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Intuitionistic Fuzzy G_δ-e-locally Continuous and Irresolute Functions

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Abstract. The purpose of this paper is to introduce the concepts of an intuitionistic fuzzy G_δ-e-locally neighborhood, intuitionistic fuzzy G_δ-e-locally quasi neighborhood, intuitionistic fuzzy G_δ-e-locally continuous and intuitionistic fuzzy G_δ-e-locally irresolute functions in intuitionistic fuzzy topological spaces. Also some interesting properties are established..

Keywords: intuitionistic fuzzy G_δ-e-locally neighborhood, intuitionistic fuzzy G_δ-e-locally quasi neighborhood, intuitionistic fuzzy G_δ-e-locally continuous, intuitionistic fuzzy G_δ-e-locally irresolute function.

AMS Mathematics Subject Classification (2010): 54A40, 54A99, 03E72, 03E99

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [10] and later Atanassov [1] generalized the idea to intuitionistic fuzzy sets. On the other hand, Coker [3] introduced the notions of an intuitionistic fuzzy topological spaces, intuitionistic fuzzy continuity and some other related concepts. The concept of an intuitionistic fuzzy e-closed set was introduced by Sobana et. al., [8]. Ganster and Relly used locally closed sets in [5] to define LC-continuity and LC-irresoluteness. Balasubramanian [2] introduced and studied the concept of fuzzy G_δ set in a fuzzy topological space. In this paper, the concepts of an intuitionistic fuzzy G_δ-e-locally quasi neighborhood, intuitionistic fuzzy G_δ-e-locally continuous and intuitionistic fuzzy G_δ-e-irresolute function are introduced and studied. Some interesting properties among continuous function are discussed.

2. Preliminaries

Definition 2.1. [1] Let X be a nonempty fixed set and I be the closed interval $[0,1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, where the mapping $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form, $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \gamma_A\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2. [1] Let X be a nonempty set and the IFSs A and B in the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$;
2. $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in X\}$;
3. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$;
4. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$;

Definition 2.3. [1] The IFS's $0_{\sim}$ and $1_{\sim}$ are defined by , $0_{\sim} = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_{\sim} = \{\langle x, 1, 0\rangle : x \in X\}$.

Definition 2.4. [3] An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family T of intuitionistic fuzzy sets in X satisfying the following axioms:

1. $0_{\sim}, 1_{\sim} \in T$; 2. $G_1 \cap G_2 \in T$, for any $G_1, G_2 \in T$; 3. $\cup G_i \in T$ for arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this paper by (X, T) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS which belongs to T is called an intuitionistic fuzzy open set (IFOS) in X. The complement $\overline{A}$ of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS) in X.

Definition 2.5. [3] Let (X, T) be an IFTS and $A = \{\langle x, \mu_A, \nu_A\rangle : x \in X\}$ be an IFS in X. Then the intuitionistic fuzzy closure and intuitionistic fuzzy interior of A are defined by

1. $IFcl(A) = \bigcap\{C : C \text{ isanIFCSin } X \text{and } C \supseteq A\}$;
2. $IFint(A) = \bigcup\{D : D \text{ isanIFOSin } X \text{and } D \subseteq A\}$;

Proposition 2.1. [1] For any IFS A in (X, T) we have

1. $cl(\overline{A}) = \overline{int(A)}$ 2. $int(\overline{A}) = \overline{cl(A)}$

Corollary 2.1. [3] Let A, $A_i (i \in J)$ be IFSs in X, B, $B_j (j \in K)$ IFSs in Y and $f : X \to Y$ a function. Then
1. $A \subseteq f^{-1}(f(A))$ (If f is injective, then $A = f^{-1}(f(A))$). 2. $f(f^{-1}(B)) \subseteq B$ (If f is surjective, then $f(f^{-1}(B)) = B$). 3. $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ 4. $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$. 5. $f^{-1}(1_{\sim}) = 1_{\sim}$. 6. $f^{-1}(0_{\sim}) = 0_{\sim}$. 7. $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

Definition 2.6. [4] Let X be a nonempty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the IFS $x_{r,s} = \langle x, x_r, 1 - x_{1-s} \rangle$ is called and intuitionistic fuzzy point (IFP) in X, where r denotes the degree of membership of $x_{r,s}$, s denotes the degree of non membership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$. The IFP $x_{r,s}$ is contained in the IFS $A (x_{r,s} \in A)$ if and only if $r < \mu_A(x)$, $s > \gamma_A(x)$.

