Intern. J. Fuzzy Mathematical Archive
Vol. 14, No. 2, 2017, 365-378
ISSN: 2320 –3242 (P), 2320 –3250 (online)
Published on 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/ijfma.v14n2a18

International Journal of
Fuzzy Mathematical Archive

Some Types of Nodal and Edge Regular Anti Fuzzy Graph

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Received 25 November 2017; accepted 7 December 2017

Abstract. In this paper, we define e-nodal, v-nodal, uninodal and binodal anti fuzzy graph. We introduce the concept of edge regular and totally edge regular on anti fuzzy graph. We applied such concepts on some types of anti fuzzy graphs and obtained the results on them. The relations between fuzzy graph and anti fuzzy graph are discussed and the bounds on them are obtained.

Keywords: Anti fuzzy graph, fuzzy graph, vertex degree, edge degree.

AMS Mathematics Subject Classification (2010): 05C62, 05E99, 05C07

I. Introduction

Kaufmann [1] was first to introduce the concept of fuzzy graph from the fuzzy relation introduced by Zedah [9]. Although Rosenfield [7] introduced another elaborated definition, including fuzzy vertex and fuzzy edge, and also introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. Nagoorgani, Chandrasekaran [4] introduced the concept of more adjacency in fuzzy graphs. Nagoorgani and Radha [5] discussed the various concepts of degrees in fuzzy graph. Radha et.al [6] conferred the concept of edge regular on fuzzy graph. Seethalakshmi and Gnanajothi [8] introduced the definition of anti fuzzy graph. Muthuraj and Sasireka [2] discussed some more concepts of anti fuzzy graph. Muthuraj and Sasireka [3] discussed the concept of some operations on anti fuzzy graphs such as union, join, Cartesian product and composition. In this paper, we introduce the concept of edge regular and totally edge regular on anti fuzzy graphs. The results are examined and some theorems are derived from them. We introduce the concept of path matrix on anti fuzzy graph and using this we find the shortest path length. The relationship between the fuzzy graph and anti fuzzy graph is discussed. We derived some theorems and results on them.

2. Preliminaries

In this section, basic concepts of anti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [4, 8, 9].

Definition 2.1. [8] A fuzzy graph $G=(\sigma, \mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Note. μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

Notation. Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

Definition 2.2. [8] The underlying crisp graph of an anti fuzzy graph $G_A = (\sigma, \mu)$ is denoted by $G_A^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V / \mu(u,v) > 0\}$.

Definition 2.3. [8] The order p and size q of an anti fuzzy graph $G_A = (V, \sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in V} \mu(xy)$. It is denoted by O(G) and S(G).

Definition 2.4. [8] Two vertices u and v in G_A are called adjacent if $(½)[\sigma(u) \vee \sigma(v)] \leq \mu(u,v)$.

Definition 2.5. [8] An anti fuzzy graph $G_A=(\sigma,\mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and G_A is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \sigma^*$. Two vertices u and v are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.6. [8] u is a vertex in an anti fuzzy graph G_A then N(u) = {v: (u,v) is a strong edge} is called the neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u.

Definition 2.7. [8] The strong neighborhood of an edge e_i in an anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G) / e_j$ is a strong edge in G_A and adjacent to $e_i\}$.

Definition 2.8. [9] The anti complement of anti fuzzy graph $G_A(\sigma,\mu)$ is an anti fuzzy graph $\overline{G_A} = (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u,v) = \mu(u,v) - (\sigma(u) \vee \sigma(v))$ for all u,v in $V(G_A)$.

Definition 2.9. [9] A path P_A in an anti fuzzy graph is a sequence of distinct vertices $u_0,u_1,u_2\ldots u_n$ such that $\mu(u_{i-1},u_i)>0$, $1\leq i \leq n$. Here $n \geq 0$ is called the length of the path P_A. The consecutive pairs (u_{i-1}, u_i) are called the edges of the path.

Definition 2.10. [8] A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest edge.

Definition 2.11. An edge e = {u,v} of an anti fuzzy graph G_A is called an effective edge if $\mu_A(u,v) = \sigma(u) \vee \sigma(v)$. An effective degree of a vertex u in anti fuzzy graph is defined to be the sum of the weights of an effective edges incident at u and is denoted by $d_E(u)$.

3. More adjacent on anti fuzzy graph

Definition 3.1. A vertex z is called a fuzzy end vertex of $G_A = (\sigma,\mu)$ if it has at most one strong neighbor in G_A.

Definition 3.2. A strongest path of (u,v) is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^{\infty}(u,v)$.

Definition 3.3. In anti fuzzy graph, the path matrix is defined for a specific pair of vertices say (u, v) and it is denoted by P(u,v).

In P(u, v), the rows correspond to different path between u and v. the columns correspond to edges within the path of u and v in G_A.

$$P(u, v) = \begin{cases} w(e), & edge\ weight\ of\ i^{th}\ path \\ 0\ , & otherwise \end{cases}$$

Example 3.4.

