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Interval Valued Intuitionistic Fuzzy Bi-ideals in Gamma Near-rings

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Abstract. The aim of this paper is to introduce the notion of interval valued intuitionistic fuzzy bi-ideal of gamma near-ring, and to study the related properties.

Keywords: Fuzzy set, Intuitionistic fuzzy set, I-V fuzzy set, I-V intuitionistic fuzzy set, Near-ring and Γ-near-ring.

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1. Introduction

Zadeh [18] introduced the concept of fuzzy set in 1965 and after that he also introduced the notion of interval valued fuzzy subset [17] (in short i-v fuzzy subset) in 1975, where the values of membership functions are intervals of numbers instead of a single number as in fuzzy set. The fuzzy set theory has been developed in many directions by the research scholars. Rosenfeld [14] first introduced the fuzzification of the algebraic sturctures and defined fuzzy subgroups. Jun and Kim [9] discussed interval-valued R-subgroups in terms of near-rings. Davvaz [6] introduced fuzzy ideals of near-rings with interval-valued membership functions. Thillaigovindan et al. [16] have studied interval valued fuzzy ideals and anti fuzzy ideals of near-rings. Abou-Zaid [1] proposed the concept of fuzzy sub near-rings and ideals.

In 1986, Atanassov [2] premised the concept of an intuitionistic fuzzy set (IFS). An Atanassov intuitionistic fuzzy set is deliberated as a generalization of fuzzy set [18] and has been found to be useful to deal with vagueness.In the sense of Atanassov an IFS is characterized by a pair of functions valued in [0,1]: the membership function and the non-membership function. The evaluation degrees of membership and non-membership are independent. Thus, an Atanassov intuitionistic fuzzy set is more material and concise to describe the essence of fuzziness, and Atanassov intuitionistic fuzzy set theory may be more suitable than fuzzy set theory for dealing with imperfect knowledge in many problems. The concept has been applied to various algebraic structure, Biswas [4] introduced the notion of intuitionistic fuzzy subgroup of a group by using the notion of intuitionistic fuzzy sets. Atanassov [3] proposed interval valued intuitionistic fuzzy set. The concept of Γ-near-ring, a generalization of both the concepts near-ring and Γ-ring, was introduced by Satyanarayana [15]. Later Jun et al. [11, 12] considered the fuzzification of left (resp. right) ideals of Γ-near-rings.

In this paper, we have introduced the notion of interval valued intuitionistic fuzzy bi-ideal in a Γ-near-ring and some properties of intuitionistic fuzzy bi-ideals are investigated.

2. Preliminaries

In this section, we include some elementry aspects that are necessary for this paper.

Definition 2.1. [7] A nonempty set N with two binary operations "$+$" (addition) and "$.$" (multiplication) is called a near-ring if it satisfies the following axioms:
(i) $(N,+)$ is a group,
(ii) $(N,.)$ is a semigroup,
(iii) $(x+y).z = x.z + y.z$, for all $x, y, z \in N$. It is a right near-ring because it satisfies the right distributive law.

Definition 2.2. [7] *A* Γ -near-ring is a triple $(M,+,\Gamma)$ *where,*
(i) $(M,+)$ is a group,
(ii) Γ is a nonempty set of binary operations on M such that for each $\alpha \in \Gamma, (M,+,\alpha)$ is a near-ring.
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Definition 2.3. [7] A subset A of a Γ-near-ring M is called a left (resp. right) ideal of M if
(i) $(A,+)$ is a normal divisor of $(M,+)$,
(ii) $u\alpha(x+v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) for all $x \in A, \alpha \in \Gamma$ and $u, v \in M$.

Definition 2.4. [7] Let M be Γ near-ring. A subgroup A of M is called a bi-ideal of M if $(A\Gamma M\Gamma A) \cap (A\Gamma M)\Gamma * A \subseteq A$, where the operation "$*$" is defined by,
$A\Gamma * B = \{a\gamma(a'+b) - a\gamma a' / a, a' \in A, \gamma \in \Gamma, b \in B\}$.

Definition 2.5. [7] Let M be Γ-near-ring. A subgroup Q of M is called a quasi-ideal of M if $(Q\Gamma M) \cap (M\Gamma Q) \cap (M\Gamma) * Q \subseteq Q$.

Definition 2.6. [7] Let M and N be Γ-near-rings. A mapping $f : M \to N$ is said to be ahomomorphism if $(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in M$ and $\alpha \in \Gamma$.

Definition 2.7. [7] A fuzzy set μ in a Γ-near-ring M is called a fuzzy left (resp.right) idealof M if,
(i) $\mu(x-y) \geq min\{\mu(x), \mu(y)\}$
(ii) $\mu(y+x-y) \geq \mu(x)$, for all $x, y \in M$,

(iii) $\mu(u\alpha(x+v)-u\alpha v)\geq \mu(x)$ (resp. $\mu(x\alpha u)\geq \mu(x)$) for all $x,u,v\in M$ and $\alpha\in\Gamma$.

