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Fuzzy Neutrosophic Soft Block Matrices and its Some Properties

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Abstract. In this article different types of fuzzy neutrosophic soft block matrices (FNSBMs) are introduced. The operations direct sum, Kronecker sum, Kronecker product of FNSMs are defined and shown that their resultant matrices are FNSBMs. Finally, some relational operations are presented and some properties of FNSBMs are discussed.

Keywords: Fuzzy neutrosophic soft block matrix, fuzzy neutrosophic soft submatrix, Kronecker Product, Kronecker sum.

AMS Mathematics Subject Classification (2010): 03E72, 15B15

1. Introduction

The development of computers and the related attempt to automate human reasoning and inference have posed a challenge to researchers. Humans, and in many cases machines, are not always operating under strict and well defined two-valued logic or discrete multi-valued logic. Their perception of sets and classes is not as crisp as implied by the traditional set and class theory. To capture this perception, Zadeh has introduced the theory of fuzzy sets and fuzzy logic [8, 9, 19,20,21,22,23]. The seminal paper [20], published by Zadeh in 1965, ignited tremendous interest among a large number of researchers. The traditional fuzzy sets is characterised by the membership value or the grade of membership value. Sometimes it may be very difficult to assign the membership value for a fuzzy sets. Consequently the concept of interval valued fuzzy set was proposed [15] to capture the uncertainty of grade of membership value. In some real life problems in expert systems, belief system, information fusion and so on, we must consider the truth-membership as well as the falsity-membership for proper description of an object in uncertain, ambiguous environment. Neither the fuzzy sets nor the interval valued fuzzy sets is appropriate for such a situation.

Intuitionistic fuzzy sets introduced by Atanassov [6] is appropriate for such a situation. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief -system. Soft set theory has enriched its potentiality since its introduction by Molodtsov [12]. Based on the several operations on soft sets introduced in [10,11] some more properties and algebra may be found in [1]. Smarandache [14] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data.

Maji et al.,[11] extended soft sets to intuitionistic fuzzy soft sets and neutrosophic soft sets. The general rectangular or square array of the numbers are known as matrix and if the elements are neutrosophic number then the matrix is called fuzzy neutrosophic soft matrix. If we delete some rows or some columns or both or neither then the fuzzy neutrosophic soft matrix is called fuzzy neutrosophic soft submatrix. The fuzzy neutrosophic soft matrix is divided or partitioned into smaller FNSMs called cells or blocks with consecutive rows and columns by drawing dotted horizontal lines of full width between rows and vertical lines of full height between columns, then the FNSMs is called fuzzy neutrosophic soft block matrix.

Arokiarani and Sumathi [4] introduced fuzzy neutrosophic soft matrix approach in decision making. Kumar et al.,[5] introduced fuzzy block matrix and its some properties. Uma et al., [17,18] introduced determinant theory for fuzzy neutrosophic soft matrices and generalized inverse of fuzzy neutrosophic soft matrix. The structure of this paper is organized as follows . In section 2, some basic definitions are defined. In section 3, different kinds of fuzzy neutrosophic soft submatrix and FNSBM are given. Section 4 discuss directsum, Kronecker sum and Kronecker product of FNSBM. In section 5, some relational operations on FNSBM are studied.

2. Preliminaries

Definition 2.1. [14] A neutrosophic set A on the universe of discourse X is defined as
$A=\left\{\langle x,T_A(x),I_A(x),F_A(x)\rangle, x\in X\right\}$,

where $T,I,F : X \rightarrow\ ]^{-}0,1^{+}[$ and $^{-}0 \le T_A(x)+I_A(x)+F_A(x) \le 3^{+}$ (1)

From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. But in real life application especially in scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-}0,1^{+}[$. Hence we consider the neutrosophic set which takes the value from the subset of [0,1] . Therefore we can rewrite the equation (1) as $0 \le T_A(x)+I_A(x)+F_A(x) \le 3.$

In short an element $\tilde{a}$ in the neutrosophic set A, can be written as $\tilde{a}=\langle a^T, a^I, a^F\rangle$, where a^T denotes degree of truth, a^I denotes degree of indeterminacy, a^F denotes degree of falsity such that $0 \le a^T + a^I + a^F \le 3.$

Definition 2.2. [12] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

Definition 2.3. [3] Let U be an initial universe set and E be a set of parameters. Consider a non empty set A, A $\subseteq$ E. Let P(U) denotes the set of all fuzzy neutrosophic sets of U. The collection (F,A) is termed to be the Fuzzy Neutrosophic Soft Set (FNSS) over U, Where F is a mapping given by $F : A \to P(U)$. Hereafter we simply consider A as FNSS over U instead of (F,A).

Definition 2.4. [4] Let $U = \{c_1, c_2, ...c_m\}$ be the universal set and E be the set of parameters given by $E = \{e_1, e_2, ...e_m\}$. Let $A \subset E$. A pair (F, A) be a FNSS over U. Then the subset of $U \times E$ is defined by $R_A = \{(u,e); e \in A, u \in F_A(e)\}$ which is called a relation form of (F_A, E). The membership function, indeterminacy membership function and non membership function are written by

$$T_{R_A} : U \times E \to [0,1], I_{R_A} : U \times E \to [0,1] \text{ and } F_{R_A} : U \times E \to [0,1]$$

where $T_{R_A}(u,e) \in [0,1], I_{R_A}(u,e) \in [0,1]$ and $F_{R_A}(u,e) \in [0,1]$ are the membership value, indeterminacy value and non membership value respectively of $u \in U$ for each $e \in E$. If $[(T_{ij}, I_{ij}, F_{ij})] = [T_{ij}(u_i, e_j), I_{ij}(u_i, e_j), F_{ij}(u_i, e_j)]$ we define a matrix

$$[\langle T_{ij}, I_{ij}, F_{ij}\rangle]_{m\times n} = \begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{bmatrix}.$$

This is called an $m \times n$ $FNSM$ of the FNSS (F_A, E) over U.

Definition 2.5. [16] Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle), B = (\langle b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle) \in \mathcal{F}_{m\times n}$ the component wise addition and component wise multiplication is defined as

$A \oplus B =$ $(sup\{a_{ij}^T, b_{ij}^T\},\ sup\{a_{ij}^I, b_{ij}^I\},\ inf\ \{a_{ij}^F, b_{ij}^F\})$.

$A \odot B = (inf\{a_{ij}^T, b_{ij}^T\},\ inf\{a_{ij}^I, b_{ij}^I\},\ sup\{a_{ij}^F, b_{ij}^F\})$.