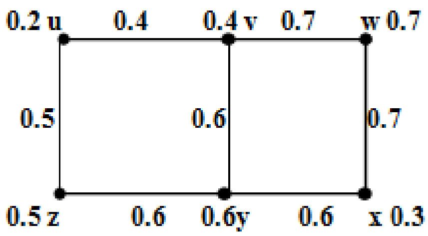

Fig. 1. Anti fuzzy graph G_A

Consider a path matrix P(u, z). The list of different paths between u and z are

1. (u, z)
2. (u, v, y, z)
3. (u, v, w, x, y, z)

$$P(u, z) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0.4 & 0 & 0 & 0 & 0.6 & 0.6 & 0 \\ 0.4 & 0.7 & 0.7 & 0.6 & 0 & 0.6 & 0 \end{bmatrix}$$

The weight of each path of u,z is the sum of each rows in P(u,z) and is denoted by $w[P_i(u,z)]$.

$W[P_1(u, z)] = 0.5$

$W[P_2(u, z)] = 0.4 + 0.6 + 0.6 = 1.6$

$W[P_3(u, z)] = 0.4 + 0.7 + 0.7 + 0.6 + 0.6 = 3$

Shortest path length of u,z = min $\{W[P_i(u, z)]\}$ = min { 0.5, 1.6, 3.0 } = 0.5

Definition 3.5. Let u and v be two vertices in $V(G_A)$ then the vertex u is more adjacent to v if $\mu(u, v) \geq \mu(u, w)$ for any $w \in V$.

Theorem 3.6. In an anti fuzzy graph G_A, every vertex is either isolated or more adjacent to some other vertex of an anti fuzzy graph.

Proof: Consider that G_A is an anti fuzzy graph and u is a vertex in G_A. If u is not isolated vertex then it must be adjacent to some other vertices in G_A. $v \neq u$ is a vertex in G_A and v is adjacent to u. if $\mu(u, v) = \max(\mu(u, x) : x \in V)$ then u is more adjacent to v.

Theorem 3.7. Let u and v be two vertices in an anti fuzzy graph G_A and u be more adjacent to v then the fuzzy values of more adjacent vertices is strength of the shortest path of (u, v).

Proof: In G_A, if u is more adjacent to v then $\mu(u, v) = \max\{\mu(u, w) : w \in V\}$. Suppose some other paths are exists from u to v. From path matrices, the sum of each rows show the strength of each path. The minimum value occurs in path which has more adjacent vertices. Such that $\mu(u, v) = \min\{\mu(u, w): w \in V\}$. Hence the fuzzy value of more adjacent vertices is strength of the shortest path of (u, v).

Relation 3.8.

1. More adjacent pair of vertices is not same in G and G_A.

Generally in G, edge weight assigns the minimum of its fuzzy incident vertices. In G_A, edge weight assigns the maximum of its fuzzy incident vertices. Hence the same pair of vertices is not more adjacent to fuzzy graph and anti fuzzy graph.

Theorem 3.9. If u and v are more adjacent vertices to only w in an anti fuzzy graph G_A, then w is an fuzzy cut node of G_A.

Proof: Let us consider that w is the only vertex which is more adjacent to u and v. if the removal of the vertex w then it reduced the strength of connectedness between u and v. Hence w is a fuzzy cut node of G_A.

Note. Let us consider the figure 1, y is the only node which is more adjacent with v, x and z. if the deletion of the vertex w increase the components and reduce the strength of connectedness between v, x and z. Hence w is the fuzzy cut node of G_A. The converse of above statement is not true.

4. Degrees of an edge in an anti fuzzy graph

Definition 4.1. Let G_A (σ,μ) be an anti fuzzy graph, the degrees of an edge $uv \in E$ is defined by $d_{G_A}(uv) = d_{G_A}(u) + d_{G_A}(v) - 2\mu(u,v)$ or $d_{G_A}(uv) = \sum_{\substack{uw \in E \\ w \neq v}}(uw) + \sum_{\substack{wv \in E \\ w \neq u}}(wv) - 2\mu(u,v)$

Definition 4.2. Let G_A (σ,μ) be an anti fuzzy graph, the total degrees of an edge $uv \in E$ is defined by $td_{G_A}(uv) = d_{G_A}(u) + d_{G_A}(v) - \mu(u,v) = d_{G_A}(uv) + \mu(u,v)$

Example 4.3.