Definition 2.8. [7] Let X be a nonempty fixed set. An IFS A in X is an object having the form $A=\{<x,\mu_A,\nu_A(x)>| x\in X\}$, where the functions $\mu_A : X\rightarrow[0,1]$ and $\nu_A : X\rightarrow[0,1]$ denote the degree of membership and degree of non membership of each element $x\in X$ to the set A, respectively, and $0\leq\mu_A(x)+\nu_A(x)\leq 1$.

Notation: For the sake of simplicity, we shall use the symbol $A=<\mu_A,\nu_A>$ for the IFS, $A=\{<x,\mu_A(x),\nu_A(x)>| x\in X\}$.

Definition 2.9. [7] Let X be a nonempty set and let $A=<\mu_A,\nu_A>$ and $B=<\mu_B,\nu_B>$ be IFSs in X . Then:

1. $A\subset B$ if $\mu_A\leq\nu_A$ and $\nu_A\geq\nu_B$
2. $A=B$ if $A\subset B$ and $B\subset A$
3. $A^c=<\nu_A,\mu_A>$
4. $A\cap B=(\mu_A\wedge\mu_B,\nu_A\vee\nu_B)$
5. $A\cup B=(\mu_A\vee\mu_B,\nu_A\wedge\nu_B)$
6. $\Box A=(\mu_A,1-\mu_Q),\Diamond A=1-\nu_A,\nu_A$

Definition 2.10. [7] Let A be an IFS in a Γ-near-ring M . For each pair $<t,s>\in[0,1]\in$ with $t+s\leq 1$, the set $A_{<t,s>}$ $\{x\in X/\mu_A(x)\geq t$ and $\nu_A(x)\leq s\}$ level subset of A.

Definition 2.11. [7] Let $A=<\mu_A,\nu_A>$ be an IFS in M and let $t\in[0,1]$. Then the sets $U(\mu_A;t)=\{x\in M:\mu_A(x)\geq t\}$ and $L(\nu_A:t)=\{x\in M:\nu_A(x)\leq t\}$ are called upper level and lower level set of A, respectively.

Definition 2.12. [16] An interval number $\bar{a}$ on [0,1] is a closed subinterval of [0,1], that is, $\bar{a}=[a^-,a^+]$ such that $0\leq a^-\leq a^+\leq 1$ where a^- and a^+ are the lower and upper endlimits of $\bar{a}$ respectively. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$.

We also identify the interval $[a,a]$ by the number $a\in[0,1]$. For any interval numbers $\bar{a}_i=[a_i^-,a_i^+],\bar{b}_i=[b_i^-,b_i^+]\in D[0,1], i\in I$, we define

$max^i\{\bar{a}_i,\bar{b}_i\}=[max^i\{a_i^-,b_i^-\},max^i\{a_i^+,b_i^+\}]$,

$min^i\{\bar{a}_i,\bar{b}_i\}=[min^i\{a_i^-,b_i^-\},min^i\{a_i^+,b_i^+\}]$,

$inf^i \overline{a}_i = [\bigcap_{i\in I} a_i^-, \bigcap_{i\in I} a_i^+], sup^i \overline{a}_i = [\bigcup_{i\in I} a_i^-, \bigcup_{i\in I} a_i^+]$

In this notation $\overline{0} = [0,0]$ and $\overline{1} = [1,1]$. For any interval numbers $\overline{a} = [a^-, a^+]$ and $\overline{b} = [b^-, b^+]$ on $[0,1]$, define

(1) $\overline{a} \leq \overline{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$.

(2) $\overline{a} = \overline{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$.

(3) $\overline{a} < \overline{b}$ if and only if $\overline{a} \leq \overline{b}$ and $\overline{a} \neq \overline{b}$

(4) $k\overline{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.13. [16] Let X be any set. A mapping $\overline{A}: X \to D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of X where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\overline{A}(x) = [A^-(x), A^+(x)]$ for all $x \in X$, where A^- and A^+ arefuzzy subsets of X such that $A^-(x) \leq A^+(x)$ for all $x \in X$.
Note that $\overline{A}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.

Definition 2.14. [16] A mapping $min^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $min^i(\overline{a}, \overline{b}) = [min\{a^-, b^-\}, min\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval min-norm.

Definition 2.15. [16] A mapping $max^i : D[0,1] \times D[0,1] \to D[0,1]$ defined by $max^i(\overline{a}, \overline{b}) = [max\{a^-, b^-\}, max\{a^+, b^+\}]$ for all $\overline{a}, \overline{b} \in D[0,1]$ is called an interval max-norm. Let min^i and max^i be the interval min-norm and max-norm on $D[0,1]$ respectively. Then the following are true.

