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Regular and Isometric Fuzzy Graphs

***K.Radha*[1] and *P.Indumathi*[2]**

PG &Research Department of Mathematics
Periyar E.V.R College, Trichy-23, Tamilnadhu, India.
[1]Email:radhagac@yahoo.com; [2]Email:indujune23@gmail.com

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Abstract. In this paper, a condition for two regular fuzzy graphs to be isometric is obtained. Also a condition for two fuzzy graphs to be isometric is obtained using degree set.

Keywords: Regular fuzzy graph, isometric fuzzy graphs, degree set.

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1. Introduction

Fuzzy graph theory was introduced by Rosenfeld in 1975. During the same time Yeh and bang have also introduced various connectedness concepts in fuzzy graphs. Though it is very young, it has been growing fast and has numerous application in various fields. Nagoorgani and Malarvizhi discussed the concept of isometry in fuzzy graphs and studied its properties. Radha and Kumaravel introduced the concept of edge regular fuzzy graphs. Radha and Rosemine introduced the concept of degree set in fuzzy graphs. Tom and Sumitha introduced sum distance in fuzzy graph. In this paper, we obtain conditions for regular fuzzy graphs to be isometric. Also we introduce isometric fuzzy subgraphs and distance preserving fuzzy graphs and prove that any fuzzy graph on a tree is distance preserving.

2. Preliminaries

In this section, we go through some basic definitions which can be found in [1-11].

Definition 2.1. Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(x,y) \le \sigma(x) \Lambda \sigma(y)$ for all $x, y \in V$.

Definition 2.2. The order and size of a fuzzy graph $G:(\sigma,\mu)$ are defined by $O(G) = \sum_{x\in V} \sigma(x)$ and $S(G) = \sum_{xy\in E} \mu(xy)$.

Definition 2.3 A fuzzy graph $G:(\sigma,\mu)$ is strong, if $\mu(xy) = \sigma(x)\Lambda\sigma(y)$ for all $xy \in E$.

Definition 2.4. A fuzzy Graph $G:(\sigma,\mu)$ is complete, if $\mu(xy)=\sigma(x)\Lambda\sigma(y)$ for all $x,y\in V$.

Definition 2.5. Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The degree of vertex x is $d_G(x)=\sum_{x\neq y}\mu(xy)$. If each vertex in G has same degree k, then G is said to be a regular fuzzy graph or k – regular fuzzy graph.

Definition 2.6. Let $G^*:(V,E)$ be a graph and let $e = uv$ be an edge in G^*. Then the degree of an edge $e=uv\in E$ is defined by $d_{G^*}(uv)=d_{G^*}(u)+d_{G^*}(v)-2$. If each edge in G^* has same degree, then G^* said to be edge regular. The degree of an edge $xy\in E$ is $d_G(xy)=\sum_{x\neq z}\mu(xz)+\sum_{z\neq y}\mu(zy)-2\mu(xy)$. If each edge in G has same degree k, then G is said to be an edge regular fuzzy graph or k - edge regular fuzzy graph.

Definition 2.7. If $\mu(x,y)>0$ then x and y are called neighbours, x and y are said to lie on the edge $e=xy$. A path ρ in a fuzzy graph $G:(\sigma,\mu)$ is a sequence of distinct nodes $v_0,v_1,v_2,.....v_n$ such that $\mu(v_i,v_{i-1})>0, 1\leq i\leq n$. Here'n' is called the length of the path. The consecutive paris (v_i,v_{i-1}) are called arcs of the path.

Definition 2.8. If u,v are nodes in $G:(\sigma,\mu)$ and if they are connected by means of a path then the strength of that path is defined as $\bigwedge_{i=1}^{m}\mu(v_{i-1},v_i)$ i.e., it is the strength of the weakest arc.

If u,v are connected by means of paths of length $'k'$ then $\mu^k(u,v)$ is defined as $\mu^k(u,v)=\sup\{\mu(u,v_1)\Lambda\mu(v_1,v_2)\Lambda\mu(v_2,v_3)\Lambda..\Lambda\mu(v_{k-1},v)/u,v_1,v_2,v_{3.....},v_{k-1},v\in V\}$. If $u,v\in V$ the strength of connectedness between u and v is, $\mu^\infty(u,v)=\sup\{\mu^k(u,v)/k=1,2,3,......\}$.

Definition 2.9. A fuzzy graph $G:(\sigma,\mu)$ is connected if $\mu^\infty(u,v)>0$ for all $u,v\in V$. An arc uv is said to be a strong arc if $\mu(u,v)\geq\mu^\infty(u,v)$. A node u is said to be an isolate node if $\mu(u,v)=0,\forall u\neq v$.