Definition 2.7. [6] An IFS U of an IFTS X is called
1. neighborhood of an IFP $c(a,b)$, if there exists an $IFOSG$ in X such that $c(a,b) \in G \leq U$. 2. q-neighborhood of an IFP $c(a,b)$, if there exists an $IFOSG$ in X such that $c(a,b) q G \leq U$.

Definition 2.8. [3] Let X and Y be two nonempty sets and $f : X \to Y$ be a function.
1. If $B = \{\langle y, \mu_B(y), \gamma_B(y) \rangle : y \in Y\}$ is an IFS in Y, then the preimage of B under f (denoted by $f^{-1}(B)$) is defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X\}$
2. If $A = \{\langle x, \lambda_A(x), \nu_A(x) \rangle : x \in X\}$ is an IFS in X, then the image of A under f (denoted by $f(A)$) is defined by $f(A) = \{\langle y, f(\lambda_A(y)), (1 - f(1 - \nu_A))(y) \rangle : \in X\}$.

Definition 2.9. [9] Let A be IFS in an IFTS (X, T). A is called an
1. intuitionistic fuzzy regular open set (briefly $IFROS$) if $A = intcl(A)$
2. intuitionistic fuzzy regular closed set (briefly $IFRCS$) if $A = clint(A)$

Definition 2.10. [2] Let (X, T) be a fuzzy topological space and λ be a fuzzy set in X λ is called G_δ set if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$. The complement of fuzzy G_λ is fuzzy F_σ

Definition 2.11. [5] A subset A of a space (X,T) is called locally closed (briefly lc) if $A = C \cup D$, where C is open and D is closed in (X,T).

Definition 2.12. [9] Let (X,T) be an IFTS and $A = \langle x, \mu_A(x), \nu_A(x) \rangle$ be a IFS in X. Then the fuzzy δ closure of A are denoted and defined by $cl_\delta(A) = \cap\{K : K$ is an IFRCS in X and $A \subseteq K\}$ and $int_\delta(A) = \cup\{G : G$ is an IFROS in X and $G \subseteq A\}$.

Definition 2.13. [8] Let A be an IFS in an IFTS(X,T). A is called an intuitionistic fuzzy e-open set (IFeOS, for short) in X if $A \subseteq clint_\delta(A) \cup intcl_\delta(A)$

Definition 2.14. [3] Let (X,T) and (Y,S) be two IFT's and let $f : X \to Y$ be a function. Then f is said to be intuitionistic fuzzy continuous iff the preimage of each IFS in S is an IFS in T.

Definition 2.15. [8] Let (X,T) and (Y,S) be two IFT's and let $f : X \to Y$ be a function. Then f is said to be intuitionistic fuzzy e-continuous iff the preimage of each IFS in S is an $IFeOS$ in T.

Definition 2.16. [7] Let (X,T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X,T). Then A is said to be intuitionistic fuzzy e- locally closed set (in short, IF-e-lcs) if $A = C \cap D$, where $C = \{\langle x, \mu_C(x), \gamma_C(x) \rangle : x \in X\}$ is an intuitionistic fuzzy e-open set and $D = \{\langle x, \mu_D(x), \gamma_D(x) \rangle : x \in X\}$ is an intuitionistic fuzzy e-closed set in (X,T).

Definition 2.17. [7] Let (X,T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space X. Then A is said to be an intuitionistic fuzzy eG_δ- set if $A = \bigcap_{i=1}^{\infty} A_i$, where $A_i = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x) \rangle : x \in X\}$ is an intuitionistic fuzzy e-open set in an intuitionistic fuzzy topological space (X,T).

Definition 2.18. [7] Let (X,T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X,T). Then A is said to be an intuitionistic fuzzy eG_δ-locally closed set (in short,IF-eG_δ-lcs) if $A = C \cap D$, where $C = \{\langle x, \mu_C(x), \gamma_C(x)\rangle : x \in X\}$ is an intuitionistic fuzzy eG_δ set and $D = \{\langle x, \mu_D(x), \gamma_D(x)\rangle : x \in X\}$ is an intuitionisitic fuzzy e-closed set in (X, T). The complement of an intuitionistic fuzzy eG_δ-locally closed set is said to be an intuitionistic fuzzy eG_δ-locally open set (in short, IFeG_δ-los).