1. $min^i\{\overline{a}, \overline{a}\} = \overline{a}$ and $max^i\{\overline{a}, \overline{a}\} = \overline{a}$ for all $\overline{a} \in D[0,1]$.
2. $min^i\{\overline{a}, \overline{b}\} = min^i\{\overline{b}, \overline{a}\}$ and $max^i\{\overline{a}, \overline{b}\} = max^i\{\overline{b}, \overline{a}\}$ for all $\overline{a}, \overline{b} \in D[0,1]$.
3. If $\overline{a} \geq \overline{b} \in D[0,1]$, then $min^i\{\overline{a}, \overline{c}\} \geq min^i\{\overline{b}, \overline{c}\}$ and $max^i\{\overline{a}, \overline{c}\} \geq max^i\{\overline{b}, \overline{c}\}$ for all $\overline{c} \in D[0,1]$.

Definition 2.16. An interval valued intuitionistic fuzzy set (i-v IFS,shortly) “$\overline{A}$" over X isan object having the form $\overline{A} = \{(x, \overline{\mu}_A, \overline{\lambda}_A) : x \in X\}$, where $\overline{\mu}_A(x) : X \to D[0,1]$ and $\overline{\lambda}_A(x) : X \to D[0,1]$, the intervals $\overline{\mu}(x)$ and $\overline{\lambda}_A(x)$ denotes the intervals of the degree of membership and the degree of the non-membership of the element x to the set $\overline{A}$, where $\overline{\mu}_A(x) = [\overline{\mu}_A^-, \overline{\mu}_A^+(x)]$ and $\overline{\lambda}_A(x) = [\overline{\lambda}_A^-, \overline{\lambda}_A^+(x)]$ for all $x \in X$ with the condition

$[0,0] \le [\overline{\mu}_A(x), \overline{\lambda}_A(x)] \le [1,1]$ for all $x \in X$. For the sake of simplicity, we use the symbol $\overline{A} = (\overline{\mu}_A, \overline{\lambda}_A)$, where $D[0,1]$ is the set of all closed sub interval of $[0,1]$.

3. Interval valued intuitionistic fuzzy bi-ideals of Γ-near-ring

In what follows, M will denote a Γ-near-ring unless otherwise specified.

Definition 3.1. An interval valued intuitionistic fuzzy ideal $A = <\overline{\mu}_A, \overline{\nu}_A>$ of M is called an interval valued intuitionistic fuzzy bi-ideal of M if,

(i) $\overline{\mu}_A(x-y) \ge \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(y)\}$,(ii) $\overline{\mu}_A(y+x-y) \ge \overline{\mu}_A(x)$,

(iii) $\overline{\mu}_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \ge \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(z)\}$ for al $x, y, z \in M, \alpha, \beta \in \gamma$

(iv) $\overline{\nu}_A(x-y) \le \{\overline{\nu}_A(x) \vee \overline{\nu}_A(y)\}$,

(v) $\overline{\nu}_A(y+x-y) \le \overline{\nu}_A(x)$,

(vi) $\overline{\nu}_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \le \vee\{\overline{\nu}_A(z)\}$ for all $x, y, z \in M, \alpha, \beta \in \Gamma$.

Example 3.2. Let N be the set of all integers then N is a ring. Take $M = \Gamma = N$. Let $a, b \in M, \alpha \in \Gamma$, suppose $a\alpha b$ is the product of $a, \alpha, b \in N$.

Then M is a Γ-near-ring. Define an IFS $A = <\overline{\mu}_A, \overline{\nu}_A>$ in N as follows.

$\overline{\mu}_A(0) = [1,1]$

$\overline{\nu}_A(0) = [0,0]$ where, $t \in [0,1), s \in (0,1]$ and $t + s \le 1$.by routine calculations, Clearly, $\overline{A}$ is an i-v intuitionistic fuzzy bi-ideal of a Γ-near-ring M.

Lemma 3.3. If B is a bi-ideal of M then for any $0 < t, s < 1$, there exists an i-v intuitionistic fuzzy bi-ideal $C = <\overline{\mu}_c, \overline{\nu}_c>$ of M such that $C_{<t,s>} = B$.