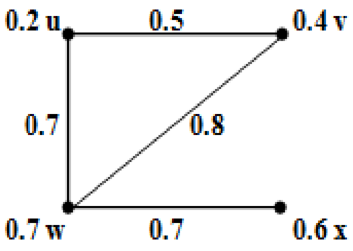

Fig. 2. Anti fuzzy graph G_A

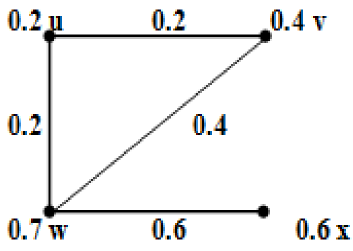

Fig. 3. Fuzzy graph G

From figure 2. In G_A,

d_{G_A}(u) = 1.2, d_{G_A}(v) = 1.3, d_{G_A}(w) = 1.4, d_{G_A}(x) = 0.7. $\therefore$ $\delta(G_A)$= 0.7, $\Delta(G_A)$=1.4

d_{G_A}(uv) = 1.2 + 1.3 - 2(0.5) = 2.5 - 1 = 1.5,

d_{G_A}(uw) = 1.2 + 1.4 - 2(0.4) = 2.6 - 0.8 = 1.8,
d_{G_A}(vw) =1.3 + 1.4 - 2(0.8) = 2.7 - 1.6 = 1.1,
d_{G_A}(wx) = 1.4 + 0.7 - 2(0.7) = 2.1 – 1.4 = 0.7,
$\delta_E(G_A)$ = d(wx) = 0.7, $\Delta_E(G_A)$ = d(uw) = 1.8
td_{G_A}(u) = 1.4, td_{G_A}(v)= 1.7, td_{G_A}(w)= 2.1, td_{G_A}(x)= 1.3
td_{G_A}(uv) = 2, td_{G_A}(uw) = 2.5, td_{G_A}(vw) = 1.9, td_{G_A}(wx)= 1.4
From figure 3.
In G, d_G(u) = 0.4, d_G (v) =0.6 , d_G (w) =1.2 , d_G (x) =0.6 .
$\therefore$ δ(G)= 0.4, Δ(G)=1.2
d_G(uv)= 0.4 + 0.6 - 2(0.2) = 1 – 0.4 = 0.6 , d_G(uw) = 1.2, d_G(vw) =1.0, d_G(wx) = 0.6,
δ_E(G) = d(wx)= 0.6, Δ_E(G) = d(uw)= 1.2
td_G(u) = 0.6 , td_G(v) = 1.0 , td_G(w) = 1.9 , td_G(x) = 1.2
td_G(uv) = 0.8, td_G(uw) = 1.4, td_G(vw) = 1.4, td_G(wx) = 1.2

Note. In G and G_A, the same edges may get the minimum and maximum degree of (anti) fuzzy graph and the same pair of edges assigned minimum and maximum total edge degrees.

1. $\delta_E(G) \neq \delta_E(G_A)$
2. $\Delta_E(G) \neq \Delta_E(G_A)$

Relation 4.4. [5]

1. $\sum d_G(v) = 2\,S(G)$
2. $\sum td_G(v) = 2\,S(G) + O(G)$
3. $\sum d_G(v) = \sum d_G(uv)$ where G is cycle

Relation 4.5.

1. $\sum d_{G_A}(v) \neq 2\,S(G_A)$
2. $\sum td_{G_A}(v) \neq 2\,S(G_A) + O(G_A)$
3. $\sum d_{G_A}(v) + \delta_E(G_A) \leq 2S(G_A) \leq \sum d_{G_A}(v) + \Delta_E(G_A)$
4. $\sum td_{G_A}(v) \leq 2S(G_A) + O(G_A) \leq \sum td_{G_A}(uv)$
5. $\sum d_{G_A}(v) = 2\,S(G_A)$, if G_A is k-regular anti fuzzy graph
6. $\sum td_{G_A}(v) = 2\,S(G_A) + O(G_A)$, if G_A is k-regular anti fuzzy graph

Definition 4.6. Let G_A (σ,μ) be an anti fuzzy graph. If each edge in G_A has same degree k then G_A is said to be an edge regular anti fuzzy graph or k-edge regular anti fuzzy graph.

Definition 4.7. Let G_A (σ,μ) be an anti fuzzy graph. If each edge in G_A has same total degree k then G_A is said to be totally edge regular anti fuzzy graph or k- totally edge regular anti fuzzy graph.

Example 4.8. From figure 4.
d_{G_A}(u) = 0.9, d_{G_A}(v) =0.8, d_{G_A}(w) =0.9, d_{G_A}(x) =1, d_{G_A}(uv) = 0.9
d_{G_A}(uw) = 1.2, d_{G_A}(vw) = 0.9, d_{G_A}(wx) = 0.9, d_{G_A}(vx) = 0.8, d_{G_A}(xu) = 0.9
td_{G_A}(uv) = 1.3, td_{G_A}(uw) = 1.5, td_{G_A}(vw) = 1.3, td_{G_A}(wx) = 1.4, td_{G_A}(vx) = 1.3, td_{G_A}(xu) = 1.4

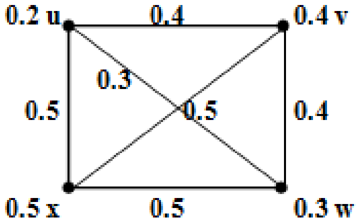

Fig. 4. Anti fuzzy graph G_A

Here G_A is a complete anti fuzzy graph but not edge regular anti fuzzy graph and not totally edge regular anti fuzzy graph.

Remark.

1. Every totally edge regular anti fuzzy graph is edge regular anti fuzzy graph.
2. Every k-regular anti fuzzy graph need not be a k-regular anti fuzzy graph.

