Intern. J. Fuzzy Mathematical Archive
Vol. 14, No. 2, 2017, 385-391
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v14n2a20

International Journal of **Fuzzy Mathematical Archive**

Truncations of Double Vertex Fuzzy Graphs

***K.Radha*[1] and *A.Uma*[2]**

[1]PG &Research Department of Mathematics
Periyar E.V.R College, Trichy-23, Tamilnadu, India.
Email: radhagac@yahoo.com
[2]Govt. Hr. Sec. School, Varusanadu-625579, Theni.(Dt), India.
Email: umabrte@gmail.com

Received 28 November 2017; accepted 9 December 2017

Abstract. In this paper, truncations of double vertex fuzzy graph of given graph is discussed. The relation between lower (upper) truncation of double vertex fuzzy graph of a given fuzzy graph and the double vertex fuzzy graph of the lower (upper) truncation of the given fuzzy graph is obtained.

Keywords: Fuzzy graph, double vertex fuzzy graph

AMS Mathematics Subject Classification (2010): 03E72, 05C07

1. Introduction

Fuzzy graph theory was introduced by Rosenfeld in 1975.The properties of fuzzy graphs have been studied by Rosenfeld [9]. Later on, Bhattacharya [7] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [3]. The conjunction of two fuzzy graphs was defined by Nagoor Gani and Radha [4]. Properties of truncations on fuzzy graphs were introduced and studied by Nagoorgani and Radha [5]. The concept of double vertex fuzzy graph and complete double vertex fuzzy graph were studied by Radha and Arumugam [10]. In this paper we discussed about some properties of truncations on double vertex fuzzy graphs and complete double vertex fuzzy graphs. Many works have been published by Pal et al. [11-16].

2. Preliminaries

In this section, let us recall some preliminary definitions that can be found in [1-10].

A fuzzy graph G is a pair of functions (σ,μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$.

Let $G(\sigma,\mu)$ be a fuzzy graph. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*:(V,E)$ where $E\subseteq V\times V$.A fuzz graph G is an effective fuzzy graph if $\mu(u,v) =\sigma(u)\wedge\sigma(v)$ for all $u,v\in E$ and G is a complete fuzzy graph if $\mu(u,v) =\sigma(u)\wedge\sigma(v)$ for all $u,v\in V$. Therefore G is a complete fuzzy graph if and only if G is an effective fuzzy graph and G* is complete.

The degree of a vertex u of a fuzzy graph $G:(\sigma,\mu)$ with underlying crisp graph $G^*:(V,E)$ is defined as $d_G(u) = \sum\mu(uv)$ where the summation runs over al, the edges uv.

Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$ with order $n\geq 2$.The double vertex graph of G denoted by $D(G)$: (σ_d, μ_d) is the fuzzy graph on $D(G^*)$: (V_d, E_d) where vertex set V_d consists of all ${}_nC_2$ unordered pairs of V such that two vertices {x,y} and {u,v} are adjacent, that is, {x,y}{u,v} is an element of E_d if and only if $|\{x,y\}\cap\{u,v\}| = 1$ and if x=u, then y and v are adjacent in G*, defined by

$$\sigma_d(\{u_i,u_j\}) = \sigma(u_i) \wedge \sigma(u_j) \text{ for all} \{u_i,u_j\} \text{ in } V_d$$

and $\mu_d(\{u, u_i\}\{u, u_j\}) = \sigma(u) \wedge \mu\{u_i,u_j\}$ for all $\{u, u_i\}\{u, u_j\}$ in E_d.

The complete double vertex fuzzy graph of G, denoted by $CD(G)$: (σ_{cd}, μ_{cd}) is the fuzzy graph on $CD(G^*)$: (V_{cd}, E_{cd}) where vertex set V_{cd} consists of ${}_{n+1}C_2$ unordered pairs of V, that is, it consists of all the vertices of D(G) and all 2-element mutisets of the form {a,a}such that two vertices {x,y} and {u,v} are adjacent if and only if $|\{x,y\}\cap\{u,v\}| = 1$ and if x=u, then y and v are adjacent in G, defined by

$\sigma_{cd}(\{u_i,u_j\}) = \sigma(u_i) \wedge \sigma(u_j)$for all$\{u_i,u_j\}$ in V_{cd}

and $\mu_{cd}(\{u, u_i\}\{u, u_j\}) = \sigma(u) \wedge \mu\{u_i,u_j\}$ for all$\{u, u_i\}\{u, u_j\}$ in E_{cd}.