Definition 2.19. [7] Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). Then A is said to be an intuitionistic fuzzy G_δ-e-locally closed set (in short,IFG_δ-e-lcs) if $A = B \cap C$, where $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$ is an intuitionistic fuzzy G_δ set and $C = \{\langle x, \mu_C(x), \gamma_C(x)\rangle : x \in X\}$ is an intuitionisitic fuzzy e-closed set in (X, T). The complement of an intuitionistic fuzzy G_δ-e-locally closed set is said to be an intuitionistic fuzzy G_δ-e-locally open set (in short, IFG_δ-e-los).

Definition 2.20. [7] Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy G_δ-e-locally closure of A is denoted and defined by $IFG_\delta\text{-}e\text{-}lcl(A) = \bigcap\{B : B = \langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X$ is an intuitionistic fuzzy G_δ-e-locally closed set in X and $A \subseteq B\}$.

Definition 2.21. [7] Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set on an intuitionistic fuzzy topological space (X, T). The intuitionistic fuzzy G_δ-e-locally interior of A is denoted and defined by $IFG_\delta\text{-}e\text{-}lint(A) = \bigcup\{B : B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$ is an intuitionistic fuzzy G_δ-e-locally open set in X and $B \subseteq A$.

Proposition 2.2. [7] Let (X, T) be an intuitionistic fuzzy topological space. For any two intuitionistic fuzzy sets $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle : x \in X\}$ of an intuitionistic fuzzy topological space (X, T) then the following statements are true.

1. $IFG_\delta\text{-}e\text{-}lcl(0_{\sim}) = 0_{\sim}$
2. $A \subseteq B \Rightarrow IFG_\delta\text{-}e\text{-}lcl(A) \subseteq IFG_\delta\text{-}e\text{-}lcl(B)$

3. $IFG_{\delta}\text{-}e\text{-}lcl(IFG_{\delta}\text{-}e\text{-}lcl(A)) = IFG_{\delta}\text{-}e\text{-}lcl(A)$
4. $IFG_{\delta}\text{-}e\text{-}lcl(A \cup B) = (IFG_{\delta}\text{-}e\text{-}lcl(A)) \cup (IFG_{\delta}\text{-}e\text{-}lcl(B))$

Remark 2.1. [7]
1. $IFG_{\delta}\text{-}e\text{-}lcl(A) = A$ if and only if A is an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally closed set. 2. $IFG_{\delta}\text{-}e\text{-}lint(A) \subseteq A \subseteq IFG_{\delta}\text{-}e\text{-}lcl(A)$ 3. $IFG_{\delta}\text{-}e\text{-}lint(1_{\sim}) = 1_{\sim}$
4. $IFG_{\delta}\text{-}e\text{-}lint(0_{\sim}) = 0_{\sim}$ 5. $IFG_{\delta}\text{-}e\text{-}lcl(1_{\sim}) = 1_{\sim}$

3. Intuitionistic fuzzy $G_{\delta}\text{-}e$-locally continuous functions

Definition 3.1. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Then A is said to be an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ if there exists an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally open set B in an intuitionistic fuzzy topological space (X, T) such that $x_{r,s} \in B, B \subseteq A$. It is denoted by $IFG_{\delta}\text{-}e$-lnbd.

Definition 3.2. Let (X, T) be an intuitionistic fuzzy topological space. Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Then A is said to be an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally quasi neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ if there exists an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally open set B in an intuitionistic fuzzy topological space (X, T) such that $x_{r,s} q B, B \subseteq A$. It is denoted by $IFG_{\delta}\text{-}e$-lqnbd.

Remark 3.1.
1. The family of all intuitionistic fuzzy $G_{\delta}\text{-}e$-locally neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ is denoted by $N^{IFG_{\delta}-e-l}(x_{r,s})$.
2. The family of all intuitionistic fuzzy $G_{\delta}\text{-}e$-locally quasi neighbourhood of an intuitionistic fuzzy point $x_{r,s}$ is denoted by $N^{IFG_{\delta}-e-lq}(x_{r,s})$.

Definition 3.3. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then f is said to be an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally continuous function, if for each intuitionistic fuzzy point $x_{r,s}$ in X and $B \in Nf(x_{r,s})$, there exists $A \in N^{IFG_{\delta}-e-lq}(x_{r,s})$ such that $f(A) \subseteq B$.

Theorem 3.1. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy mapping. Then the following are equivalent.