Note. In crisp graph, every complete graph is a edge regular but in an anti crisp fuzzy graph, complete anti fuzzy graph need not be a edge regular and totally edge regular anti fuzzy graph.

Theorem 4.9. Let G_A be an anti fuzzy graph.μ is a constant function and d_{G_A} (u) =k for all u$\in$E(G_A) if and only if the following are equivalent.

i. G_A is an edge regular anti fuzzy graph
ii. G_A is a totally edge regular anti fuzzy graph

Proof: Let G_A be an anti fuzzy graph and consider that μ is a constant function.
That is μ(uv) = c, for every uv$\in$E (G_A) and d_{G_A} (u) = k where k = nc for all u $\in$ V(G_A)

$$\therefore d_{G_A}(uv) = d_{G_A}(u) + d_{G_A}(v) - 2\mu(uv)$$
$$= k + k - 2c$$
$$= k_1 \text{ (say)}$$

Then G_A is a k_1-edge regular anti fuzzy graph.
Now G_A is a k_1-edge regular anti fuzzy graph.

$$td_{G_A}(uv) = d_{G_A}(uv) + \mu(uv)$$
$$= k_1+c = k_2 \text{ (say)} \rightarrow (1) \text{ for all } uv\in E(G_A).$$

Hence G_A is (k_1+c) totally edge regular anti fuzzy graph.
(i) $\Rightarrow$ (ii) is proved.
Suppose that G_A is a k_2-totally edge regular anti fuzzy graph. ie., td_{G_A} (uv) = k_2 for all uv$\in$ E(G_A).
We know that $td_{G_A}(uv) = d_{G_A}(uv) + \mu(uv)$

$$k_2 = d_{G_A}(uv) + \mu(uv)$$
$$d_{G_A}(uv) = k_2 - \mu(uv)$$
$$= k_2 - c \text{ for all } uv\in E(G_A)$$
$$= k_1 \text{ (by (1))}$$

Hence G_A is k_1-edge regular anti fuzzy graph with d_{G_A} (u) = k
(ii) $\Rightarrow$ (i) is proved.
Conversely, G_A is edge regular and totally edge anti fuzzy graph.

ie., d_{G_A} (uv) = k, $\forall$ uv$\in$ E(G_A).

To prove that μ(uv) is a constant function.

Suppose μ(uv) is not a constant function. ie. some pair of edges does not have a constant value. μ(ab)=c_1, for ab$\in$ E (G_A). Therefore μ(uv) $\neq$ μ(ab).

But d_{G_A} (uv) = d_{G_A} (ab) = k (Given)

td_{G_A} (uv) = d_{G_A} (uv) + μ(uv) = k +c

td_{G_A} (ab) = d_{G_A} (ab) + +μ(ab) = k +c_1

$\therefore$ td_{G_A} (uv) $\neq$ td_{G_A} (ab).

Then G_A is not totally edge regular anti fuzzy graph which is contradicted to our assumption. Hence μ is a constant function.

Suppose td_{G_A} (uv) = td_{G_A} (ab), for all uv, ab $\in$ E (G_A).

$\Rightarrow$ d_{G_A} (uv) + μ(uv) = d_{G_A} (ab) + +μ(ab)

$\Rightarrow$ d_{G_A} (uv) - d_{G_A} (ab) = μ(ab) - μ(uv)

$\Rightarrow$ d_{G_A} (uv) - d_{G_A} (ab) = c_1 – c {Since $c_1 \neq$ c}

$\Rightarrow$ d_{G_A} (uv) - d_{G_A} (ab) $\neq 0$

$\therefore$ d_{G_A} (uv) $\neq$ d_{G_A} (ab) .

Therefore, G_A is not edge regular anti fuzzy graph which is also contradicted to our assumption. Hence G_A is an edge regular anti fuzzy graph and totally edge regular anti fuzzy graph then μ is a constant function.

Theorem 4.10. Let μ(uv) = c is a constant function in G_A. if G_A is regular anti fuzzy graph then G_A is edge regular anti fuzzy graph.

Proof: Let μ(uv) = c is a constant function in G_A.

Consider that G_A is regular anti fuzzy graph then d_{G_A} (u) = k for all u$\in$V (G_A).

To prove that G_A is edge regular anti fuzzy graph.

By definition, d_{G_A} (uv) = d_{G_A} (u) + d_{G_A} (v) - μ(uv)

= k + k – c = 2k - c

d_{G_A} (uv) = k_1 (say), for all uv $\in$ E (G_A).

Hence G_A is k_1-edge regular anti fuzzy graph.

Theorem 4.11. Let G_A is a k-regular anti fuzzy graph with μ is a constant function then G_A is totally edge regular anti fuzzy graph.

Note: The converse of above all theorem may not be true.

5. Some types of nodal anti fuzzy graph

Definition 5.1. Every vertex in G_A has unique fuzzy values then G_A is said to be v-nodal anti fuzzy graph. i.e. σ(u) = c for all u$\in$V(G_A).