The lower and upper truncations of σ at a level t,$0<t\leq 1$,are the fuzzy subsets $\sigma_{(t)}$ and $\sigma^{(t)}$ defined respectively by

$\sigma_{(t)}(u) = \sigma(u)$ if $u\in\sigma_{(t)}$; $\sigma_{(t)}(u) = 0$ if $u\notin\sigma_{(t)}$

and $\sigma^{(t)} = t$ if $u\in\sigma^{(t)}$; $\sigma^{(t)} = \sigma(u)$, if $u\notin\sigma^{(t)}$

An isomorphism between two fuzzy graphs G_1:(V_1, σ_1,μ_1) and G_2:(V_2, σ_2,μ_2) is a bijective map h from V_1 to V_2 such that$\sigma_1(u) = \sigma_2(h(u))$for all u in V_1

and $\mu_1(uv) = \mu_2(h(uv))$ for all u, v in V_1.

3. Lower and upper truncations of a double vertex fuzzy graph

In the first four theorems, we obtain the relationship between the lower (upper) truncation of the double vertex fuzzy graph of a given fuzzy graph G andthe double vertex fuzzy graph of the lower (upper) truncation of G.

Theorem 3.1. The lower truncation of the double vertex fuzzy graph of a fuzzy graph $G(\sigma,\mu)$ is the double vertex fuzzy graph of the lower truncation of G.That is,$[D(G)]_{(t)} = D(G_{(t)})$

Proof: First we prove that $(\sigma_d)_{(t)}=(\sigma_{(t)})_d$

Let $\{u_i,u_j\}$ be any element in V_d

Without loss of generality assume that $\sigma(u_i)\leq \sigma(u_j)$.

Then we have the following three cases:

$t \leq\sigma(u_i)\leq \sigma(u_j)$, $\sigma(u_i)\leq t \leq\sigma(u_j)$ or $\sigma(u_i)\leq \sigma(u_j) \leq t$

Case 1: $t\leq\sigma(u_i)\leq \sigma(u_j)$

Then $\sigma_{(t)}(u_i) = \sigma(u_i)$, $\sigma_{(t)}(u_j) = \sigma(u_j)$and $\sigma(u_i) \wedge \sigma(u_j)= \sigma(u_i) \geq t$

$\sigma(u_i) \wedge \sigma(u_j)\geq t \Rightarrow \sigma_d(\{u_i,u_j\}) \geq t \Rightarrow (\sigma_d)_t(\{u_i,u_j\}) = \sigma_d(\{u_i,u_j\})$

Also $(\sigma_{(t)})_d(\{u_i,u_j\}) = \sigma_{(t)}(u_i) \wedge \sigma_{(t)}(u_j)$

$= \sigma(u_i) \wedge \sigma(u_j)$

$= \sigma_d(\{u_i,u_j\})$

$\therefore\ (\sigma_d)_t(\{u_i,u_j\}) = (\sigma_{(t)})_d\{u_i,u_j\}$

Case 2: $\sigma(u_i) \le t \le \sigma(u_j)$

Then $\sigma_{(t)}(u_i)= 0$, $\sigma_{(t)}(u_j) = \sigma(u_j)$ and $\sigma(u_i) \wedge \sigma(u_j) = \sigma(u_i) \le t$

$\therefore\ \sigma_d(\{u_i,u_j\}) = \sigma(u_i) \wedge \sigma(u_j) \le t \Rightarrow (\sigma_d)_{(t)}(\{u_i,u_j\}) = 0$

Also $(\sigma_{(t)})_d\{u_i,u_j\} = \sigma_{(t)}(u_i) \wedge \sigma_{(t)}(u_j)= 0 \wedge \sigma(u_j)= 0$