1. f is an intuitionistic fuzzy G_δ-e-locally continuous function.
2. $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-e-locally open set in an intuitionistic fuzzy topological space (X,T), for each intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space (Y,S).
3. $f^{-1}(B)$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X,T), for each intuitionistic fuzzy closed set B in an intuitionistic fuzzy topological space (Y,S).
4. $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))\subseteq f^{-1}(IFcl(A))$, for each intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S).
5. $f^{-1}(IFint(A))\subseteq IFG_\delta\text{-}e\text{-}lint(f^{-1}(A))$, for each intuitionistic fuzzy set A in an intuitionistic fuzzy topological space (Y,S).

Proof: (i)$\Rightarrow$ (ii): Let A be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y,S). Let $x_{r,s}$ be an intuitionistic fuzzy point in an intuitionistic fuzzy topological space (X,T) such that $x_{r,s}qf^{-1}(A)$. Since f is an intuitionistic fuzzy G_δ-e-locally continuous function, there exists $B\in N^{IFG_\delta-e-lq}(x_{r,s})$ such that $f(B)\subseteq A$. Then

$$x_{r,s}\in B \tag{1}$$

$$B\subseteq f^{-1}(f(B)) \tag{2}$$

Thus, $x_{r,s}\in B\subseteq f^{-1}(f(B))\subseteq f^{-1}(A)$. This implies $x_{r,s}\in f^{-1}(A)$ which shows that $f^{-1}(A)\in N^{IFG_\delta-e-lq}(x_{r,s})$. Hence $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-e-locally open set in an intuitionistic fuzzy topological space (X,T).

(ii)$\Rightarrow$ (i): This can be proved by taking complement of (i)

(iii)$\Rightarrow$ (iv): Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y,S). Since $A\subseteq IFcl(A), f^{-1}(A)\subseteq f^{-1}(IFcl(A))$. By (iii), $f^{-1}(IFcl(A))$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X,T). Thus, $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))\subseteq f^{-1}(IFcl(A))$.

(iv)$\Rightarrow$(v): Using (iv), $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))\subseteq f^{-1}(IFcl(A))$. Then $\overline{IFG_\delta-e-lcl(f^{-1}(A))}\supseteq \overline{f^{-1}(IFcl(A))}, IFG_\delta-lint(\overline{f^{-1}(A)})\supseteq f^{-1}(IFint(\overline{A})), IFG_\delta-e-lint(f^{-1}(\overline{A}))\supseteq f^{-1}(IFint(\overline{A}))$ implies that $f^{-1}(IFint(\overline{A}))\subseteq IFG_\delta-e-lint(\overline{f^{-1}(A)})$, putting $\overline{A}=A$, we have $f^{-1}(IFint(\overline{A}))\subseteq IFG_\delta-e-lint(f^{-1}(A))$.

(v)$\Rightarrow$ (i): Let A be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y,S). Then $IFintA = A$. Using (v), $f^{-1}(IFint(A)) \subseteq IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(A))$ implies that $f^{-1}(A) \subseteq IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(A))$. But, $IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(A)) \subseteq f^{-1}(A)$ implies that $f^{-1}(A) = IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(A))$ that is, $f^{-1}(A)$ is an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally open set in an intuitionistic fuzzy topological space (X,T). Let $x_{r,s}$ be any intuitionistic fuzzy point in $f^{-1}(A)$. Then $x_{r,s} \in f^{-1}(A)$. We have $x_{r,s} q f^{-1}(A)$ implies that $f(x_{r,s}) q f(f^{-1}(A))$. But $f(f^{-1}(A)) \subseteq A$. Thus, for any intuitionistic fuzzy point $x_{r,s}$ and $A \in Nf(x_{r,s})$, there exists $B = f^{-1}(A) \in N^{IFG_{\delta}-e-lq}(x_{r,s})$ such that $f^{-1}(f(A)) \subseteq A$. Therefore, $f(B) \subseteq A$. Thus, f is an intuitionisic fuzzy $G_{\delta}\text{-}e$-locally continuous function.

Theorem 3.2. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T) \to (Y,S)$ be an intuitionistic fuzzy bijective function. Then f is an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally continuous function if and only if $IFint(f(A)) \subseteq f(IFG_{\delta}\text{-}e\text{-}lint(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T).