Definition 2.6 . [16] Let $A \in \mathcal{F}_{m\times n}, B \in \mathcal{F}_{n\times p}$, the composition of A and B is defined as

$A \circ B = \left(\sum_{k=1}^{n} (a_{ik}^T \wedge b_{kj}^T),\ \sum_{k=1}^{n} (a_{ik}^I \wedge b_{kj}^I),\ \prod_{k=1}^{n} (a_{ik}^F \vee b_{kj}^F) \right)$ equivalently we can write the same as $= \left(\vee_{k=1}^{n} (a_{ik}^T \wedge b_{kj}^T),\ \vee_{k=1}^{n} (a_{ik}^I \wedge b_{kj}^I),\ \wedge_{k=1}^{n} (a_{ik}^F \vee b_{kj}^F) \right)$.

The product $A \circ B$ is defined if and only if the number of columns of A is same as the number of rows of B. A and B are said to be conformable for multiplication. We shall use AB instead of $A \circ B$.

3. Main results

In this section we define different types of fuzzy neutrosophic soft sub matrix and FNSBMs. Futher some properties are discussed.

Definition 3.1. A fuzzy neutrosophic soft submatrix of an FNSM of order ≥ 1 is obtained by deleting some rows or some columns or both (not necessarily consecutive) or neither. The FNSM itself is its fuzzy neutrosophic soft submatrix(FNSSM).The maximum number of fuzzy neutrosophic soft submatrix of an $n \times m$ fuzzy neutrosophic soft matrix is $(2^n - 1)(2^m - 1)$.

Definition 3.2. The fuzzy neutrosophic soft submatrix of order (n-r) obtained by deleting r rows and columns of an n square FNSM is called fuzzy neutrosophic soft principal submatrix. The first order principal fuzzy neutrosophic soft submatrices obtained from the following third order FNSM

$$\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F \rangle & \langle a_{12}^T, a_{12}^I, a_{12}^F \rangle & \langle a_{13}^T, a_{13}^I, a_{13}^F \rangle \\ \langle a_{21}^T, a_{21}^I, a_{21}^F \rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F \rangle & \langle a_{23}^T, a_{23}^I, a_{23}^F \rangle \\ \langle a_{31}^T, a_{31}^I, a_{31}^F \rangle & \langle a_{32}^T, a_{32}^I, a_{32}^F \rangle & \langle a_{33}^T, a_{33}^I, a_{33}^F \rangle \end{bmatrix}$$ are $[\langle a_{11}^T, a_{11}^I, a_{11}^F \rangle]$, $[\langle a_{12}^T, a_{12}^I, a_{12}^F \rangle]$

and $[\langle a_{13}^T, a_{13}^I, a_{13}^F \rangle]$. Second order fuzzy neutrosophic soft submatrices are

$$\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F \rangle & \langle a_{12}^T, a_{12}^I, a_{12}^F \rangle \\ \langle a_{21}^T, a_{21}^I, a_{21}^F \rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F \rangle \end{bmatrix}, \begin{bmatrix} \langle a_{22}^T, a_{22}^I, a_{22}^F \rangle & \langle a_{23}^T, a_{23}^I, a_{23}^F \rangle \\ \langle a_{32}^T, a_{32}^I, a_{32}^F \rangle & \langle a_{33}^T, a_{33}^I, a_{33}^F \rangle \end{bmatrix},$$

$$\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F \rangle & \langle a_{13}^T, a_{13}^I, a_{13}^F \rangle \\ \langle a_{31}^T, a_{31}^I, a_{31}^F \rangle & \langle a_{33}^T, a_{33}^I, a_{33}^F \rangle \end{bmatrix}.$$

Third order fuzzy neutrosophic soft submatrix is the matrix itself.

Definition 3.3. The fuzzy neutrosophic soft submatrix of order(n-r) obtained by deleting last r rows and columns of an n Fuzzy Neutrosophic Soft Square Matrix (FNSSM) A is called leading principal fuzzy neutrosophic soft submatrix. The first order leading principal fuzzy neutrosophic soft submatrix is $[\langle a_{11}^T, a_{11}^I, a_{11}^F \rangle]$.

The second order leading principal fuzzy neutrosophic soft submatrix of the above FNSM is

$$\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F \rangle & \langle a_{12}^T, a_{12}^I, a_{12}^F \rangle \\ \langle a_{21}^T, a_{21}^I, a_{21}^F \rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F \rangle \end{bmatrix}.$$

Definition 3.4. If an FNSM is divided or partitioned into smaller FNSMs called blocks or cells with consecutive rows and columns separated by dotted horizontal lines of full width between rows and vertical lines of full height between columns, then the FNSM is called fuzzy neutrosophic soft partition matrix.

The elements of fuzzy neutrosophic soft partition matrix are smaller FNSMs.

It is also called the fuzzy neutrosophic soft block matrix.
Advantages of fuzzy neutrosophic soft partitioning.

The following advantages may be noted in partitioning an FNSM A into blocks or cells:

1. The partitioning may simplify the writing or printing in compact form thus saves space.
2. It exhibits some smaller structure of A which is of great interest.

Definition 3.5. Two FNSMs of same order are said to be conformally or identically partitioned if

1. Both the FNSMs are partitioned in such way that the number of columns of two fuzzy neutrosophic soft partition matrices are same.
2. The corresponding blocks are of same order.

Definition. 3.6. The FNSM whose elements are blocks obtained by partitioning is called fuzzy neutrosophic soft block matrix (FNSBM)
Thus
A=

$$\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F\rangle & \langle a_{12}^T, a_{12}^I, a_{12}^F\rangle & \ldots & \langle a_{13}^T, a_{13}^I, a_{13}^F\rangle & \langle a_{14}^T, a_{14}^I, a_{14}^F\rangle \\ \langle a_{21}^T, a_{21}^I, a_{21}^F\rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F\rangle & \ldots & \langle a_{23}^T, a_{23}^I, a_{23}^F\rangle & \langle a_{24}^T, a_{24}^I, a_{24}^F\rangle \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \langle a_{31}^T, a_{31}^I, a_{31}^F\rangle & \langle a_{32}^T, a_{32}^I, a_{32}^F\rangle & \ldots & \langle a_{33}^T, a_{33}^I, a_{33}^F\rangle & \langle a_{34}^T, a_{34}^I, a_{34}^F\rangle \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

where

$$B_{11} = [\langle a_{11}^T, a_{11}^I, a_{11}^F\rangle\langle a_{12}^T, a_{12}^I, a_{12}^F\rangle], B_{12} = \langle a_{13}^T, a_{13}^I, a_{13}^F\rangle\langle a_{14}^T, a_{14}^I, a_{14}^F\rangle,$$

$$B_{21} = \begin{bmatrix} \langle a_{21}^T, a_{21}^I, a_{21}^F\rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F\rangle \\ \langle a_{31}^T, a_{31}^I, a_{31}^F\rangle & \langle a_{32}^T, a_{32}^I, a_{32}^F\rangle \end{bmatrix} \text{and } B_{22} = \begin{bmatrix} \langle a_{23}^T, a_{23}^I, a_{23}^F\rangle & \langle a_{24}^T, a_{24}^I, a_{24}^F\rangle \\ \langle a_{33}^T, a_{33}^I, a_{33}^F\rangle & \langle a_{34}^T, a_{34}^I, a_{34}^F\rangle \end{bmatrix}$$

The FNSM $B = \begin{bmatrix} B_{11} & \vdots & B_{12} \\ \cdots & \cdots & \cdots \\ B_{21} & \vdots & B_{22} \end{bmatrix}$ is an example of fuzzy neutrosophic soft block matrix.