Proof: Let $C \to [0,1]$ be a function defined by

$$\overline{\mu}_B(x) = \begin{cases} \overline{t} & \text{if } x \in B, \\ \overline{0} & \text{if } x \notin B, \end{cases} \quad \overline{\nu}_B(x) = \begin{cases} \overline{s} & \text{if } y \in B, \\ \overline{0} & \text{if } y \notin B. \end{cases}$$

For all $x \in M$ and the pair $s, t \in [0,1]$. Then $C_{<t,s>} = B$ is an i-v intuitionistic fuzzy bi-ideal of M with $t + s \le 1$. Now suppose that B is a bi-ideal of M. For all $x, y \in B$, such that $x - y \in B$, we have,

$\overline{\mu}_c(x-y) \ge t = \{\overline{\mu}_c(x) \wedge \overline{\mu}_c(y)\}$, $\overline{\nu}_c(x-y) \le s = \{\overline{\nu}_c(x) \vee \overline{\nu}_c(y)\}$,

$\overline{\mu}_c(y+x-y) \ge t = \overline{\mu}_c(x)$, $\overline{\nu}_c(y+x-y) \le s = \overline{\nu}_c(x)$,

Also, for all $x, y, z \in B$ and $\alpha, \beta \in \Gamma$ such that $x\alpha y\beta z \in B$, we have,

$\overline{\mu}_c((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))) \ge t = \{\overline{\mu}_c(x) \wedge \overline{\mu}_c(z)\}$,

$\overline{\nu}_c((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) \le s = \{\overline{\nu}_c(x) \vee \overline{\nu}_c(z)\}$, Thus $C_{<t,s>}$ is an i-v intuitionistic fuzzy bi-ideal of M.

Lemma 3.4. Let B be a nonempty subset of M. Then B is a bi-ideal of M if and only if the IFS $\overline{B} =< \overline{\lambda}_B, \overline{\lambda'}_B >$ is an i-v intuitionistic fuzzy ideal of M.

Proof: Let $x, y \in B$ from the hypothesis, $x - y \in B$.

(i) If $x, y \in B$, then $\overline{\lambda}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(y) = \bar{1}$ and $\overline{\lambda'}_B(y) = \bar{0}$. In this case, $\overline{\lambda}_B(x-y) = \bar{1} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(y)\}$. $\overline{\lambda'}_B(x-y) = \bar{0} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda'}_B(y)\}$.

(ii) If $x \in B, y \notin B$, then $\overline{\lambda}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(y) = \bar{0}$, and $\overline{\lambda'}_B(y) = \bar{1}$.

Thus, $\overline{\lambda}_B(x-y) = \bar{0} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(y)\}$. $\overline{\lambda'}_B(x-y) = \bar{1} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda'}_B(y)\}$.

(iii) If $x \notin B, y \in B$, then $\overline{\lambda}_B(x) = \bar{0}, \overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda}_B(x) = \bar{1}$ and $\overline{\lambda'}_B(y) = \bar{0}$.

Thus, $\overline{\lambda}_B(x-y) = \bar{0} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(y)\}$. $\overline{\lambda'}_B(x-y) = \bar{1} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda}_B(y)\}$.

Thus (i) of Definition 3.1 holds good.

Let $x, y, \in B$. From the hypothesis, $y + x - y \in B$.

(i) If $x, y \in B$, then $\overline{\lambda}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(y) = \bar{1}$ and $\overline{\lambda'}_B(y) = \bar{0}$.

In this case, $\overline{\lambda}_B(y + x - y) = \bar{1} \geq \overline{\lambda}_B(x)$. $\overline{\lambda'}_B(y + x - y) = \bar{0} \leq \overline{\lambda'}_B(x)$.

(ii) If $x \in B, y \notin B$, then $\overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(y) = \bar{0}$ and $\overline{\lambda'}_B(y) = \bar{1}$.

Thus, $\overline{\lambda}_B(y + x - y) = \bar{0} \geq \overline{\lambda}_B(x)$. $\overline{\lambda'}_B(y + x - y) = \bar{1} \leq \overline{\lambda'}_B(x)$.

(iii) If $x \notin B, y \in B$, then $\overline{\lambda}_B(x) = \bar{0}, \overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda}_B(y) = \bar{1}$ and $\overline{\lambda'}_B(y) = \bar{0}$.

Thus, $\overline{\lambda}_B(y + x - y) = \bar{0} \geq \overline{\lambda}_B(x)$. $\overline{\lambda'}_B(y + x - y) = \bar{1} \leq \overline{\lambda'}_B(x)$

(iv) If $x \notin B, y \notin B$, then $\overline{\lambda}_B(x) = \bar{0}, \overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda}_B(y) = \bar{0}$ and $\overline{\lambda'}_B(y) = \bar{1}$.

Thus, $\overline{\lambda}_B(y + x - y) \geq \bar{0} = \overline{\lambda}_B(x)$. $\overline{\lambda'}_B(y + x - y) = \bar{1} \leq \overline{\lambda'}_B(x)$.

Thus (ii) of Definition 3.1 holds good.

Let $x, y, z \in B$ and $\alpha, \beta \in \Gamma$. From the hypothesis, $x\alpha y\beta, x\alpha(y+z) - x\alpha z \in B$.

(i) If $x, z \in B$, then $\overline{\lambda}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(z) = \bar{1}$ and $\overline{\lambda'}_B(z) = \bar{0}$.