$\therefore\ (\sigma_d)_t(\{u_i,u_j\}) = (\sigma_{(t)})_d\{u_i,u_j\}$

Case 3: $\sigma(u_i) \le \sigma(u_j) \le t$

Then $\sigma_{(t)}(u_i)= 0$, $\sigma_{(t)}(u_j) = 0$ and $\sigma(u_i) \wedge \sigma(u_j) = \sigma(u_i) \le t$

$\therefore\ \sigma_d(\{u_i,u_j\}) \le t \Rightarrow (\sigma_d)_t(\{u_i,u_j\}) = 0$

Also $(\sigma_{(t)})_d(\{u_i,u_j\}) = \sigma_{(t)}(u_i) \wedge \sigma_{(t)}(u_j)= 0 \wedge 0 = 0$

$\therefore\ (\sigma_d)_{(t)}(\{u_i,u_j\}) = (\sigma_{(t)})_d(\{u_i,u_j\})$

Hence in all the three cases $(\sigma_d)_t(\{u_i,u_j\}) = (\sigma_{(t)})_d\{u_i,u_j\}$.

This is true for all $\{u_i,u_j\} \in (V_d)_{(t)}$

$\therefore\ (\sigma_d)_t = (\sigma_{(t)})_d$

Next we prove that $(\mu_{(d)})_{(t)} = (\mu_{(t)})_{(d)}$

Let $\{u, u_i\}\{u, u_j\}$ be any element in E_d.

Without loss of generality assume that $\sigma(u) \le \mu\{u_i,u_j\}$.

Then we have the following three cases to consider:

$t \le \sigma(u) \le \mu\{u_i,u_j\}$, $\sigma(u) \le t \le \mu\{u_i,u_j\}$ or $\sigma(u) \le \mu\{u_i,u_j\} \le t$

Case 1: $t \le \sigma(u) \le \mu\{u_i,u_j\}$

Then $\sigma_{(t)}(u) = \sigma(u)$, $\mu_{(t)}\{u_i,u_j\} = \mu\{u_i,u_j\}$ and $\sigma(u) \wedge \mu\{u_i,u_j\} = \sigma(u) \ge t$

$\therefore \mu_d(\{u, u_i\}\{u, u_j\}) \ge t \Rightarrow (\mu_d)_{(t)}(\{u, u_i\}\{u, u_j\})= \mu_d(\{u, u_i\}\{u, u_j\})$

Also $(\mu_{(t)})_d(\{u, u_i\}\{u, u_j\}) = \sigma_{(t)}(u) \wedge \mu_{(t)}\{u_i,u_j\}$

$= \sigma(u) \wedge \mu\{u_i,u_j\}$

$= \mu_d(\{u, u_i\}\{u, u_j\})$

$\therefore (\mu_d)_{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu_{(t)})_d(\{u, u_i\}\{u, u_j\})$

Case 2: $\sigma(u) \le t \le \mu\{u_i,u_j\}$

Then $\sigma(u) \le t \Rightarrow \sigma_{(t)}(u)= 0$ and $\mu_{(t)}\{u_i,u_j\} = \mu\{u_i,u_j\}$

Now $\mu_d(\{u, u_i\}\{u, u_j\})= \sigma(u) \wedge \mu\{u_i,u_j\}= 0 \wedge \mu\{u_i,u_j\}= 0$

Also $(\mu_{(t)})_d(\{u, u_i\}\{u, u_j\}) = \sigma_{(t)}(u) \wedge \mu_{(t)}\{u_i,u_j\}= 0 \wedge \mu\{u_i,u_j\}=0$

Hence $(\mu_d)_{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu_{(t)})_d(\{u, u_i\}\{u, u_j\})$

Case 3: $\sigma(u) \le \mu\{u_i,u_j\} \le t$

Then $\sigma(u) \le t \Rightarrow \sigma_{(t)}(u)= 0$ and $\mu_{(t)}\{u_i,u_j\}=0$