Proof: Assume that f is an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally continuous function and A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Hence, $f^{-1}(IFint(f(A)))$ is an intuitionistic fuzzy $G_{\delta}\text{-}e$-locally open set in an intuitionistic fuzzy topological space (X,T). From Theorem (v) of (0.1) $f^{-1}(IFintf(A)) \subseteq IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(f(A)))$, $f^{-1}(IFintf(A)) \subseteq IFG_{\delta}\text{-}e\text{-}lint(A)$. Since f is an intuitionistic fuzzy surjective function, $f(f^{-1}(IFintf(A))) \subseteq f(IFG_{\delta}\text{-}e\text{-}lint(A))$. That is, $IFint(f(A)) \subseteq f(IFG_{\delta}\text{-}e\text{-}lint(A))$.

Conversely, assume that $IFint(f(A)) \subseteq f(IFG_{\delta}\text{-}e\text{-}lint(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T). Let B be an intuitionistic fuzzy open set in an intuitionistic fuzzy topological space (Y,S). Then $B = IFint(B)$. Since f is an intuitionistic fuzzy surjective function, $B = IFint(B) = IFint(f(f^{-1}(B))) \subseteq f(IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(B)))$.

Since f is an intuitionistic fuzzy injective function, $f^{-1}(B) \subseteq f^{-1}(f(IFG_{\delta}\text{-}e\text{-}lint(f^{-1}(B))))$. From the fact that f, is an intuitionistic fuzzy injective function, we have

$$f^{-1}(B) \subseteq IFG_{\delta} - e - lint(f^{-1}(B)) \tag{3}$$

but

$$IFG_\delta - e - lint(f^{-1}(B)) \subseteq f^{-1}(B) \qquad (4)$$

From (3) and (4) implies that $f^{-1}(B) = IFG_\delta\text{-}e\text{-}lint(f^{-1}(B))$. That is, $f^{-1}(B)$ is an intuitionistic fuzzy G_δ-e-locally open set in an intuitionistic fuzzy topological space (X,T). Thus, f is an intuitionistic fuzzy G_δ-e-locally continuous function.

Theorem 3.3. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy bijective fuction. Then f is an intuitionistic fuzzy G_δ-e-locally continuous function if and only if $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq IFcl(f(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T).

Proof: Assume that f is an intuitionistic fuzzy G_δ-e-locally continuous function and A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X,T). Hence, $f^{-1}(IFcl(f(A)))$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X,T). From Theorem (iv) of (0.1) $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(f(A))) \subseteq f^{-1}(IFclf(A))$. Since f is an intuitionistic fuzzy injective function, $IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq f^{-1}(IFclf(A))$. Taking f on both sides, $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq f(f^{-1}(IFclf(A)))$. Since f is an intuitionistic fuzzy surjective function, $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq IFcl(f(A))$.

Conversely, assume that $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq IFcl(f(A))$, for each intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X,T). Let B be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y,S). Then $B = IFcl(B)$. Since f is an intuitionistic fuzzy surjective function, and by assumption $B = IFcl(B) = IFcl(f(f^{-1}(B))) \supseteq f(IFG_\delta\text{-}e\text{-}lcl(f^{-1}(B)))$, $f^{-1}(B) \supseteq f^{-1}(f(IFG_\delta\text{-}e\text{-}lcl(f^{-1}(B))))$. Since f is an intuitionistic fuzzy injective function,

$$f^{-1}(B) \supseteq IFG_\delta - e - lcl(f^{-1}(B)) \qquad (5)$$

But

$$(f^{-1}(B)) \subseteq IFG_\delta - e - cl(f^{-1}(B)) \qquad (6)$$

From (5) and (6) implies that $f^{-1}(B) = IFG_\delta\text{-}e\text{-}lcl(f^{-1}(B))$. That is, $f^{-1}(B)$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X,T). Thus, f is an intuitionistic fuzzy G_δ-e-locally continuous function.

Theorem 3.4. Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy bijective fuction. If f is an intuitionistic fuzzy G_δ-e-locally continuous function. Then if $A \in I^Y$ is an intuitionistic fuzzy closed set, then $f^{-1}(A) = IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))$.

Proof: Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Y, S). By Theorem(iv)of (0.1).

$$IFG_\delta - e - lcl(f^{-1}(A)) \subseteq f^{-1}(IFcl(A)) = f^{-1}(A) \tag{7}$$

Since $A = IFcl(A)$. But

$$f^{-1}(A) \subseteq IFG_\delta - e - lcl(f^{-1}(A)) \tag{8}$$

From (7) and (8) implies that $f^{-1}(A) = IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))$.