Definition 3.7. The transpose of FNSBM is the transpose of both blocks and constituents blocks.

$$A^T = \begin{bmatrix} B_{11}^T & \vdots & B_{21}^T \\ \cdots & \cdots & \cdots \\ B_{12}^T & \vdots & B_{22}^T \end{bmatrix}$$

Definition 3.8. The blocks along the diagonal of the FNSBM are called diagonal blocks. The blocks B_{ij} for which i=j are diagonal blocks. Thus B_{11} and B_{22} are diagonal blocks of the FNSBM $A=\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$.

Definition 3.9. If the number of rows and the number of columns of blocks are equal then the matrix is called Fuzzy Neutrosophic Soft Square Block Matrix (FNSSBM).
thus the partitioned FNSM

$$A=\begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F\rangle & \langle a_{12}^T, a_{12}^I, a_{12}^F\rangle & \dots & \langle a_{13}^T, a_{13}^I, a_{13}^F\rangle & \langle a_{14}^T, a_{14}^I, a_{14}^F\rangle \\ \langle a_{21}^T, a_{21}^I, a_{21}^F\rangle & \langle a_{22}^T, a_{22}^I, a_{22}^F\rangle & \dots & \langle a_{23}^T, a_{23}^I, a_{23}^F\rangle & \langle a_{24}^T, a_{24}^I, a_{24}^F\rangle \\ \dots & \dots & \dots & \dots & \dots \\ \langle a_{31}^T, a_{31}^I, a_{31}^F\rangle & \langle a_{32}^T, a_{32}^I, a_{32}^F\rangle & \dots & \langle a_{33}^T, a_{33}^I, a_{33}^F\rangle & \langle a_{34}^T, a_{34}^I, a_{34}^F\rangle \\ \langle a_{41}^T, a_{41}^I, a_{41}^F\rangle & \langle a_{42}^T, a_{42}^I, a_{42}^F\rangle & \dots & \langle a_{43}^T, a_{43}^I, a_{43}^F\rangle & \langle a_{44}^T, a_{44}^I, a_{44}^F\rangle \end{bmatrix}$$

$$=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

is a fuzzy neutrosophic soft square block matrix (FNSSBM), since all A_{ij} are square blocks.

Definition 3.10. If a FNSSBM is such that the blocks $A_{ij}=[\langle 0,0,1\rangle]$ for all $i\neq j$, the FNSBM A is said to be a Diagonal Fuzzy Neutrosophic Soft Block Matrix (DFNSBM). Here FNSBM A needs not be square but it must be partitioned as a FNSSBM.

Thus $\begin{bmatrix} A_{11} & O_1 & O_1 \\ O_1 & A_{22} & O_1 \end{bmatrix}$, where $O_1=[\langle 0,0,1\rangle]$ is a DFNSBM.

Definition 3.11. If the square or rectangular blocks above(or below) the square diagonal blocks of a FNSSBM are all zero, then the FNSM is said to be the lower (or upper) triangular FNSBM.

Definition 3.12. It is a FNSBM whose diagonal blocks are FNSSBMs of different orders and off diagonal blocks are zero FNSMs.
Thus,

$$A=\begin{bmatrix} \langle D_{11}^T, D_{11}^I, D_{11}^F\rangle & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle \\ \cdots & \langle D_{22}^T, D_{22}^I, D_{22}^F\rangle & \cdots & \vdots \\ \cdots & \cdots & \cdots & \vdots \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle D_{nn}^T, D_{nn}^I, D_{nn}^F\rangle \end{bmatrix}$$

is a quasi diagonal matrix whose diagonal blocks $\langle D_{ii}^T, D_{ii}^I, D_{ii}^F\rangle$, i=1,2,...,n are FNSSMs of different orders.

Theorem 3.13. If $A=[a_{ij}^T, a_{ij}^I, a_{ij}^F]_{m\times n}$ and $B=[b_{ij}^T, b_{ij}^I, b_{ij}^F]_{n\times p}$ are two FNSBM such that $AB=C=[c_{ij}^T, c_{ij}^I, c_{ij}^F]_{m\times p}$ then the j-th column of C is AB_j, where

$$B_j=\begin{bmatrix}\langle b_{1j}^T, b_{1j}^I, b_{1j}^F\rangle\\ \langle b_{2j}^T, b_{2j}^I, b_{2j}^F\rangle\\ \vdots\\ \langle b_{nj}^T, b_{nj}^I, b_{nj}^F\rangle\end{bmatrix}$$ are the column partition of FNSM B.

Proof: Let FNSM B of order $n\times p$ be partition into p column vectors $(n\times 1)$ FNSMs as

B= $\begin{bmatrix}B_1 & B_2 & B_3 & \cdots & B_j & \cdots & B_p\end{bmatrix}$

where j=1,2,...,p to find a column of the product AB.

From the product rule of the FNSMs, the elements of the product is

$\langle c_{ij}^T, c_{ij}^I, c_{ij}^F\rangle=\langle\sum_k(a_{ik}^T\wedge b_{kj}^T),\sum_k(a_{ik}^I\wedge b_{kj}^I),\prod_k(a_{ik}^F\vee b_{kj}^F)\rangle, i=1,2,...,m$ and j=1,2,...,n,

where $A=[\langle a_{ik}^T, a_{ik}^I, a_{ik}^F\rangle]_{m\times n}, B=[\langle b_{kj}^T, b_{kj}^I, b_{kj}^F\rangle]_{n\times p}, C=[\langle c_{ij}^T, c_{ij}^I, c_{ij}^F\rangle]_{m\times n}$ and C=AB.