Definition 5.2. Every edge in G_A has unique fuzzy values then G_A is said to be e-nodal anti fuzzy graph. i.e. μ(u) = c for all uv$\in$ E (G_A)

Example 5.3. Let us consider anti fuzzy graph (G_A), V (G_A) = {a = b = c = d = e = 0.6}and the edge set E(G_A) = {ab=0.6, ac=0.7, ae=0.6, bc=0.8, bd=0.6, cd=0.8, ce=0.7}. d_{G_A} (a) = 1.9, d_{G_A} (b) = 2, d_{G_A} (c) = 2.2, d_{G_A} (d) = 0.6, d_{G_A} (e) = 1.3. σ(a)=0.6 $\forall$ a$\in$V (G_A). G_A is a v-nodal anti fuzzy graph

Let us consider anti fuzzy graph (G_A), V (G_A) = {a =0.5, b = 0.4, c =0.3 d = 0.3} and the edge set consider as E (G_A) = {ab=0.5, ac=0.5, ad=0.5, bd=0.5, cd=0.5}. μ(ab) = 0.5 $\forall$ ab$\in$E(G_A). G_A is a e-nodal anti fuzzy graph

Definition 5.4. Every vertices and edges in an anti fuzzy graph have the unique fuzzy values then G_A is called as uninodal anti fuzzy graph.

Definition 5.5. If σ(u) or μ(uv) has the unique fuzzy values in an anti fuzzy graph $G_{A,}$ then G_A is said to be partially uninodal anti fuzzy graph.

Definition 5.6. If $\sigma(u)=c_1$ and $\mu(uv)= c_2$ in an anti fuzzy graph $G_{A,}$ then G_A is called as binodal anti fuzzy graph.

Example 5.7.

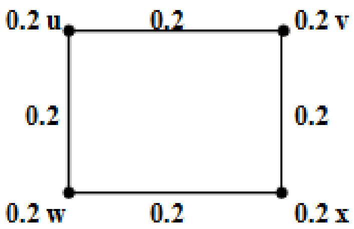

Fig. 5. Uninodal anti fuzzy graph G_A

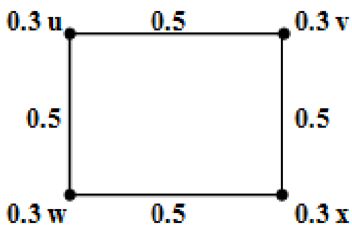

Fig. 6. Binodal anti fuzzy graph G_A

From the figure 5, σ(u) = μ (uv) = 0.2 $\forall$ u$\in$V (G_A) and uv$\in$E (G_A).
d_{G_A} (u) = d_{G_A} (v) = d_{G_A} (w) = d_{G_A} (x) = 0.4
td_{G_A} (u) = td_{G_A} (v) = td_{G_A} (w) = td_{G_A} (x) = 0.4
d_{G_A} (uv) = d_{G_A} (uw) = d_{G_A} (vx) = d_{G_A} (xw) = 0.4,
td_{G_A} (uv) = td_{G_A} (uw) = td_{G_A} (v x) = d_{G_A} (xw) = 0.6
From the figure 6, σ(u) =0.3, μ (uv) = 0.5 $\forall$ u$\in$V (G_A) and uv$\in$E (G_A).
d_{G_A} (u) = d_{G_A} (v) = d_{G_A} (w) = d_{G_A} (x) = 1
td_{G_A} (u) =td_{G_A} (v) = td_{G_A} (w) = td_{G_A} (x) = 1.3
d_{G_A} (uv) = d_{G_A} (uw) = d_{G_A} (vx) = d_{G_A} (xw) = 1,
td_{G_A} (uv) = td_{G_A} (uw) = td_{G_A} (v x) = d_{G_A} (xw) = 1.5

Proposition 5.8.

1. Every uninodal anti fuzzy graph is a strong anti fuzzy graph but the converse is not true.
2. Every uninodal anti fuzzy graph is a k-regular anti fuzzy graph
3. Every uninodal anti fuzzy graph need not be a k-edge regular anti fuzzy graph
4. Every uninodal anti fuzzy graph is also a fuzzy graph
5. Every uninodal anti fuzzy graph is a strong fuzzy graph but the converse is not true.
6. Every binodal cycle and binodal complete anti fuzzy graph is k-regular and k_1-totally regular anti fuzzy graph.
7. Every binodal cycle and binodal complete anti fuzzy graph is k-edge regular and k_1-totally edge regular anti fuzzy graph.

Theorem 5.9. If G_A is complete and v-nodal anti fuzzy graph, then G_A is k-regular and k_1-edge regular anti fuzzy graph.