Thus, $\overline{\lambda}_B(\overline{\mu}((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))) = \bar{1} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(z)\}$

$\overline{\lambda'}_B(\overline{\mu}((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) = \bar{0} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda'}_B(z)\}$.

(ii)If $x \in B, z \notin B$, then $\overline{\lambda}_B(x) = \bar{1}, \overline{\lambda'}_B(x) = \bar{0}, \overline{\lambda}_B(z) = \bar{0}$ and $\overline{\lambda'}_B(z) = \bar{1}$,

Thus, $\overline{\lambda}_B(\overline{\mu}((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))) = \bar{0} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(z)\}$

$\overline{\lambda'}_B(\overline{\mu}((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) = \bar{1} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda'}_B(z)\}$.

(iii) If $x \notin B, z \in B$, then $\overline{\lambda}_B(x) = \bar{0}, \overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda}_B(z) = \bar{1}$ and $\overline{\lambda'}_B(z) = \bar{0}$.

Thus, $\overline{\lambda}_B(\overline{\mu}((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))) = \bar{0} \geq \{\overline{\lambda}_B(x) \wedge \overline{\lambda}_B(z)\}$.

$\overline{\lambda'}_B(\overline{\mu}((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) = \bar{1} \leq \{\overline{\lambda'}_B(x) \vee \overline{\lambda'}_B(z)\}$.

(iv) If $x \notin B, z \notin B$, then $\overline{\lambda}_B(x) = \bar{0}, \overline{\lambda'}_B(x) = \bar{1}, \overline{\lambda}_B(z) = \bar{0}$ and $\overline{\lambda'}_B(z) = \bar{1}$.

Thus, $\overline{\lambda}_B(\overline{\mu}((x\alpha y\beta z)\wedge(x\alpha(y+z)-x\alpha z)))\geq\overline{0}=\{\overline{\lambda}_B(x)\wedge\overline{\lambda}_B(z)\}$

$\overline{\lambda'}_B(\overline{\mu}((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z)))\leq\bar{1}=\{\overline{\lambda'}_B(x)\vee\overline{\lambda'}_B(z)\}$.

Thus (iii) of Definition 3.1 holds good.

Conversely, suppose that IFS $\overline{B}=<\overline{\lambda}_B,\overline{\lambda'}_B>$ is an i-v intuitionistic fuzzy ideal of M. Then by Lemma 3.3, $\overline{\lambda}_B$ is two-valued, Hence B is a bi-ideal of M.

Theorem 3.5. If $\{\overline{A}_i\}_{i\in A}$ is a family of i-v intuitionistic fuzzy bi-ideals of M then $\cap A_i$ is an i-v intuitionistic fuzzy bi-ideals of M, where

$\cap A_i=\{\wedge\overline{\mu}_{A_i},\vee\overline{\nu}_{Ai}\}$, $\wedge\overline{\mu}_{Ai}(x)=inf\{\overline{\mu}_{Ai}(x)\mid i\in\wedge, x\in M\}$, and

$\vee\overline{\nu}_{Ai}(x)=sup\{\overline{\nu}_{Ai}(x)\mid i\in\vee, x\in M\}$.

Proof: Let $x,y\in M$. Then we have,

$\wedge\overline{\mu}_{Ai}(x-y)=inf\{\{\overline{\mu}_{Ai}(x)\wedge\overline{\mu}_{Ai}(y)\}\mid i\in\wedge, x,y\in M\}$

$=\{\{inf(\overline{\mu}_{Ai}(x))\wedge inf(\overline{\mu}_{Ai}(y))\}\mid i\in\wedge, x,y\in M\}$

$=\{\{inf(\overline{\mu}_{Ai}(x))\mid i\in\wedge, x\in M\}\wedge\{inf(\overline{\mu}_{Ai}(y))\mid i\in\wedge, y\in M\}\}$

$=\{(\wedge\overline{\mu}_{Ai}(x))\wedge(\wedge\overline{\mu}_{Ai}(y))\}$.

Let $x,y\in M$. Then we have,

$\wedge\overline{\mu}_{Ai}(y+x-y)=inf\{\overline{\mu}_{Ai}(x)\mid i\in\wedge, x,y\in M\}=\wedge\overline{\mu}_i(x)$.

Let $x,y,z\in M$ and $\alpha,\beta\in\Gamma$.

$\wedge\overline{\mu}_{Ai}((x\alpha y\beta z)\wedge(x\alpha(y+z)-x\alpha z))=inf\{\{\overline{\mu}_{Ai}(x)\wedge\overline{\mu}_{Ai}(z)\}\mid i\in\wedge, x,z\in M\}$

$=\{\{inf(\overline{\mu}_{Ai}(x)\mid i\in M)\}\wedge\{inf(\overline{\mu}_{Ai}\mid i\in\wedge, z\in M)\}\}=\{(\wedge\overline{\mu}_{Ai}(x))\wedge(\wedge\overline{\mu}_{Ai}(z))\}$.