Therefore j-th column of C is obtained by giving the values 1,2,...,m to i and it is

$$C_j=\begin{bmatrix}\sum_k a_{1k}^T\wedge b_{kj}^T & \sum_k a_{1k}^F\wedge b_{kj}^F & \prod_k a_{1k}^F\vee b_{kj}^F\\ \sum_k a_{2k}^T\wedge b_{kj}^T & \sum_k a_{2k}^F\wedge b_{kj}^F & \prod_k a_{2k}^F\vee b_{kj}^F\\ \vdots & \vdots & \vdots\\ \sum_k a_{mk}^T\wedge b_{kj}^T & \sum_k a_{mk}^F\wedge b_{kj}^F & \prod_k a_{mk}^F\vee b_{kj}^F\end{bmatrix}$$

$$=\begin{bmatrix}\langle a_{11}^T a_{11}^I a_{11}^F\rangle & \langle a_{12}^T a_{12}^I a_{12}^F\rangle & \ldots & \langle a_{13}^T a_{13}^I a_{13}^F\rangle & \langle a_{1n}^T a_{1n}^I a_{1n}^F\rangle\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ \langle a_{21}^T a_{21}^I a_{21}^F\rangle & \langle a_{22}^T a_{22}^I a_{22}^F\rangle & \ldots & \langle a_{23}^T a_{23}^I a_{23}^F\rangle & \langle a_{2n}^T a_{2n}^I a_{2n}^F\rangle\\ \langle a_{31}^T a_{31}^I a_{31}^F\rangle & \langle a_{32}^T a_{32}^I a_{32}^F\rangle & \ldots & \langle a_{33}^T a_{33}^I a_{33}^F\rangle & \langle a_{3n}^T a_{3n}^I a_{3n}^F\rangle\end{bmatrix}\begin{bmatrix}\langle b_{1j}^T, b_{1j}^I, b_{1j}^F\rangle\\ \langle b_{2j}^T, b_{2j}^I, b_{2j}^F\rangle\\ \vdots\\ \langle b_{nj}^T, b_{nj}^I, b_{nj}^F\rangle\end{bmatrix},$$

j=1,2,...,p

$=AB_j$

Hence the proof.

Theorem 3.14. Let A be an $m\times n$ FNSM and B be an $n\times p$ FNSM. Let A (or B) be partitioned into two blocks by column partitioning only. Then the product AB is also partitioned into two blocks of same column (row) partitioning.

Proof: Let $A=[A_1 \quad A_2]$ where A_1 is of order $m\times t$ and A_2 is of order $m\times(n-t)$ FNSM. Then

$AB=[a_1 \quad a_2 \quad \cdots \quad a_t \vdots \quad a_{t+1} \quad a_n]B \quad =[a_1B \quad a_2B \quad \cdots \quad a_tB \vdots \quad a_{t+1}B \vdots \quad a_nB]$

$=[[A_1B \quad \vdots A_2B]$.

Hence the proof.

3.15. Operations on FNSBM

Addition:

The conformal fuzzy neutrosophic soft matrices can be added by block as addition of two fuzzy neutrosophic soft matrices of the same dimensions.

$$A \oplus B = \begin{bmatrix} A_{11} \oplus B_{11} & A_{12} \oplus B_{12} & \cdots A_{1n} \oplus B_{1n} \\ A_{21} \oplus B_{21} & A_{22} \oplus B_{22} & \cdots A_{2n} \oplus B_{2n} \\ \cdots & \cdots & \cdots \\ A_{m1} \oplus B_{m1} & A_{m2} \oplus B_{m2} & \cdots A_{mn} \oplus B_{mn} \end{bmatrix}$$

Scalar multiplication:

As in scalar multiplication of an FNSM by a scalar, each block of partition FNSM is multiplied by scalar.

Thus, $$\alpha A = \begin{bmatrix} \alpha A_{11} & \alpha A_{12} & \cdots & \alpha A_{1n} \\ \alpha A_{21} & \alpha A_{22} & \cdots & \alpha A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha A_{m1} & \alpha A_{m1} & \cdots & \alpha A_{mn} \end{bmatrix}$$

Here $\alpha A_{ij} = \alpha\{a_{ij}^T, a_{ij}^I, a_{ij}^F\} = inf\{\alpha, a_{ij}^T\}, inf\{\alpha, a_{ij}^I\}, sup\{\alpha, a_{ij}^F\}$.

Multiplication of fuzzy neutrosophic soft partition matrices:

Let $A = [\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle]_{m\times n}$ and $B = [\langle b_{jk}^T, b_{jk}^I, b_{jk}^F \rangle]_{n\times p}$ be two FNSMs conformable for multiplication. Let A_{ij}° and B_{ij}° denote blocks of A and B. For FNSM multiplication AB by partition to be conformable, the number of column partitioning of A must be equal to that of row partitioning of B and in addition blocks must be conformable for multiplication of FNSM and

$$AB = [\langle c_{ik}^T, c_{ik}^I, c_{ik}^F \rangle]_{m\times n} = [\sum_{j=1}^{s} A_{ij}^{\circ} B_{jk}^{\circ}]$$

which is again a partition FNSM with m horizontal and n vertical dotted lines.

Theorem 3.16. If $AB = C$ the fuzzy neutrosophic soft submatrix containing rows $i_1, i_2, ..., i_r$ and columns $j_1, j_2, ..., j_s$ of C is equal to the product of the fuzzy neutrosophic soft submatrix with these rows of A and the fuzzy neutrosophic soft sub matrix with these columns of B.

Proof: Let $A = (\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle)_{m\times n}$ and $B = (\langle b_{jk}^T, b_{jk}^I, b_{jk}^F \rangle)_{n\times p}$ be two fuzzy neutrosophic soft sub matrix. Then

$$C = (\langle c_{ik}^T, c_{ik}^I, c_{ik}^F \rangle) = \langle \sum_{j=1}^{n} (a_{ij}^T \wedge b_{jk}^T), \sum_{j=1}^{n} (a_{ij}^I \wedge b_{jk}^I), \prod_{j=1}^{n} (a_{ij}^F \vee b_{jk}^F) \rangle$$

where i=1,2,...,m and k=1,2,...,p. Now the fuzzy neutrosophic soft sub matrix C_{ik} of FNSM C with rows $i_1, i_2, ..., i_r$ and columns $j_1, j_2, ..., j_s$ is obtained by replacing row i and column k of C by these rows and columns. It is

$$C_{ik} = \langle \sum_{j=1}^{n} (a_{ij}^{T} \wedge b_{jk}^{T}),\ \sum_{j=1}^{n} (a_{ij}^{I} \wedge b_{jk}^{I}),\ \prod_{j=1}^{n} (a_{ij}^{F} \vee b_{jk}^{F}) \rangle \text{ .....(1)}$$

where $i = i_1, i_2, ..., i_r$ and columns $k = j_1, j_2, ..., j_s$. Again the product of the given fuzzy neutrosophic soft sub matrices A_i and B_k of FNSM A and B respectively is