Proof: Consider that G_A is v-nodal anti fuzzy graph. By definition $\sigma(u) = c\ \forall\ u \in V(G_A)$.
Given G_A is complete anti fuzzy graph. By definition $\mu(uv) = \sigma(u) \vee \sigma(v)$
Therefore, $\mu(uv) = c$
i.e. All the vertices and edge have the unique degree values.
To prove that G_A is k-regular.

$$\begin{aligned} d_{G_A}(u) &= \sum \mu(uv) \\ &= (n-1)c\ \forall\ uv \in E(G_A) \\ &= k \text{ (say)} \end{aligned}$$

Therefore G_A is k-regular anti fuzzy graph.
Now to prove that G_A is k-edge regular.

$$\begin{aligned} d_{G_A}(uv) &= d_{G_A}(u) + d_{G_A}(v) - 2\mu(uv) \\ &= (n-1)c + (n-1)c - 2c\ \forall\ uv \in E(G_A) \\ &= 2n - 4c = k_1 \text{ (say)} \end{aligned}$$

Therefore G_A is k_1-edge regular anti fuzzy graph.

Theorem 5.10. If G_A is cycle and e-nodal anti fuzzy graph then G_A is k-regular and k_1-edge regular anti fuzzy graph.

Proof: Consider that G_A is e-nodal anti fuzzy graph. by definition $\mu(uv) = c\ \forall\ uv \in E(G_A)$.
To prove that G_A is k-regular.
Given G_A is cycle. Every vertex in cycle is incident with two edges.

$$\begin{aligned} d_{G_A}(u) &= c + c = 2c \\ &= k \text{ (say)} \end{aligned}$$

Therefore G_A is k-regular anti fuzzy graph.
Now to prove that G_A is k-edge regular.

$$\begin{aligned} d_{G_A}(uv) &= d_{G_A}(u) + d_{G_A}(v) - 2\mu(uv) \\ &= 2c + 2c - 2c\ \forall\ uv \in E(G_A) \\ &= 2c = k_1 \text{ (say)} \end{aligned}$$

Therefore G_A is k_1-edge regular anti fuzzy graph.

Theorem 5.11. If G_A is cycle and k-regular anti fuzzy graph then G_A is e-nodal anti fuzzy graph.

Proof: Consider that G_A is cycle and k-regular anti fuzzy graph. $\Rightarrow d(u) = k, \forall\ u \in V(G_A)$

If G_A is cycle then every vertex exactly incident with two edges.
i.e., $d(u) = \mu(e_1) + \mu(e_2)$, $d(v) = \mu(e_2) + \mu(e_3)$,
$\therefore\ \mu(e_1) + \mu(e_2) = \mu(e_2) + \mu(e_3) = \ldots\ldots = k$ (since G_A is k-regular anti fuzzy graph)

Consider, $\mu(e_1) + \mu(e_2) = \mu(e_2) + \mu(e_3)$
$\Rightarrow \mu(e_1) = \mu(e_3)$.
Consider, $\mu(e_2) + \mu(e_3) = \mu(e_3) + \mu(e_4)$
$\Rightarrow \mu(e_2) = \mu(e_4)$.....
Similarly, $\mu(e_1) = \mu(e_3)$, $\mu(e_2) = \mu(e_4)$, $\mu(e_2) = \mu(e_3)$, ..., $\mu(e_1) = \mu(e_n)$
Therefore, $\mu(e_1) = \mu(e_2) = \mu(e_3) = \mu(e_4) = \ldots\ldots\ldots = \mu(e_n)$.
Hence G_A is an e-nodal anti fuzzy graph.

Theorem 5.12. If G_A is k-regular and k-edge regular anti fuzzy graph then G_A is not uninodal anti fuzzy graph

Proof: Consider that G_A is k-regular and k-edge regular anti fuzzy graph. i.e. d_{G_A} (u) = d_{G_A} (uv) = k.

To prove that G_A is uninodal anti fuzzy graph.

We know that d_{G_A} (uv) = d_{G_A} (u) + d_{G_A} (v) – 2μ(uv)

$$k = k + k - 2\mu(uv)$$

$$2\,\mu(uv) = 2k - k$$

$$\mu(uv) = k / 2, \text{ for all } uv \in E\,(G_A)$$

This is contradicting to the definition of uninodal anti fuzzy graph. Hence G_A is not uninodal anti fuzzy graph.

Example 5.13.

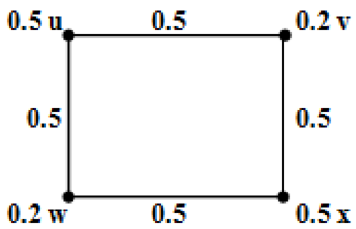

Fig. 7. Anti fuzzy graph G_A

d_{G_A}(u) = d_{G_A}(v) = d_{G_A}(w) = d_{G_A}(x) = 1, d_{G_A}(uv) = d_{G_A}(vx) = d_{G_A}(xw) = d_{G_A}(wu) = 1. The above graph is 1- regular anti fuzzy graph and 1-edge regular anti fuzzy graph but not a uninodal anti fuzzy graph because σ(u) ≠ σ(v).

Theorem 5.14. Every strong cycle is a uninodal anti fuzzy graph.

Note: The converse of the above theorem is not true.

Theorem 5.15. If G_A is uninodal anti fuzzy graph and a strong cycle then G_A is k-totally regular anti fuzzy graph and k-totally edge regular anti fuzzy graph.