Definition 2.10. The μ - distance $\delta(u,v)$ is the smallest μ - length of any $u-v$ path, where the μ - length of path $\rho: u_0,u_1,u_2,.......u_n$ is $l(\rho)=\sum_{i=1}^{n}\frac{1}{\mu(u_{i-1},u_i)}$. The eccentricity of a node

v is defined as $e(v)=\max_u(\delta(u,v))$. The diameter $diam(G)=\vee\{e(v)\mid v\in V\}$, radius $r(G)=\wedge\{e(v)\mid v\in V\}$. A node whose eccentricity is minimum in a connected fuzzy graph is called a central node. A connected fuzzy graph is called self – centered if each node is a central node.

Definition 2.11. Let $G_i:(\sigma_i,\mu_i)$ be the fuzzy graphs with underlying crisp graphs $G_i^*:(V_i, E_i)$ for i= 1,2. G_2 is said to be isometric from G_1 if for each $v\in V_1$ there is a bijection $\phi_v:V_1\to V_2$ such that $\delta_1(u,v)=\delta_2(\phi_v(u),\phi_v(v))$, for every $u\in V_1$. If they are isometric from each other they are said to be isometric. This relation is termed as an isometry relation.

Definition 2.12. Let G:(σ, μ) be a connected fuzzy graph. For any path P, the length of the path L(P) is defined as the sum of the weights of the arcs in P. The sum distance between two vertices u and v is $d_s(u, v)$ = min{L(P) / P is a u-v path}.

3. Main results

Theorem 3.1. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $G_1{*}$ and $G_2{*}$ are k-regular graphs, μ_1 and μ_2 are constant functions of same constant value c and $|V_1|=|V_2|=n$. If $k\ge\frac{n-1}{2}$, then G_1 and G_2 are isometric fuzzy graphs.

Proof: Let us construct an isometric mapping from G_1 to G_2 as follows

Choose $u\in V_1$ and $v\in V_2$ arbitrarily.

Define : $\phi_u:V_1\to V_2$ by $\phi_u(u)=v$

Since $G_1{*}$ and $G_2{*}$ are k-regular, k- vertices say, $u_1,u_2,\ldots\ldots,u_k$ are at distance 1 from u in $G_1{*}$ and K vertices say, $v_1,v_2,\ldots\ldots,v_k$ are at distance 1 from v in $G_2{*}$

Define : $\phi_u(u_i)=v_i$, for $i=1,2,\ldots\ldots\ldots,k$

Since $G_1{*}$ and $G_2{*}$ are k – regular with $k\ge\frac{n-1}{2}$, $d(G_i{*})\le 2$. Therefore, the remaining $n-1-k$, vertices say, $u_{k+1},u_{k+2},\ldots\ldots,u_{n-1}$ in $G_1{*}$ are at distance 2 from u and the remaining $n-1-k$ vertices , say $v_{k+1},v_{k+2},\ldots\ldots,v_{n-1}$ are at distance 2from$v$ in G_2*.

Define $\phi_u(u_i)=v_i, i=k+1,\ldots\ldots n-1$

Since μ_1 and μ_2 are constant functions of same constant value c, using sum distance,

$$d_{G_1}(u,u_i)=\mu_1(u\,u_i)=c,\qquad i=1,2,\ldots\ldots\ldots,k$$

$$d_{G_2}(v,v_i)=\mu_1(v\,v_i)=c,\qquad i=1,2,\ldots\ldots\ldots,k$$

$$/\ d_{G_1}(u,u_i)=d_{G_2}(v,v_i),\qquad \forall i=1,2,\ldots\ldots\ldots,k$$

Since $u_{k+1},\ldots\ldots\ldots,u_{n-1}$ are at distance 2 in $G_1{*}$

$d_{G_1}(u,u_i) = 2C, \qquad i = k+1,.......,n$ -1

Similarly $d_{G_2}(v,v_i) = 2C, \qquad i = k+1,.......,n$ -1

/ $d_{G_1}(u,u_i) = d_{G_2}(v,v_i), \qquad i = k+1,.......,n$ -1

Therefore $d_{G_1}(u,u_i) = d_{G_2}(v,v_i), \qquad i = 1,2,.......,n-1$

Since u and v are arbitrary, G_2 is isometric from G_1.