$$\mathrm{A_i B_k} = \begin{bmatrix} \langle a^T_{i_1 1} a^I_{i_1 1} a^F_{i_i 1} \rangle & \langle a^T_{i_1 2} a^I_{i_1 2} a^F_{i_1 2} \rangle & \cdots & \langle a^T_{i_1 n} a^I_{i_1 n} a^F_{i_1 n} \rangle \\ \langle a^T_{i_2 1} a^I_{i_2 1} a^F_{i_2 1} \rangle & \langle a^T_{i_2 2} a^I_{i_2 2} a^F_{i_2 2} \rangle & \cdots & \langle a^T_{i_2 n} a^I_{i_2 n} a^F_{i_i n} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle a^T_{i_r 1} a^I_{i_r 1} a^F_{r1} \rangle & \langle a^T_{i_r 2} a^I_{i_r 2} a^F_{r2} \rangle & \cdots & \langle a^T_{i_r 1} a^I_{[i_1]1} a^F_{i_i 1} \rangle \end{bmatrix}_{\mathrm{rxn}}$$

$$\begin{bmatrix} \langle b^T_{1j_1}, b^T_{1j_1}, b^T_{1j_1} \rangle & \langle b^T_{1j_2}, b^T_{1j_2}, b^T_{1j_2} \rangle & \cdots & \langle b^T_{1j_s}, b^T_{1j_s}, b^T_{1j_s} \rangle \\ \langle b^T_{2j_1}, b^T_{2j_1}, b^T_{2j_1} \rangle & \langle b^T_{2j_1}, b^T_{2j_1}, b^T_{2j_1} \rangle & \cdots & \langle b^T_{2j_1}, b^T_{2j_1}, b^T_{2j_1} \rangle \\ \cdots & \cdots & \cdots & \cdots \\ \langle b^T_{nj_1}, b^T_{nj_1}, b^T_{nj_1} \rangle & \langle b^T_{nj_1}, b^T_{nj_1}, b^T_{nj_1} \rangle & \cdots & \langle b^T_{nj_1}, b^T_{nj_1}, b^T_{nj_1} \rangle \end{bmatrix}_{\mathrm{nxs}}$$

$$= \left[\sum_{j=1}^{n} (a_{ij}^{T} \wedge b_{jk}^{T}), \sum_{j=1}^{n} (a_{ij}^{I} \wedge b_{jk}^{I}), \prod_{j=1}^{n} (a_{ij}^{F} \wedge b_{jk}^{F}) \right]_{r \times s} \text{ ...... (2)}$$

where $i = i_1, i_2, ..., i_r$ and columns $k = j_1, j_2, ..., j_s$.
Therefore from equation (1) and (2) give the results.

Theorem 3.17. If the $m \times n$ FNSM A is partitioned by consecutive groups of rows into blocks A_i, i=1,2,...,r and $n \times p$ FNSM B is partitioned by consecutive groups of columns into blocks B_j, $j = 1, 2, ..., s$. Then the product AB=C of order $m \times p$ is partitioned into blocks by row groups exactly as A and column groups exactly as B. The ik-th block C_{ik} of C is given by $(c_{ik}^T, c_{ik}^I, c_{ik}^F) = (A_i^T B_k^T, A_i^I B_k^I, A_i^F B_k^F)$.

Proof:

Let $i = i_1, i_2, ..., i_r$ be the consecutive rows of A and $k = j_1, j_2, ..., j_s$ be the consecutive columns of B. Then $C_{ik} = \left[\langle \sum_{j=1}^{n} (a_{ij}^{T} \wedge b_{jk}^{T}), \sum_{j=1}^{n} (a_{ij}^{I} \wedge b_{jk}^{I}), \prod_{j=1}^{n} (a_{ij}^{F} \vee b_{jk}^{F}) \rangle \right]_{r \times s}$ .....(3)

where $i = i_1, i_2, ..., i_r$ and $k = j_1, j_2, ..., j_s$

$$A_i B_k = \left[\langle \sum_{j=1}^{n} (a_{ij}^{T} \wedge b_{jk}^{T}), \sum_{j=1}^{n} (a_{ij}^{I} \wedge b_{jk}^{I}), \prod_{j=1}^{n} (a_{ij}^{F} \vee b_{jk}^{F}) \rangle \right]_{r \times s} \text{ .....(4)}$$

where i=1,2,...,r and k=1,2,...,s.
from equation (3) and (4) ik-th block C_{ik} of C is given by $C_{ik} = A_i B_k$.

4. Some algebraic operations of fuzzy neutrosophic soft matrices:

In this section we define direct sum, Kronecker product and Kronecker sum of the FNSMs and studied some properties.

Definition 4.1. Let $A_1, A_2, ..., A_s$ be FNSSMs of orders $n_1, n_2, ..., n_r$ respectively. The diagonal FNSM

$$\text{Diag } (A_1, A_2, ..., A_s) = \begin{bmatrix} A_1 & \langle 0,0,1\rangle & \cdots & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & A_2 & \cdots & \langle 0,0,1\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \cdots & A_r \end{bmatrix}_{n_1, n_2, ..., n_r}$$

is called the direct sum of the square FNSMs $A_1, A_2, ..., A_s$ and it can be written as $A_1 @ A_2 @ ... @ A_s$ of order $(n_1 + n_2 + ... + n_r)$.

Properties of direct sum of FNSM:

(a) Commutative property:

The square FNSSMs does not satisfy the commutative property.

Let A and B be two FNSSMs. Then the direct sum of A and B are

$$A @ B = \begin{bmatrix} A & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B \end{bmatrix} \text{ and } B @ A = \begin{bmatrix} B & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & A \end{bmatrix}$$

It is obvious that, $A @ B \neq B @ A.$

(b) Associative property:

Let A,B and C be three FNSSMs. Then $A @ B = \begin{bmatrix} A & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B \end{bmatrix} = D.$

Now $(A @ B) @ C = D @ C = \begin{bmatrix} D & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & C \end{bmatrix} = \begin{bmatrix} A & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & C \end{bmatrix}$

Similarly, $B @ C = \begin{bmatrix} B & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & C \end{bmatrix} = E.$

Now, $A @ (B \oplus C) = A @ E = \begin{bmatrix} A & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & E \end{bmatrix} = \begin{bmatrix} A & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & C \end{bmatrix}.$

Hence, $(A @ B) @ C = A @ (B @ C).$

(c) Transposition:

$(A @ B)^T = A^T @ B^T.$

Since, $A @ B = \begin{bmatrix} A & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B \end{bmatrix}$ then $(A @ B)^T = \begin{bmatrix} A^T & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B^T \end{bmatrix}$

$= A^T @ B^T$.