Proof: Consider that G_A is uninodal anti fuzzy graph. By definition, σ(u) = μ(uv) = c for all u∈ V(G_A) and uv∈ E(G_A)

Given G_A is a strong cycle ⇒ d_{G_A}(u) = 2c and d_{G_A}(uv) = 2c

To prove that G_A is k-totally regular anti fuzzy graph.

By the definition, td_{G_A} (u) = d_{G_A} (u) + σ(u)

= 2c + c = k(say)

Hence G_A is k-totally regular anti fuzzy graph.

To prove that G_A is a k-totally edge regular anti fuzzy graph.

By the definition,

$$td_{G_A}(uv) = d_{G_A}(uv) + \mu(uv)$$

$$= d_{G_A}(u) + d_{G_A}(v) - 2\mu(uv) + \mu(uv)$$

$$= 2c + 2c - 2c + c = 2c - c = k(\text{say})$$

Hence G_A is k-totally edge regular anti fuzzy graph.

Example 5.16.

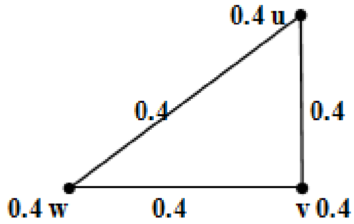

Fig. 8. Anti fuzzy graph G_A

d_{G_A} (u) = d_{G_A} (v) = d_{G_A} (w) =0.8 ,
td_{G_A} (u) = td_{G_A} (v) = td_{G_A} (w) =1.2
d_{G_A} (uv) = d_{G_A} (uw) = d_{G_A} (vw) = 0.8,
td_{G_A} (uv) = td_{G_A} (uw) = td_{G_A} (vw) = 1.2

The above graph represent as fuzzy graph and also an anti fuzzy graph. G is a complete anti fuzzy graph and also regular, edge regular and uninodal anti fuzzy graph. When uninodal anti fuzzy graph is a complete anti fuzzy graph then the anti fuzzy graph is a k-regular, k-edge regular and k_1-totally edge regular anti fuzzy graph.

Remark:

1. G_A is k-edge regular fuzzy graph if and only if $\delta_E(G_A) = \Delta_E(G_A) = k$.
2. G_A is k-totally edge regular fuzzy graph if and only if $\delta_{tE}(G_A) = \Delta_{tE}(G_A) = k$. Where $\delta_{tE}(G_A)$ and $\Delta_{tE}(G_A)$ is the minimum and maximum total edge degree of G_A.

6. Relation between fuzzy graph and anti fuzzy graph

In this chapter, to discuss the relationship between fuzzy graph (G) and anti fuzzy graph (G_A), consider the vertex set with fuzzy values are same in G and G_A. The adjacencies between the pair of vertices are also same. Depending upon the definition of fuzzy graph and anti fuzzy graph the edge set is defined. i.e. $\sigma_G(u) = \sigma_{G_A}(u)$ and $\mu_G(u) \neq \mu_{G_A}(uv)$. From our assumption, the relationship between G and G_A in degree based concepts is discussed. In such case the notation of the graph simply taken as G.

Proposition 6.1.

1. $p - \Delta_N(G_A) = p - \Delta_N(G)$; $p - \delta_N(G_A) = p - \delta_N(G)$
2. $p - \Delta_N(G_A) \geq p - \Delta_E(G_A)$
3. $p - \Delta_E(G_A) \leq p - \Delta_E(G)$
4. $p - \delta_E(G_A) \leq p - \delta_E(G)$
5. $p - \Delta_E(G_A) \leq p - \delta_E(G_A)$

For an anti fuzzy graph G_A of order p, where $\Delta_N(G_A)$ and $\Delta_E(G_A)$ denote the maximum neighborhood degrees and maximum effective degrees of G_A . $\delta_N(G_A)$ and $\delta_E(G_A)$ denote the minimum neighborhood degrees and minimum effective degrees of G_A

Example 6.2.

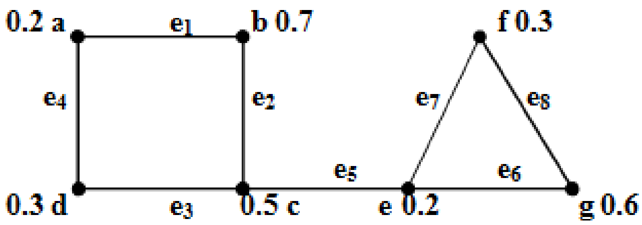

Fig. 9. Anti fuzzy graph G_A

Let us consider the figure 9, as a fuzzy graph (G) and anti fuzzy graph (G_A) with the same vertex set of fuzzy values. V={a=0.2, b=0.7, c=0.5, d=0.3, e=0.2, f=0.3, g=0.6}and the edge set E={e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8}.

In G, the edge set consider as E(G) = { e_1=0.2, e_2 =0.5,e_3 =0.3,e_4=0.2,e_5=0.2, e_6=0.2, e_7=0.2, e_8=0.3 }.
In G_A, the edge set consider as E(G_A) = { e_1=0.7, e_2 =0.7,e_3 =0.5,e_4=0.3,e_5=0.5, e_6=0.6,e_7=0.3,e_8=0.6}.Therefore p=2.8.