Proposition 3.1. Let $(X, T), (Y, S)$ and (Z, R) be any three intuitionistic fuzzy topological spaces. Let $f : (X.T) \to (Y, S)$ be an intuitionistic fuzzy G_δ-e-locally continuouus function. If $f(X) \subset Z \subset Y$ then $g : (X, T) \to (Z, R)$ where $R = S/Z$ restricting the range of f is an intuitionistic fuzzy G_δ-e-locally continuous function.

Proof: Let B be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (Z, R). Then $B = S/Z$, for some intuitionistic fuzzy closed set A of an intuitionistic fuzzy topological spaces (Y, S). If $f(X) \subset Z \subset Y$, $f^{-1}(A) = g^{-1}(B)$. Since $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $g^{-1}(B)$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X, T). Therefore, g is an intuitionistic fuzzy G_δ-e-locally continuous function.

Proposition 3.2. Let $(X, T), (X_1, T_1)$ and (X_2, T_2) be any three intuitionistic fuzzy topological spaces and $P_i : X_1 \times X_2 \to X_i$ be an intuitionistic fuzzy projection of $X_1 \times X_2$ onto X_i. If $f : X \to X_1 \times X_2$ is an intuitionistic fuzzy G_δ-e-locally continuous function. Then $P_i \circ f : X \to X_i$ is also an intuitionistic fuzzy G_δ-e-locally cotinuous function.

Proof: Let A be an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological spaces $(X_i, T_i)\,(i = 1, 2), (P_i \circ f)^{-1}(A) = f^{-1}(P_i^{-1}(A))$. Since P_i is an intuitionistic fuzzy mapping $P_i^{-1}(A)$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological spaces $X_1 \times X_2$. Hence, $f^{-1}(P_i^{-1}(A))$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X, T). Hence, $P_i \circ f$ is an intuitionistic fuzzy G_δ-e-locally continuous function.

Proposition 3.3. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If intuitionistic fuzzy graph function $g : X \to X \times Y$ is an intuitionistic fuzzy

G_δ-e-locally continuous function. Then $f:(X,T)\to(Y,S)$ is an intuitionistic fuzzy G_δ-e-locally continuous function.

Proof: Let g be an intuitionistic fuzzy G_δ-e-locally continuous function and $x_{r,s}$ be any intuitionistic fuzzy point in an intuitionistic fuzzy topological space (X,T). If $B\in N^{IFG_\delta-e-lq} f(x_{r,s})$ in an intuitionistic fuzzy topological space

$$(Y,S),\ X\times B\in N^{IFG_\delta-e-lq} g(x_{r,s})$$

in an intuitionistic fuzzy topological space $X\times Y$. Since g is an intuitionistic fuzzy G_δ-e-locally continuous function, there exists $A\in N(x_{r,s})$ such that $g(A)\subseteq X\times B$. By Definition 0.24, we have $f(A)\subseteq B$. Therefore, f is an intuitionistic fuzzy G_δ-e-locally continuous function.

Definition 3.4. Let (X,T) and (Y,S) be two intuitionistic fuzzy topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic fuzzy mapping. Then f is said to be an

1. Intuitionistic fuzzy G_δ-e-locally irresolute function, if for each intuitionistic fuzzy G_δ-e-locally closed set A in an intuitionistic fuzzy topological space (Y,S), $f^{-1}(A)$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X,T).
2. Intuitionistic fuzzy weakly G_δ-e-locally continuous function, if for each intuitionistic fuzzy G_δ-e-locally closed set A in an intuitionistic fuzzy topological space (Y,S), $f^{-1}(A)$ is an intuitionistic fuzzy closed set in an intuitionistic fuzzy topological space (X,T).

Example 3.1. Let $X=\{a,b,c\}=Y$, and

$A=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$,

$B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$, $A\vee B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.3}),(\frac{a}{0.5},\frac{b}{0.4})\right\rangle$,

$A\wedge B=\left\langle x,(\frac{a}{0.3},\frac{b}{0.1}),(\frac{a}{0.6},\frac{b}{0.6})\right\rangle$, $C=\left\langle x,(\frac{a}{0.7},\frac{b}{0.7}),(\frac{a}{0},\frac{b}{0.1})\right\rangle$

Now, the family $T=\{0_{\sim},1_{\sim},A,B,A\vee B,A\wedge B\}$ of IFS's in X is an IFT on X and the family $S=\{0_{\sim},1_{\sim},C\}$ of IFS's in Y is an IFT on Y. If we define the function $f:X\to Y$ be the identity function. Now, f is an intuitionistic fuzzy G_δ-e-locally irresolute function, because C is an IFG_δ-e-locally closed set in Y, $f^{-1}(C)$ is also an IFG_δ-e-locally closed set in X.