4.2. Kronecker product of FNSMs

A$=(a_{ij}^T, a_{ij}^I, a_{ij}^F)_{m\times n}$ and $B=(b_{ij}^T, b_{ij}^I, b_{ij}^F)_{p\times q}$

be two rectangular FNSMs. Then the Kronecker product of A and B , denoted by $A\otimes B$ is defined as the partitioned fuzzy neutrosophic soft matrix

$$A\otimes B = \begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F\rangle B & \langle a_{12}^T, a_{12}^I, a_{12}^F\rangle B & \dots & \langle a_{13}^T, a_{13}^I, a_{13}^F\rangle B & \langle a_{1n}^T, a_{1n}^I, a_{1n}^F\rangle B \\ \langle a_{21}^T, a_{21}^I, a_{21}^F\rangle B & \langle a_{22}^T, a_{22}^I, a_{22}^F\rangle B & \dots & \langle a_{23}^T, a_{23}^I, a_{23}^F\rangle B & \langle a_{2n}^T, a_{2n}^I, a_{2n}^F\rangle B \\ \dots & \dots & \dots & \dots & \dots \\ \langle a_{31}^T, a_{31}^I, a_{31}^F\rangle B & \langle a_{32}^T, a_{32}^I, a_{32}^F\rangle B & \dots & \langle a_{33}^T, a_{33}^I, a_{33}^F\rangle B & \langle a_{mn}^T, a_{mn}^I, a_{mn}^F\rangle B \end{bmatrix}$$

where $a_{ij} = \langle a_{ij}^T, a_{ij}^I, a_{ij}^F\rangle$ for i=1,2,...,m and j=1,2,...,n. It has mn blocks. The ij-th blocks $a_{ij}B$ of order $mp\times nq$.

Note. The difference between the product of FNSMs and Kronecker product of FNSMs is that in product of FNSM product AB requires equality of the number of columns in A and the number of rows in B while in Kronecker product it is free from such restriction.

Kronecker product of two fuzzy neutrosophic soft column vectors:

Let $x = \left[[\langle x_{11}^T, x_{11}^I x_{11}^F\rangle \quad \langle x_{21}^T, x_{21}^I x_{21}^F\rangle \quad \cdots \quad \langle x_{n1}^T, x_{n1}^I x_{n1}^F\rangle\right]_{n\times 1}^T$

and $y = \left[[\langle y_{11}^T, y_{11}^I y_{11}^F\rangle \quad \langle y_{21}^T, y_{21}^I x_{21}^F\rangle \quad \cdots \quad \langle y_{n1}^T, y_{n1}^I y_{n1}^F\rangle\right]_{m\times 1}^T$ be two column fuzzy neutrosophic soft vectors. Then by definition of Kronecker product , we have

$$x\otimes y = \begin{bmatrix} \langle x_{11}^T, x_{11}^I, x_{11}^F\rangle \\ \langle x_{21}^T, x_{21}^I, x_{21}^F\rangle \\ \vdots \\ \langle x_{n1}^T, x_{n1}^I, x_{n1}^F\rangle \end{bmatrix}_{nmx1} = \begin{bmatrix} \langle x_{11}^T, x_{11}^I, x_{11}^F\rangle\langle y_{11}^T, y_{11}^I, y_{11}^F\rangle \\ \langle x_{21}^T, x_{21}^I, x_{21}^F\rangle\langle y_{11}^T, y_{11}^I, y_{11}^F\rangle \\ \vdots \\ \langle x_{n1}^T, x_{n1}^I, x_{n1}^F\rangle\langle y_{m1}^T, y_{m1}^I, y_{m1}^F\rangle \end{bmatrix}_{nmx1}$$

Kronecker sum:

The Kronecker sum of two FNSSMs $A_{n\times n}$ and $B_{m\times m}$ is defined by

$A \oslash B = A\otimes (I_1)_m + (I_1)_n \otimes B,$ which is an $nm\times nm$ fuzzy neutrosophic soft matrix.

Example 1.

Let

$$A = \begin{bmatrix} \langle 1,1,0\rangle & \langle 0.2,0.3,0.1\rangle \\ \langle 0.5,0.2,0.3\rangle & \langle 1,1,0\rangle \end{bmatrix} \begin{bmatrix} \langle 1,1,0\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.7,0.3.0.2\rangle \\ \langle 0.2,0.1,0.6\rangle & \langle 1,1,0\rangle & \langle 0.2,0.3,0.5\rangle \\ \langle 0.1,0.2,0.3\rangle & \langle 0.1,0.2,0.3\rangle & \langle 1,1,0\rangle \end{bmatrix}$$

be two FNSMs. Then $A \oslash B = A \otimes (I_1)_3 + (I_1)_2 \otimes B =$

$$\left[\begin{array}{cccccccc} \langle 1,1,0\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0.7,0.3,0.2\rangle & \vdots & \langle 0.2,0.3,0.5\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0.2,0.1,0.6\rangle & \langle 1,1,0\rangle & \langle 0.2,0.3,0.5\rangle & \vdots & \langle 0,0,1\rangle & \langle 0.2,0.3,0.5\rangle & \langle 0,0,1\rangle \\ \langle 0.1,0.2,0.3\rangle & \langle 0.1,0.2,0.3\rangle & \langle 1,1,0\rangle & \vdots & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.2,0.3,0.5\rangle \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ \langle 0.5,0.2,0.3\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & 1,1,0 & \langle 0.2,0.3,0.5\rangle & \langle 0.7,0.3,0.2\rangle \\ \langle 0,0,1\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0,0,1\rangle & \vdots & \langle 0.2,0.1,0.6\rangle & \langle 1,1,0\rangle & \langle 0.2,0.3,0.5\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.5,0.2,0.3\rangle & \vdots & \langle 0.1,0.2,0.3\rangle & \langle 0.1,0.2,0.3\rangle & \langle 1,1,0\rangle \end{array}\right]$$

5. Some relational operations on fuzzy neutrosophic soft block matrices

Here we define four types of reflexivity and irrefleivity of a FNSM.

Definition 5.1. Let A be an FNSM of any order then

1. A is a reflexive of type-1 if $a_{ii}^T = 1, a_{ii}^I = 1, a_{ii}^F = 0,$ for all i=1,2,...,n.
2. A is a reflexive of type-2 if $(a_{ii}^F \vee a_{jj}^F) \le a_{ij}^F$, for each i,j =1,2,...,n.
3. A is a reflexive of type-3 if $(a_{ii}^T \wedge a_{jj}^T) \ge a_{ij}^T$ and $(a_{ii}^I \wedge a_{jj}^I) \ge a_{ij}^I$ for all i,j=1,2,...,n.
4. A is a reflexive of type-4 if $(a_{ii}^F \vee a_{jj}^F) \le a_{ij}^F$ and $(a_{ii}^T \wedge a_{jj}^T) \ge a_{ij}^T$ and $(a_{ii}^I \wedge a_{jj}^I) \ge a_{ij}^I$ for all i, j=1,2,...,n.