The neighborhood degree of vertices in G and G_A are, d_N(a)=1, d_N(b)=0.7, d_N(c)=1.2, d_N(d)=0.7, d_N(e)=1.4, d_N(f)=0.8, d_N(g)=0.5. Hence $\Delta_N(G)= \Delta_N(G_A)$=1.4. Hence $\delta_N(G)= \delta_N(G_A)$=0.5.

The effective degree of vertices in G are, d_E(a)=0.4, d_E(b)=0.7, d_E(c)=0.8, d_E(d)=0.7, d_E(e)=0.6, d_E(f)=0.5, d_E(g)=0.5. Hence $\Delta_E(G)$= 0.8, $\delta_E(G)$=0.4.
The effective degree of vertices in G_A are, d_E(a)=1, d_E(b)=1.4, d_E(c)=1.2, d_E(d)=0.8, d_E(e)=1.4, d_E(f)=0.9, d_E(g)=1.2. Hence $\Delta_E(G_A)$=1.4, $\delta_E(G_A)$=0.8.

Observation 6.3.

1. If $\overline{G_1 \text{ o } G_2} \cong \overline{G_1} \text{ o } \overline{G_2}$ then G_1 and G_2 are either complete fuzzy graphs or complete anti fuzzy graphs.
2. If G_1 and G_2 are either complete fuzzy graphs or complete anti fuzzy graphs then the following conditions are holds.
   a. $\overline{G_1 + G_2} \cong \overline{G_1} \cup \overline{G_2}$
   b. $\overline{G_1 \cup G_2} \cong \overline{G_1} + \overline{G_2}$
3. If $\sum_{i=1}^{n} d[\sigma(v_i)] = 2 \sum_{i=1}^{n} \mu(v_i, v_{i+1})$ for v_i, $v_{i+1} \in$ V then V is the vertex set of either fuzzy graph or an anti fuzzy graph.
4. If $\sigma(u_i)$ of G is a constant function then G is either regular and totally regular fuzzy graph or regular and totally regular anti fuzzy graph.
5. If G is either k-regular fuzzy graph or k-regular anti fuzzy graph then the size of G is nk/2.
6. The order of k-regular fuzzy graph is equal to the order of k-regular anti fuzzy graph.
7. A highly irregular (anti)fuzzy graph need not be a neighborly irregular (anti)fuzzy graph.

8. A neighborly irregular (anti)fuzzy graph need not be a highly irregular (anti)fuzzy graph
9. G is highly irregular (anti) fuzzy graph and neighborly irregular (anti) fuzzy graph if and only if the degrees of all vertices of G are distinct.
10. If G is k-regular fuzzy graph then G_A is not a k-regular anti fuzzy graph but G_A may be regular anti fuzzy graph for some other values (say k_1).
11. If G is k-totally edge regular fuzzy graph then G_A is not a k-totally edge regular fuzzy graph but G_A may be totally regular anti fuzzy graph for some other values (say k_1).
12. G_A is not a strong anti fuzzy graph and G_A does not have any effective edge also.

Example 6.4. Consider $G(V)=G_A(V)=\{u,v,w,x\}$ and $\sigma(u)=0.5$, $\sigma(v)=0.4$, $\sigma(w)=0.7$, $\sigma(x)=0.5$. The edge set is $E(G)=E(G_A)=\{uv,vw,wx,xu\}$.
In G, $\mu(uv)=0.2$, $\mu(vw)=0.4$, $\mu(wx)=0.2$, $\mu(xu)=0.4$ and $d_G(u)=d_G(v)=d_G(w)=d_G(x)=0.6$. Hence G is 0.6 regular fuzzy graph.
In G_A, $\mu(uv)=0.7$, $\mu(vw)=0.7$, $\mu(wx)=0.7$, $\mu(xu)=0.7$ and $d_{G_A}(u)=d_{G_A}(v)=d_{G_A}(w)=d_{G_A}(x)=1.4$. Hence G_A is 1.4 regular anti fuzzy graph.
Suppose G_A is strong anti fuzzy graph then $\mu(uv)=0.5$, $\mu(vw)=0.7$, $\mu(wx)=0.7$, $\mu(xu)=0.5$ and $d_{G_A}(u)=1.0$, $d_{G_A}(v)=1.2$, $d_{G_A}(w)=1.4$, $d_{G_A}(x)=1.2$. Hence G_A is not a regular anti fuzzy graph.

7. Conclusion

In this paper, some types of nodal anti fuzzy graph such as e-nodal, v-nodal, uninodal and binodal anti fuzzy graph are defined. The vertex degree and edge degree based concepts such as regular, totally regular, edge regular and totally edge regular are derived on some types of nodal anti fuzzy graph. Also, these concepts are applied on various types of anti fuzzy graph and results are obtained on them. Based on such concepts, the relationships between fuzzy graph and anti fuzzy graph are discussed and bounds are obtained on them.

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