Let $x,y\in M$. Then we have,

$\vee(\overline{\nu}_{Ai}(x-y))=\sup\{\{(\overline{\nu}_{Ai}(x))\vee(\overline{\nu}_{Ai}(y))\}\mid i\in\vee, x,y\in M\}$

$=\{\{(\sup(\overline{\nu}_{Ai}(x))\vee(\sup(\overline{\nu}_{Ai}(y))\}\mid i\in\vee, x,y\in M\}$

$=\{\{\sup(\overline{\nu}_{Ai}(x))\mid i\in\vee, x\in M\}\vee\{\sup(\overline{\nu}_{Ai}(y))\mid i\in\vee, y\in M\}\}$

$=\{(\vee\overline{\nu}_{Ai}(x))\vee(\vee\overline{\nu}_{Ai}(y))\}$.

Let $x,y\in M$. Then we have,

$\vee(\overline{\nu}_{Ai}(y+x-y))=sup\{\overline{\nu}_{Ai}\mid i\in\vee, x,y\in M\}=\vee(\overline{\nu}_{Ai}(x))$.

Let $x,y,z\in M$ and $\alpha,\beta\in\Gamma$.

$\vee\overline{\nu}_{Ai}((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))=sup\{\{\overline{\nu}_{Ai}(x)\vee\overline{\nu}_{Ai}(z)\}\mid i\in\vee, x,z\in M\}$

$=\{\{sup(\overline{\nu}_{Ai}(x))\vee sup(\overline{\nu}_{Ai}(z))\}\mid i\in\vee, x,z\in M\}$

$=\{\{sup(\overline{\nu}_{Ai}(x)\mid i\in\vee, x\in M)\}\vee\{sup(\overline{\nu}_{Ai}(z)\mid i\in\wedge, z\in M)\}\}$

$=\{(\vee\overline{\nu}_{Ai}(x))\vee(\vee\overline{\nu}_{Ai}(z))\}$.

Hence, $\cap A_i = \{\wedge\overline{\mu}_{Ai}, \vee\overline{\nu}_{Ai}\}$ is an i-v intuitionistic fuzzy bi-ideal of M.

Theorem 3.6. If $\overline{A}$ is an i-v intuitionistic fuzzy bi-ideal of M then $\overline{A}'$ is also an i-v intuitionistic fuzzy bi-ideal of M.

Proof: Let $x, y \in M$. We have,

$\overline{\mu'}_A(x-y) = \bar{1} - \overline{\mu}_A(x-y) = \bar{1} - \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(y)\}$,

$\overline{\nu'}_A(x-y) = \bar{1} - \overline{\nu}_A(x-y) = \bar{1} - \{\overline{\nu}_A(x) \vee \overline{\nu}_A(y)\}$.

Let $x, y \in M$. We have,

$\overline{\mu'}_A(y+x-y) = \bar{1} - \overline{\mu}_A(y+x-y) = \bar{1} - \overline{\mu}_A(x) = \overline{\mu'}_A(x)$,

$\overline{\nu'}_A(y+x-y) = \bar{1} - \overline{\nu}_A(y+x-y) = \bar{1} - \overline{\nu}_A(x) = \overline{\nu'}_A\overline{\nu}(x)$.

Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. We have

$\overline{\mu'}_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z) = \bar{1} - \overline{\mu}_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))$

$= \bar{1} - \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(z)\} = \bar{1} - \{\overline{\mu}_A(x) \wedge \bar{1} - \overline{\mu}_A(z)\} = \{\overline{\mu}_A'(x) \wedge \overline{\mu}_A'(z)\}$,

$\overline{\nu}'_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z) = \bar{1} - \overline{\nu}_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))$

$= \bar{1} - \{\overline{\nu}_A(x) \vee \overline{\nu}_A(z)\} = \{\overline{\nu}_A'(x)\overline{\nu}(x) \wedge \overline{\nu}_A'\overline{\nu}(z)\}$.

Therefore, $\overline{A'}$ is also an i-v intuitionistic fuzzy bi-ideal of M.

Theorem 3.7. An IVIFS $\overline{A}$ of M is an i-v intuitionistic fuzzy bi-ideal of M if and only if the level sets. $U(\overline{\mu}_A;t) = \{x \in M \mid \overline{\mu}(x) \geq t\}$ and $L(\overline{\nu}_A;t) = \{x \in M : \overline{\nu}_A(x) \leq t\}$ are bi-ideal of M when it is non-empty.

Proof: Let $\overline{A}$ be an i-v intuitionistic fuzzy bi-ideal of M.