For irreflexivity:

1. A is a irreflexive of type-1 if $a_{ii}^T = 0, a_{ii}^I = 0, a_{ii}^F = 1,$ for all i=1,2,...,n.
2. A is a irreflexive of type-2 if $(a_{ii}^F \wedge a_{jj}^F) \ge a_{ij}^F$, for each i,j =1,2,...,n.
3. A is a irreflexive of type-3 if $(a_{ii}^T \vee a_{jj}^T) \le a_{ij}^T$ and $(a_{ii}^I \vee a_{jj}^I) \le a_{ij}^I$ for all i,j=1,2,...,n.
4. A is a irreflexive of type-4 if $(a_{ii}^F \wedge a_{jj}^F) \ge a_{ij}^F$ and $(a_{ii}^T \vee a_{jj}^T) \le a_{ij}^T$ and, $(a_{ii}^I \vee a_{jj}^I) \le a_{ij}^I$ for all i,j=1,2,...,n.

Theorem 5.2. If FNSM A and B be reflexive of any type then direct sum of these FNSMs is also reflexive of the same type.

Proof: (i) Let FNSMs A and B be reflexive of type-1, then $\langle a_{ij}^T, a_{ij}^I, a_{ij}^F \rangle = \langle 1,1,0\rangle$ and $\langle b_{ij}^T, b_{ij}^I, b_{ij}^F \rangle = \langle 1,1,0\rangle$. Then the direct sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix S= A@ B= $\begin{bmatrix} A & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & B \end{bmatrix}$

Now $\langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle = \langle 1,1,0 \rangle$, since diagonal elements in fuzzy neutrosophic soft block matrix S are FNSMs A and B and diagonal elements in A and B are $\langle 1,1,0 \rangle$.
Hence direct sum S of the FNSMs A and B is reflexive of type-1.
(ii) Let FNSMs A and B be reflexive of type-2, then $(a_{ii}^F \vee a_{jj}^F) \leq a_{ij}^F$, and $(b_{ii}^F \vee b_{jj}^F) \leq b_{ij}^F$. Then the direct sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix $S = A@B.$
fuzzy neutrosophic soft block matrix S contains four blocks, diagonal blocks are FNSMs A and B off diagonal blocks are fuzzy neutrosophic soft zero matrices.
Now for A blocks we have $s_{ij}^F = a_{ij}^F$ [i=i,2,...,m, j=1,2,...,m]
$\geq a_{ii}^F \vee a_{jj}^F$ [as A is reflexive of type-2]
= $s_{ii}^F \vee s_{jj}^F$
Now for B blocks we have $s_{(m+p)(m+q)}^F = b_{pq}^F$ [p=1,2,...,n q=1,2,...,n]
$\geq b_{pp}^F \vee b_{qq}^F$ [as B is reflexive of type-2]
$= s_{(m+p)(m+p)}^F \vee s_{(m+q)(m+q)}^F$ [as off diagonal blocks are FNS zero matrices.]
Therefore, $s_{kl}^F \geq s_{kk}^F \vee s_{ll}^F, k = 1,2,...,m+n, l = 1,2,...,m+n.$
Hence direct sum S of the FNSMs A and B is reflexive of type-2.
(iii) Let FNSMs A and B be reflexive of type-3, then $(a_{ii}^T \wedge a_{jj}^T) \geq a_{ij}^T, (a_{ii}^I \wedge a_{jj}^I) \geq a_{ij}^I, (b_{ii}^T \wedge b_{jj}^T) \geq b_{ij}^T$ and $(b_{ii}^I \wedge b_{jj}^I) \geq b_{ij}^I$. Then the direct sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix $S = A@B.$
Now for A blocks we have
$s_{ij}^T = a_{ij}^T [i = 1,2,...,m, j = 1,2,...,m]$ and
$\leq a_{ii}^T \wedge a_{jj}^T$
$s_{ij}^I = a_{ij}^I [i = 1,2,...,m, j = 1,2,...,m]$
$\leq a_{ii}^I \wedge a_{jj}^I$ [as A is reflexive of type-3]
$= s_{ii}^T \wedge s_{jj}^T$
Now for B blocks we have $s_{(m+p)(m+q)}^T = b_{pq}^T$ [p=1,2,...,n q=1,2,...,n]
$\geq b_{pp}^T \wedge b_{qq}^T$ [as B is reflexive of type-2]
$= s_{(m+p)(m+p)}^T \wedge s_{(m+q)(m+q)}^T$ [as off diagonal blocks are fuzzy neutrosophic soft zero matrices.]
Therefore, $s_{kl}^T \leq s_{kk}^T \wedge s_{ll}^T$, k=1,2,...,m+n.
Hence direct sum S of the FNSMs A and B is reflexive of type-3.
(iv) Let FNSMs A and B be reflexive of type-4, then

$(a_{ii}^F \vee a_{jj}^F) \leq a_{ij}^F, (a_{ii}^T \wedge a_{jj}^T) \geq a_{ij}^T, (a_{ii}^I \wedge a_{jj}^I) \geq a_{ij}^I$ and
$(b_{ii}^F \vee b_{jj}^F) \leq b_{ij}^F, (b_{ii}^T \wedge b_{jj}^T) \geq b_{ij}^T, (b_{ii}^I \wedge b_{jj}^I) \geq b_{ij}^I$. Then the direct sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix reflexive of type-4, by using the results (ii) and (iii).

Hence direct sum of FNSMs reflexive of any type is also reflexive of the same type.

Theorem 5.3. If FNSMs A and B be reflexive of type-1 then Kronecker product of these FNSMs is also reflexive of type-1.

Proof: Let FNSMs A and B be reflexive of type-1, then $\langle a_{ii}^T, a_{ii}^I, a_{ii}^F \rangle = \langle 1,1,0 \rangle$ and $\langle b_{ii}^T, b_{ii}^I, b_{ii}^F \rangle = \langle 1,1,0 \rangle$. Then the Kronecker product of these FNSMs A and B be fuzzy neutrosophic soft block matrix

S= $A \otimes B$

$$= \begin{bmatrix} \langle a_{11}^T, a_{11}^I, a_{11}^F \rangle B & \langle a_{12}^T, a_{12}^I, a_{12}^F \rangle B & \dots & \langle a_{13}^T, a_{13}^I, a_{13}^F \rangle B & \langle a_{1m}^T, a_{1m}^I, a_{1m}^F \rangle B \\ \langle a_{21}^T, a_{21}^I, a_{21}^F \rangle B & \langle a_{22}^T, a_{22}^I, a_{22}^F \rangle B & \dots & \langle a_{23}^T, a_{23}^I, a_{23}^F \rangle B & \langle a_{2m}^T, a_{2m}^I, a_{2m}^F \rangle B \\ \dots & \dots & \dots & \dots & \dots \\ \langle a_{31}^T, a_{31}^I, a_{31}^F \rangle B & \langle a_{32}^T, a_{32}^I, a_{32}^F \rangle B & \dots & \langle a_{33}^T, a_{33}^I, a_{33}^F \rangle B & \langle a_{mm}^T, a_{mm}^I, a_{mm}^F \rangle B \end{bmatrix}$$

Here $\langle a_{ii}^T, a_{ii}^I, a_{ii}^F \rangle = \langle 1,1,0 \rangle$ for all i=1,2,...,m as A is an FNSM reflexive of type-1 and diagonal elements of B are $\langle b_{jj}^T, b_{jj}^I, b_{jj}^F \rangle = \langle 1,1,0 \rangle$ for all j=1,2,...,n as B is an FNSM reflexive of type-1.