Now $\sigma(u) \wedge \mu\{u_i,u_j\} = \sigma(u) \le t \Rightarrow \mu_d(\{u, u_i\}\{u, u_j\}) \le t \Rightarrow (\mu_d)_{(t)}(\{u, u_i\}\{u, u_j\}) =0$

Also $(\mu_{(t)})_d(\{u, u_i\}\{u, u_j\})= \sigma_{(t)}(u) \wedge \mu_{(t)}(\{u_i,u_j\}) = 0 \wedge 0= 0$

Hence $(\mu_d)_{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu_{(t)})_d(\{u, u_i\}\{u, u_j\})$

Finally let us prove that $(V_d)_{(t)} = (V_{(t)})_d$ and $(E_d)_{(t)} = (E_{(t)})_d$.

$\{u_i,u_j\} \in (V_d)_{(t)} \Leftrightarrow \sigma_d(\{u_i,u_j\}) \geq t$

$\Leftrightarrow \sigma(u_i) \wedge \sigma(u_j) \geq t$

$\Leftrightarrow \sigma(u_i) \geq t$ and $\sigma(u_j) \geq t$

$\Leftrightarrow u_i \in V_{(t)}$ and $u_j \in V_{(t)}$

$\Leftrightarrow \{u_i,u_j\} \in (V_{(t)})_d$

Therefore $(V_d)_{(t)} = (V_{(t)})_d$

$\{u, u_i\}\{u, u_j\} \in (E_d)_{(t)} \Leftrightarrow \mu_d(\{u, u_i\}\{u, u_j\}) \geq t$

$\Leftrightarrow \sigma(u) \wedge \mu\{u_i,u_j\} \geq t$

$\Leftrightarrow \sigma(u) \geq t$ and $\mu\{u_i,u_j\} \geq t$

$\Leftrightarrow u \in V_{(t)}$ and $u_i,u_j \in E_{(t)}$

$\Leftrightarrow \{u, u_i\}\{u, u_j\} \in (E_{(t)})_d$

$\therefore \; (E_d)_{(t)} = (E_{(t)})_d$

Hence the theorem.

Theorem 3.2. The lower truncation of the double vertex fuzzy graph of a graph $G(\sigma, \mu)$ is isomorphic to the double vertex fuzzy graph of the lower truncation of the fuzzy graph

Proof: Using the above theorem 3.1, the identity mapping I from $(V_d)_{(t)}$ to $(V_{(t)})_d$ given by $I(\{u_i,u_j\}) = \{u_i,u_j\}, \forall \{u_i,u_j\}$ in $(V_d)_{(t)}$ provides the required isomorphism.

Theorem 3.3. The upper truncations of the double vertex fuzzy graph of a fuzzy graph $G(\sigma, \mu)$ is the double vertex fuzzy graph of the truncation of the fuzzy graph $G(\sigma, \mu)$.(ie) $[D(G)]^{(t)} = D(G^{(t)})$.

Proof: First we prove that $(\sigma_d)^{(t)} = (\sigma^{(t)})_d$

Let $\{u_i,u_j\}$ be any element in V_d

Without loss of generality assume that $\sigma(u_i) \leq \sigma(u_j)$.

Then we have the following three cases to consider:

$t \leq \sigma(u_i) \leq \sigma(u_j)$, $\sigma(u_i) \leq t \leq \sigma(u_j)$ or $\sigma(u_i) \leq \sigma(u_j) \leq t$

Case 1: $t \leq \sigma(u_i) \leq \sigma(u_j)$

Then $\sigma^{(t)}(u_i) = t$, $\sigma^{(t)}(u_j) = t$ and $\sigma(u_i) \wedge \sigma(u_j) = \sigma(u_i) \geq t$

Therefore $\sigma_d(\{u_i,u_j\}) \geq t \Rightarrow (\sigma_d)^{(t)}(\{u_i,u_j\}) = t$

Also $(\sigma^{(t)})_d(\{u_i,u_j\}) = \sigma^{(t)}(u_i) \wedge \sigma^{(t)}(u_j) = t \wedge t = t$