Similarly, it can be proved that G_1 is isometric from G_2

Theorem 3.2. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two m-regular fuzzy graphs such that μ_1 and μ_2 are constant functions of same constant value c and $|V_1| = |V_2| = n$. If $K \geq \frac{n-1}{2}$ then G_1 and G_2 are isometric fuzzy graphs.

Proof: Since G_1 is $m - regular$

$$d_{G_1}(u) = m, \quad \forall\ u \in v_1$$

$$\Rightarrow \sum_{uv\in E_1} \mu_1(uv) = m, \ \forall u \in v_1$$

$$\Rightarrow \sum_{uv\in E_1} c \ = m, \ \forall u \in v_1$$

$$\Rightarrow cd_{G^*{}_1}(u) = m \ , \ \forall u \in v_1$$

$$d_{G^*{}_1}(u) = \frac{m}{c}, \quad \forall\ u \in v_1$$

Similarly $d_{G^*{}_2}(u) = \frac{m}{c}, \ \forall\ u \in V_2$ / G_1* andG_2* are $\frac{m}{c}$ - regular. Then proceeding as in theorem 3.1, G_1 and G_2 are isometaic fuzzy graphs.

Theorem 3.3. Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that μ_1 and μ_2 are constant functions of same constant value *c*. If G_1 and G_2 are isometric fuzzy graphs, then they have the same degree set and the same eccentricity set.

Proof: Since G_2 is isometric from G_1 there exists a one-to-one map ϕ from V_1 to V_2 such that,

$$d_{G_2}(u,v) = d_{G_2}(\phi(u),\phi(v)), \forall\ u,v \in V_1 \qquad (2.1)$$

If u and v are adjacent in G_1, then using sum distance in fuzzy graphs, $d_{G_1}(u,v) = \mu(uv) = C$

From (2.1) $d_{G_2}(\phi(u),\phi(v)) = C = \mu(\phi(u),\phi(v))$

$\phi(u)$ and $\phi(v)$ are adjacent is G_2

$d_{G_1^*}(u) = d_{G_2^*}(\phi(u))$

$$d_{G_1}(v) = \sum_{uv \in E_1} \mu(uv)$$
$$= \sum_{uv \in E_1} C$$
$$= cd_{G_1^*}(u)$$
$$= cd_{G_2^*}(\phi(u))$$
$$= d_{G_2}(\phi(u))$$

The degree of each vertex in G_1 is the degree of some vertex in G_2

The degree set of G_1 is contained in the degree set of G_2 . (2.2)

Also by the distance preserving property,

$$e_{G_1}(v) = \max\{d(u,v) / u \in V_1\}$$
$$= \max\{d(\varphi(u), \varphi(v)) / u \in V_1\}$$
$$= e_{G_2}(\varphi(v)) \quad (2.3)$$

The eccentricity set of G_1 is contained in the eccentricity set of G_2

Since G_1 is isometric from G_2 proceeding as above.

The degree set of G_2 is contained is the degree set of G_1 and the eccentricity set of G_2 is contained in the eccentricity set of G_1.

From (2.2), (2.3)

G_1 and G_2 have the same degree set and the same eccentricity set.

Definition 3.4. Let $G:(\sigma,\mu)$ be a fuzzy graph. A fuzzy subgraph H of G is isometric if $d_H(u,v) = d_G(u,v)$ every $u, v \in H$

Definition 3.5. A fuzzy graph $G:(\sigma,\mu)$ is distance preserving if it has an isometric fuzzy subgraph with each possible number of vertices upto $|V|$.

Theorem 3.6. If $G:(\sigma,\mu)$ is a fuzzy graph on a tree, then G is a distance preserving fuzzy graph.

Proof: Let $G:(\sigma,\mu)$ be a fuzzy graph on a tree with*n* vertices.

In a tree, there exists a unique path between any two vertices.

If G_1 is a fuzzy subgraph obtained from G by removing a verter of degree 1 in G^* then using sum distance $d_{G_1}(u,v) = d_G(u,v) \; \forall u, v \in v(G_1)$

If G_2 is a fuzzy subgraph obtained from G_1 by removing a vertex of degree 1 in G_1^* then $d_{G_2}(u,v) = d_G(u,v) \; \forall u, v, \in v(G_2)$ and so on.

This procedure gives an isometric fuzzy subgraph G_i of a G with $|V|-i$, $i-1,2,.....,|V|-1,$ number of vertices.

Hence G is a distance preserving fuzzy graph.

4. Conclusion

The conditions obtained for isometric fuzzy graphs the concepts of isometric fuzzy subgraphs and distance preserving fuzzy graphs will be helpful in obtaining many more properties of isometric fuzzy graphs.

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