Example 3.2. Let $X = \{a, b, c\} = Y$, and

$A = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$, $B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$,

$A \vee B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.3}), (\frac{a}{0.5}, \frac{b}{0.4}) \right\rangle$, $A \wedge B = \left\langle x, (\frac{a}{0.3}, \frac{b}{0.1}), (\frac{a}{0.6}, \frac{b}{0.6}) \right\rangle$,

$C = \left\langle x, (\frac{a}{0}, \frac{b}{0.1}), (\frac{a}{0.7}, \frac{b}{0.7}) \right\rangle$ $D = \left\langle x, (\frac{a}{0.7}, \frac{b}{0.7}), (\frac{a}{0}, \frac{b}{0.1}) \right\rangle$

Now, the family $T = \{0_{\sim}, 1_{\sim}, A, B, A \vee B, A \wedge B, C\}$ of IFS's in X is an IFT on X and the family $S = \{0_{\sim}, 1_{\sim}, D\}$ of IFS's in Y is an IFT on Y. If we define the function $f : X \to Y$ be the identity function, Now, f is an intuitionistic fuzzy weakly G_δ-e-locally irresolute function, because D is an IFG_δ-e-locally closed set in Y, $f^{-1}(D)$ is intuitionistic fuzzy closed set in X.

Theorem 3.5. Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic fuzzy mapping. Then the following statements are equivalent

1. f is an intuitionistic fuzzy G_δ-e-locally irresolute function.
2. for every intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, T), $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq IFG_\delta\text{-}e\text{-}lcl(f(A))$.
3. for every intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (Y, S), $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta\text{-}e\text{-}lcl(A))$.

Proof: (i)$\Rightarrow$ (ii): Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). Suppose f is an intuitionistic fuzzy G_δ-e-locally irresolute function. Now, $IFG_\delta\text{-}e\text{-}lcl(f(A))$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (Y, S). By hypothesis, $f^{-1}(IFG_\delta\text{-}e\text{-}lcl(f(A)))$ is an intuitionistic fuzzy G_δ-e-locally closed set in an intuitionistic fuzzy topological space (X, T) and hence, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(IFG_\delta\text{-}e\text{-}(lcl(f(A))))$. Now, $IFG_\delta\text{-}e\text{-}lcl(A) \subseteq f^{-1}(IFG_\delta\text{-}e\text{-}lcl(f(A)))$.
That is, $f(IFG_\delta\text{-}e\text{-}lcl(A)) \subseteq IFG_\delta\text{-}e\text{-}lcl(f(A))$.

(ii)$\Rightarrow$ (iii): Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (Y, S), then $f^{-1}(A)$ is an intuitionistic fuzzy set in an intuitionistic fuzzy topological space (X, T). By (ii), $f(IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A))) \subseteq IFG_\delta\text{-}e\text{-}lcl(f(f^{-1}(A)))$. Since f is an intuitionistic fuzzy bijective function, $IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta\text{-}e\text{-}lcl(A))$

(iii) $\Rightarrow$ (i): Suppose A is intuitionistic fuzzy IFG_δ-e-locally closed set in an intuitionistic fuzzy topological space (Y, S). Then $IFG_\delta\text{-}e\text{-}lcl(A) = A$. By hypothesis,

$IFG_\delta\text{-}e\text{-}lcl(f^{-1}(A)) \subseteq f^{-1}(IFG_\delta\text{-}e\text{-}lcl(A)), IFG_\delta\text{-}e\ lcl(f^{-1}(A)) \subseteq f^{-1}(A)$.

4. Conclusion

Intuitionistic fuzzy topology is an important and a major area of mathematics. In this paper, we introduce intuitionistic fuzzy G_δ-e-locally neighborhood, intuitionistic fuzzy G_δ-e-locally quasi neighborhood, intuitionistic fuzzy G_δ-e-locally continuous and intuitionistic fuzzy G_δ-e-locally irresolute functions in intuitionistic fuzzy topological spaces are studied.

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