Then, $\overline{\mu}_A(x-y) \geq \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(y)\}$. $x, y \in U\{\overline{\mu}_A; t \Rightarrow \overline{\mu}_A(x) \geq t, \overline{\mu}_A(y) \geq t\}$

$\overline{\mu}_A(x-y) \geq t \Rightarrow x-y \in U(\overline{\mu}_A;t)$. $\overline{\mu}_A(x-y) \geq \{\overline{\mu}_A \wedge \overline{\mu}_A(y)\}$.

$x, y \in L(\overline{\nu}_A;t) \Rightarrow \overline{\nu}_A(x) \leq t, \overline{\nu}_A(y) \leq t$. $\overline{\nu}_A(x-y) \leq \{\overline{\nu}_A(x) \vee \overline{\nu}_A(y) \leq t\}$

$\overline{\nu}_A(x-y \leq t) \Rightarrow x-y \in L(\overline{\nu}_A;t)$.

Let $\overline{\mu}_A(y+x-y) \geq \overline{\mu}_A(x)$. $x, y \in U(\overline{\mu}_A;t) \Rightarrow \overline{\mu}_A(x) \geq t, \overline{\mu}_A(y) \geq t$

$\overline{\mu}_A(y+x-y) \geq \overline{\mu}_A(x) \geq t$. $\overline{\mu}_A(y+x-y) \geq t \Rightarrow y+x-y \in U(\overline{\mu};t)$.

Let $\overline{\nu}_A(y+x-y) \leq \overline{\nu}_A(x)$. $x, y \in L(\overline{\nu}_A;t) \Rightarrow \overline{\nu}_A(x) \leq t, \overline{\nu}_A(y) \leq t$

$\overline{\nu}_A(y+x-y) \leq \overline{\nu}_A(x) \leq t$. $\overline{\nu}_A(y+x-y) \leq t \Rightarrow y+x-y \in L(\overline{\nu}_A;t)$.

Also,let $\overline{\mu}_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) \geq \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(z)\}$.

$x, y, z \in U(\overline{\mu}_A;t), \alpha, \beta \in \Gamma \Rightarrow \overline{\mu}_A(x) \geq t, \overline{\mu}_A(y) \geq t, \overline{\mu}_A(z) \geq t$.

$\overline{\mu}_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq \{\overline{\mu}_A(x) \wedge \overline{\mu}_A(z) \geq t\}$

$\overline{\mu}_A((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z)) \geq t \Rightarrow (x\alpha y\beta z), (x\alpha(y+z) - x\alpha z) \in U(\overline{\mu}_A;t)$.

Thus, $U(\overline{\mu}_A;t)$ is an bi-ideal of M.

$\overline{\nu}_A((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))\leq\{\overline{\nu}_A(x)\vee\overline{\nu}_A(z)\}$.

$x,y,z\in L(\overline{\nu}_A;t),\alpha,\beta\in\Gamma\Rightarrow\overline{\nu}_A(x)\leq t,\overline{\nu}_A(y)\leq t,\overline{\nu}_A(z)\leq t$

$\overline{\nu}_A((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))\leq\{\overline{\nu}_A(x)\vee\overline{\nu}_A(z)\}\leq t$

$\overline{\nu}_A((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))\leq t\Rightarrow(x\alpha y\beta z),(x\alpha(y+z)-x\alpha z)\in L(\overline{\nu}_A;t)$.

Thus, $L(\overline{\nu}_A;t)$ is an bi-ideal of M.

Conversely, if $U(\overline{\mu}_A;t)$ is an bi-ideal of M let $t=\{\overline{\mu}_A(x)\wedge\overline{\mu}_A(y)\}$.

Then $x,y\in U(\overline{\mu}_A;t),\Rightarrow x-y\in U(\overline{\mu}_A;t)\Rightarrow\overline{\mu}_A(x-y)\geq t$.

$\overline{\mu}_A(x-y)\geq\{\overline{\mu}_A(x)\wedge\overline{\mu}_A(y)\}$.

Also, $x,y\in U(\overline{\mu}_A;t),\Rightarrow y+x-y\in U(\overline{\mu}_A;t)\Rightarrow\overline{\mu}_A(y+x-y)\geq\overline{\mu}_A(x)$.

If $L(\overline{\nu}_A;t)$ is an bi-ideal of M let $t=\{\overline{\nu}_A(x)\vee\overline{\nu}_A(y)\}$. Then

$x,y\in L(\overline{\nu}_A;t)\Rightarrow x-y\in L(\overline{\nu}_A;t)\Rightarrow\overline{\nu}_A(x-y)\leq t\Rightarrow\overline{\nu}_A(x-y)\leq\{\overline{\nu}_A(x)\vee\overline{\nu}_A(y)\}$.

Also, $x,y\in L(\overline{\nu}_A;t),\Rightarrow y+x-y\in L(\overline{\nu}_A;t)\Rightarrow\overline{\nu}_A(y+x-y)\geq\overline{\nu}_A(x)$.

