\right)^{-\,{}_{IVTr}\tilde{B}}$

(iv) $\left({}_{IVTr}\widetilde{A}^{-1} \right)^{-1} \underset{\mathcal{R}}{=} {}_{IVTr}\widetilde{A}$.

4. Numerical illustrations

(i) Consider $\sqrt{|[(-7,-6,-3,1),(-8,-6,-5,2)]|}$.
Let ${}_{IVTr}\widetilde{A}$ = $[(-7,-6,-3,1),(-8,-6,-5,2)]$, $\mathcal{R}\left({}_{IVTr}\widetilde{A}\right) = -4$.
Then, $\left| {}_{IVTr}\widetilde{A} \right|$ = $[(-1,3,6,7),(-2,5,6,8)]$
$\sqrt{\left| {}_{IVTr}\widetilde{A} \right|}$ = $\left[\left(\frac{-1}{2},\frac{3}{2},3,\frac{7}{2},\right)\left(-1,\frac{5}{2},3,4\right)\right]$.

(ii) Consider $e^{[(0,2,3,6),(-1,1,5,8)]}$. Then,
$e^{[(0,2,3,6),(-1,1,5,8)]}$
$\approx [(1,13.7237,20.0856,39.1711),(-5.3618,7.3618,32.8092,51.8948)]$.

(iii) Consider $[(1,2,5,7),(2,4,5,6)]^{[(2,4,5,6),(1,3,5,6)]}$. Then,

$[(1,2,5,7),(2,4,5,6)]^{[(2,4,5,6),(1,3,5,6)]}$
$\approx [(128.5,256,319.75,383.5),(64.75,192.25,319.75,383.5)]$.

5. Conclusion

In this paper, we discussed some types of interval valued trapezoidal fuzzy numbers and introduced some non-linear arithmetic operations like modulus, square root, exponentiation etc., of interval valued trapezoidal fuzzy numbers. By using these operations, the fuzzy transcendental equations corresponding to interval valued trapezoidal fuzzy numbers and their solutions may be utilized in future.

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