Hence $(\sigma_d)^{(t)}(\{u_i,u_j\}) = (\sigma^{(t)})_d(\{u_i,u_j\})$

Case 2: $\sigma(u_i) \leq t \leq \sigma(u_j)$

Since $\sigma(u_i) \leq t$, $\sigma^{(t)}(u_i) = \sigma(u_i)$ and since $t \leq \sigma(u_j)$, $\sigma^{(t)}(u_j) = t$

Now $\sigma(u_i) \wedge \sigma(u_j) = \sigma(u_i) \leq t \Rightarrow \sigma_d(\{u_i,u_j\}) \leq t \Rightarrow (\sigma_d)^{(t)}(\{u_i,u_j\}) = \sigma_d(\{u_i,u_j\}) = \sigma(u_i)$

Also $(\sigma^{(t)})_d(\{u_i,u_j\}) = \sigma^{(t)}(u_i) \wedge \sigma^{(t)}(u_j) = \sigma(u_i) \wedge t = \sigma(u_i)$

Hence $(\sigma_d)^{(t)}(\{u_i,u_j\}) = (\sigma^{(t)})_d(\{u_i,u_j\})$.

Case 3: $\sigma(u_i) \leq \sigma(u_j) \leq t$
Then $\sigma^{(t)}(u_i) = \sigma(u_i)$ and $\sigma^{(t)}(u_j) = \sigma(u_j)$
Now $\sigma(u_i) \wedge \sigma(u_j) = \sigma(u_i) \leq t \Rightarrow \sigma_d(\{u_i,u_j\}) \leq t \Rightarrow (\sigma_d)^{(t)}(\{u_i,u_j\}) = \sigma_d(\{u_i,u_j\})$
Also $(\sigma^{(t)})_d(\{u_i,u_j\}) = \sigma^{(t)}(u_i) \wedge \sigma^{(t)}(u_j) = \sigma(u_i) \wedge \sigma(u_j) = \sigma_d(\{u_i,u_j\})$
Hence $(\sigma_d)^{(t)}(\{u_i,u_j\}) = (\sigma^{(t)})_d(\{u_i,u_j\})$
Hence in all the three cases $(\sigma_d)^{(t)}(\{u_i,u_j\}) = (\sigma^{(t)})_d(\{u_i,u_j\})$. This is true for all $\{u_i,u_j\} \in (V_d)^{(t)}$.

$\therefore (\sigma_d)^{(t)} = (\sigma^{(t)})_d$
Next we prove that $(\mu_d)^{(t)} = (\mu^{(t)})_d$
Let $\{u, u_i\}\{u, u_j\}$ be any element E_d.
Without loss of generality assume that $\sigma(u) \leq \mu(\{u_i,u_j\})$.
Then we have the following three cases to consider:
$t \leq \sigma(u) \leq \mu\{u_i,u_j\}, \sigma(u) \leq t \leq \mu\{u_i,u_j\}$ or $\sigma(u) \leq \mu\{u_i,u_j\} \leq t$.

Case 1: $t \leq \sigma(u) \leq \mu\{u_i,u_j\}$
Then $\sigma^{(t)}(u) = t$ and $\mu^{(t)}\{u_i,u_j\} = t$
Now $\sigma(u) \wedge \mu\{u_i,u_j\} = \sigma(u) \geq t \Rightarrow \mu_d(\{u, u_i\}\{u, u_j\}) \geq t \Rightarrow (\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = t$
Also $(\mu^{(t)})_d(\{u, u_i\}\{u, u_j\}) = \sigma^{(t)}(u) \wedge \mu^{(t)}\{u_i,u_j\} = t \wedge t = t$
Hence $(\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu^{(t)})_d(\{u, u_i\}\{u, u_j\})$.