Next, define $t=\{\overline{\nu}_A(x)\vee\overline{\nu}_A(z)\}$.

Then $x,y,z\in U(\overline{\mu}_A;t),\alpha,\beta\in\Gamma\Rightarrow(x\alpha y\beta z),(x\alpha(y+z)-x\alpha z)\in L(\overline{\nu}_A;t)$

$\overline{\nu}_A((x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z))\leq t$.

$\overline{\nu}_A(x\alpha y\beta z)\vee(x\alpha(y+z)-x\alpha z)\leq\{\overline{\nu}(x)\vee\overline{\nu}_A(z)\}$.

Consequently, $\overline{A}$ is an i-v intuitionistic fuzzy bi-ideal of M.

Theorem 3.8. Let $\overline{A}$ be an i-v intuitionistic fuzzy bi-ideal of M. If M is completely regular, then $\overline{\nu}_A(a)=\overline{\mu}_A(a\alpha a),\overline{\nu}_A(a)=\overline{\nu}_A(a\alpha a)$ for all $a\in M$ and $\alpha\in\Gamma$.

Proof: Straight forward.

Let f be mapping from a set X to Y, and $\overline{A}$ be $IVIFS$ on Y. Then the preimage of $\overline{\mu}$ under f, denoted by $f^{-1}(\overline{A})$, is defined by;

$f^{-1}(\overline{\mu}_A(x))=\overline{\mu}_A(f(x)),\ f^{-1}(\overline{\nu}_A(x))=\overline{\nu}_A(f(x))$ for all $x\in X$.

Theorem 3.9. Let the pair of mapping $f:M\rightarrow N$ be a homomorphism of Γ- near rings. If $\overline{\mu}$ is an i-v intuitionistic fuzzy bi-ideal of N, then the preimage $f^{-1}(A)$ of $\overline{A}$ under f is an i-v intuitionistic fuzzy bi-ideal of M.

Proof: Let $x,y\in M$. Then we have $f^{-1}(\overline{\mu}_A)(x-y)=\overline{\mu}_A(f(x-y))$

$=\overline{\mu}_A(f(x)-f(y))\geq\{\overline{\mu}_A(f(x))\wedge\overline{\mu}_A(f(y))\}=\{f^{-1}(\overline{\mu}_A)(x)\wedge f^{-1}(\overline{\mu}_A(y))\}$.

$\{f^{-1}(\overline{\nu}_A)(x-y)=\overline{\nu}_A(f(x-y))\}=\overline{\nu}_A(f(x)-f(y))\leq\{\overline{\nu}_A(f(x))\vee\overline{\nu}_A(f(y))\}$

$=\{f^{-1}(\overline{\nu}_A(x)) \vee f^{-1}(\overline{\nu}_A(y))\}$.

Let $x, y \in M$. Then we have $f^{-1}(\overline{\mu}_A)(y+x-y) = \overline{\mu}_A(f(y+x-y))$
$\geq \overline{\mu}_A(f(x)) = f^{-1}(\overline{\mu}_A(x))$.

$f^{-1}(\overline{\nu}_A)(y+x-y) = \overline{\nu}_A(f(y+x-y)) = \overline{\nu}_A(f(x)) = f^{-1}(\overline{\nu}_A(x))$.

Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then $f^{-1}(\overline{\mu}_A)((x\alpha y\beta z) \wedge (x\alpha(y+z) - x\alpha z))$
$= \overline{\mu}_A((f(x\alpha y\beta z)) \wedge (x\alpha(y+z) - x\alpha z))) = \overline{\mu}_A((f(x\alpha y\beta z)) \wedge (f(x\alpha(y+z) - x\alpha z)))$
$\geq \overline{\mu}_A(f(x)) \wedge \overline{\mu}_A(f(z))$
$=\{f^{-1}(\overline{\mu}_A(x)) \wedge f^{-1}(\overline{\mu}_A(z))\}$. Therefore, $f^{-1}(\overline{\mu}_A)$ is an i-v intuitionistic fuzzy bi-idealof M.

$f^{-1}(\overline{\nu}_A((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z)) = \overline{\nu}_A(f((x\alpha y\beta z) \vee (x\alpha(y+z) - x\alpha z))) =$
$\overline{\nu}_A((f(x\alpha y\beta z)) \vee (f(x\alpha(y+z) - x\alpha z))) \leq \{\overline{\nu}_A(f(x)) \vee \overline{\nu}_A(f(z))\}$

Therefore, $f^{-1}(\overline{\nu}_A)$ is an i-v intuitionistic fuzzy bi-ideal of M.

4. Conclusion

In this paper, we have presented the notion of interval valued intuitionistic fuzzy bi-ideals of a Γ-near-rings and derived the properties of these ideals.

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