Case 2: $\sigma(u) \leq t \leq \mu\{u_i,u_j\}$
Then $\sigma^{(t)}(u) = \sigma(u)$ and $\mu^{(t)}\{u_i,u_j\} = t$
Now $\sigma(u) \wedge \mu\{u_i,u_j\} = \sigma(u) \leq t \Rightarrow \mu_d(\{u, u_i\}\{u, u_j\}) \leq t$
$\Rightarrow (\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = \mu_d(\{u, u_i\}\{u, u_j\}) = \sigma(u)$
Also $(\mu^{(t)})_d(\{u, u_i\}\{u, u_j\}) = \sigma^{(t)}(u) \wedge \mu^{(t)}\{u_i,u_j\} = \sigma(u) \wedge t = \sigma(u)$
Hence $(\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu^{(t)})_d(\{u, u_i\}\{u, u_j\})$

Case 3: $\sigma(u) \leq \mu\{u_i,u_j\} \leq t$
Then $\sigma^{(t)}(u) = \sigma(u)$ and $\mu^{(t)}\{u_i,u_j\} = \mu\{u_i,u_j\}$
Now $\sigma(u) \wedge \mu\{u_i,u_j\} = \sigma(u) \leq t$,
$\therefore \mu_d(\{u, u_i\}\{u, u_j\}) \leq t \Rightarrow (\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = \mu_d(\{u, u_i\}\{u, u_j\})$
Also $(\mu^{(t)})_d(\{u, u_i\}\{u, u_j\}) = \sigma^{(t)}(u) \wedge \mu^{(t)}\{u_i,u_j\}$
$= \sigma(u) \wedge \mu\{u_i,u_j\}$
$= \mu_d(\{u, u_i\}\{u, u_j\})$
Hence $(\mu_d)^{(t)}(\{u, u_i\}\{u, u_j\}) = (\mu^{(t)})_d(\{u, u_i\}\{u, u_j\})$.
Finally let us prove that $(V_d)^{(t)} = (V^{(t)})_d$ and $(E_d)^{(t)} = (E^{(t)})_d$
$\{u_i,u_j\} \in (V_d)^{(t)} \Leftrightarrow \sigma_d(\{u_i,u_j\}) \leq t \Leftrightarrow \sigma(u_i) \wedge \sigma(u_j) \leq t \Leftrightarrow \sigma(u_i) \leq t$ and $\sigma(u_j) \leq t \Leftrightarrow u_i \in V^{(t)}$ and $u_j \in V^{(t)} \Leftrightarrow \{u_i,u_j\} \in (V^{(t)})_d$. Hence $(V_d)^{(t)} = (V^{(t)})_d$,
Similarly $(E_d)^{(t)} = (E^{(t)})_d$.
Hence the theorem.

Theorem 3.4. The upper truncation of the double vertex fuzzy graph of a graph G(σ, μ) is isomorphic to the double vertex fuzzy graph of the upper truncation of the fuzzy graph.
Proof: Using the above theorem 3.3 the identity mapping from $(V_d)^{(t)}$ to $(V^{(t)})_d$ provides the required isomorphism.

In the following theorems, we obtain the relationship between the lower (upper) truncation of the complete double vertex fuzzy graph of a given fuzzy graph G and the complete double vertex fuzzy graph of the lower (upper) truncation of G.

Theorem 3.5. The lower truncation of the Complete double vertex fuzzy graph of a fuzzy graph G(σ,μ) is the Complete double vertex fuzzy graph of the lower truncations of G. That is, $[CD(G)]_{(t)} = CD(G_{(t)})$
Proof: Proceeding as in theorem 3.1, it can be proved that $(\sigma_{cd})_{(t)}=(\sigma_{(t)})_{cd}$, $(\mu_{(cd)})_{(t)}=(\mu_{(t)})_{(cd)}$, $(V_{cd})_{(t)}=(V_{(t)})_{cd}$ and $(E_{cd})_{(t)}=(E_{(t)})_{cd}$. Hence the theorem.

Theorem 3.6. The lower truncation of the complete double vertex fuzzy graph of a graph G(σ, μ) is isomorphic to the complete double vertex fuzzy graph of the lower truncation of the fuzzy graph.
Proof: Using the above theorem 3.5, the identity mapping from $(V_{cd})_{(t)}$ to $(V_{(t)})_{cd}$ provides the required isomorphism.

Theorem 3.7. The upper truncations of the Complete double vertex fuzzy graph of a fuzzy graph G(σ, μ) is the Complete double vertex fuzzy graph of the truncation of the fuzzy graph G(σ, μ). i.e, $[CD(G)]^{(t)}=CD(G^{(t)})$
Proof: Proceeding as in theorem 3.3, it can be proved that $(\sigma_{cd})^{(t)} = (\sigma^{(t)})_{cd}$, $(\mu_{cd})^{(t)}=(\mu^{(t)})_{cd}$, $(V_{cd})^{(t)}=(V^{(t)})_{cd}$ and $(E_{cd})^{(t)} = (E^{(t)})_{cd}$. Hence the theorem.

Theorem 3.8. The upper truncation of the Complete double vertex fuzzy graph of a graph G:(σ, μ) is isomorphic to the Complete double vertex fuzzy graph of the upper truncation of the fuzzy graph.
Proof: Using the above theorem 3.7, the identity mapping from $(V_{cd})_{(t)}$ to $(V_{(t)})_{cd}$ provides the required isomorphism.

4. Conclusion

In this paper, we have discussed the truncation properties of double vertex fuzzy graphs and complete double vertex fuzzy graphs. Also we have studied the isomorphic property of truncations of double vertex fuzzy graph and complete double vertex fuzzy graph. These properties will certainly be helpful in studying various properties of fuzzy graphs in detail.

REFERENCES

1. P.Battacharya, Some research on fuzzy graphs, *Pattern Recognition Letter,* 6 (1987) 297-302.
2. F.Harary, *Graph Theory*, Narosa/Addition Wesley, Indian Student Edition, 1988.
3. J.N.Modeson and P.S.Nair, *Fuzzy Graph and Fuzzy Hypergraphs*, Physica-verlag Heidelberg, 2000.
4. J. N.Modeson and C.S.Perg, Operations on fuzzy graphs, *Information Sciences,* 79 (1994) 159-170.

5. A.Nagoorgani and K.Radha, Conjunctions of two fuzzy graphs, *International Review of Fuzzy Mathematics*, 3 (2008) 95-105.
6. A.Nagoorgani and K.Radha, Some properties of truncations of graphs, *Advance in Fuzzy Sets and Systems*, 4(2) (2009) 215-227.
7. A.Nagoorgani and K.Radha, Regular property of fuzzy graphs, *Bulletin of Pure and Applied Sciences*, 27E(2) (2008) 411-419.
8. K.Radha and S.Arumugam, On direct sum of two fuzzy graphs, *International Journal of Scientific and Research Publications*, 3(3) (2013) .
9. K.Radha and S.Arumugam, Double vertex graph and complete double vertex graph of a fuzzy graph, *International Journal of Multidisciplinary Research and Development*, 2(10) (2015)
10. A.Rosenfeld, Fuzzy Graphs, In.Zadeh L.A., Fu,K.S. Tanaka.K, Shimura.M.(eds) *Fuzzy Sets and their Applications to cognitive and Decision Process*, Academic Press, New York, (1975) pp.77-95.
11. S.Samanta and M.Pal, Fuzzy tolerance graphs. *Int. J. Latest Trends Math.*, 192 (2011) 57-67.
12. S.Samanta,T .Pramanik and M.Pal, Fuzzy colouring of fuzzy graphs, *Afrika Matematika*, 27(1-2) (2016) 37-50.
13. Sk.Md.Abu Nayeem and M.Pal, Genetic algorithm to solve the *p*-centre and *p*-radius problem on a network, *Intern. Journal of Computer Mathematics*, 82(5) (2005) 541-550.
14. Sk.Md.Abu Nayeem and M.Pal, Shortest path problem on a network with imprecise edge weight, *Fuzzy Optimization and Decision Making*, 4 (2005) 293-312.
15. S.Samanta and M.Pal, Fuzzy *k*-competition graphs and *p*-competition fuzzy graphs, *Fuzzy Inf. Eng.*, 5(2) (2013) 191-204.
16. S.Samanta, M.Akram and M.Pal, m-step fuzzy competition graphs, *Journal of Applied Mathematics and Computing*, 47 (2015) 461-472.