Therefore $\langle s_{pp}^T, s_{pp}^I, s_{pp}^F \rangle = \langle 1,1,0 \rangle$, for p=1,2,...,mn, where m and n are the order of the FNSMs A and B respectively.

Hence Kronecker product of FNSMs reflexive of type-1 is also reflexive of type-1.

Theorem 5.4. If FNSMs A and B be reflexive of type-2 then Kronecker product of these FNSMs is also reflexive of type-2.

Proof: Let FNSMs A and B be reflexive of type-2, then $(a_{ii}^F \vee a_{jj}^F) \leq a_{ij}^F$ and $(b_{ii}^F \vee b_{jj}^F) \leq b_{ij}^F$. Then the Kronecker product of these FNSMs A and B be fuzzy neutrosophic soft block matrix $S = A \otimes B.$

Fuzzy neutrosophic soft block matrix S contains mm blocks, diagonal blocks are FNSMs $(a_{ii}^T, a_{ii}^I, a_{ii}^F)B$ and off diagonal blocks are $(a_{ij}^T, a_{ij}^I, a_{ij}^F)B$ when $i \neq j$. Now for the diagonals blocks we have

$s_{pq}^F = max\{a_{ii}^F, b_{pq}^F\}$ [i=1,2,...,m, p,q=1,2,...,n]

$\geq max\{a_{ii}^F, b_{pp}^F \vee b_{qq}^F\}$ [as B is reflexive of typ.e-2]= $s_{pp}^F \vee s_{qq}^F$.

Now for off diagonal blocks we have

$s_{pq}^F = max\{a_{ij}^F, a_{pq}^F\}$ [i=1,2,...,m,p,q=1,2,...,n]

$\geq max\{a_{ij}^F, b_{pp}^F \vee b_{qq}^F\}$ [as B is reflexive of type-2]

$= s_{pp}^F \vee s_{qq}^F$

Again, $(a_{ii}^F \vee a_{jj}^F) \leq a_{ij}^F$ for all i,j=1,2,...,m.

Therefore, $s_{kl}^F \geq (s_{kk}^F \vee s_{ll}^F), k = 1, 2, ..., m, l = 1, 2, ..., n.$

Hence Kronecker product of two reflexive FNSMs of type-2 is fuzzy neutrosophic soft block matrix S , which is also reflexive of type-2.

Theorem 5.5. If FNSMs A and B be reflexive of type-3 then Kronecker product of these FNSMs is also reflexive of type-3.

Proof: Let FNSMs A and B be reflexive of type-3,

then $(a_{ii}^T \wedge a_{jj}^T) \geq a_{ij}^T, (a_{ii}^I \wedge a_{jj}^I) \geq a_{ij}^I,$

and $(b_{ii}^T \wedge b_{jj}^T) \geq b_{ij}^T, (b_{ii}^I \wedge b_{jj}^I) \geq b_{ij}^I$ Then the Kronecker product of these FNSMs A and B be fuzzy neutrosophic soft block matrix $S = A \otimes B.$

Fuzzy neutrosophic soft block matrix S contains mm blocks , diagonal blocks are FNSMs $(a_{ii}^T, a_{ii}^I, a_{ii}^F)B$ and off diagonal blocks are $(a_{ij}^T, a_{ij}^I, a_{ij}^F)B$ when $i \neq j.$

Now for the diagonal blocks

$s_{pq}^T = min\{(a_{ii}^T, b_{pq}^T), (a_{ii}^I, b_{pq}^I)\}$ [where i=1,2,...,m, p,q=1,2,...,n]

$\leq min\{(a_{ii}^T, b_{pp}^T \vee b_{qq}^T), (a_{ii}^I, b_{pp}^I \vee b_{qq}^I)\}$ [as B is reflexive of type-3]

$= (s_{pp}^T \vee s_{qq}^T, s_{pp}^I \vee s_{qq}^I).$

Now for off diagonal blocks

$s_{pq}^T = min\{(a_{ii}^T, b_{pq}^T), (a_{ii}^I, b_{pq}^I)\}$ [where i=1,2,...,m, p,q=1,2,...,n]

$\leq max\{(a_{ii}^T, b_{pp}^T \vee b_{qq}^T), (a_{ii}^I, b_{pp}^I \vee b_{qq}^I)\}$ [as B is reflexive of type-3]

$= (s_{pp}^T \vee s_{qq}^T, s_{pp}^I \vee s_{qq}^I).$

Again, $(a_{ii}^T \wedge a_{jj}^T) \geq a_{ij}^T$ for all i,j=1,2,...,m.

$(a_{ii}^I \wedge a_{jj}^I) \geq a_{ij}^I$ for all i,j=1,2,...,m.

Therefore, $s_{kl}^F \leq s_{kk}^T \vee s_{ll}^T$, k=1,2,...,m, l=1,2,...,n.

$s_{kl}^F \leq s_{kk}^T \vee s_{ll}^T, k = 1, 2, ..., m, l = 1, 2, ..., n. s_{kl}^F \leq s_{kk}^I \vee s_{ll}^I, k = 1, 2, ..., m, l = 1, 2, ..., n.$ Hence Kronecker product of reflexive FNSMs of type-2 is fuzzy neutrosophic soft block matrix S, reflexive of type-3.

Theorem 5.6. If FNSMs A and B be reflexive of type-4 then Kronecker product of these FNSMs is also reflexive of type-4.

Proof: If FNSMs A and B be reflexive of type-4 then

$(a_{ii}^F \vee a_{jj}^F) \leq a_{ij}^F, (a_{ii}^T \wedge a_{jj}^T) \geq a_{ij}^T, (a_{ii}^I \wedge a_{jj}^I) \geq a_{ij}^I$ and

$(b_{ii}^{F} \vee b_{jj}^{F}) \le b_{ij}^{F}, (b_{ii}^{T} \wedge b_{jj}^{T}) \ge b_{ij}^{T}, (b_{ii}^{I} \wedge b_{jj}^{I}) \ge b_{ij}^{I}$ Then from the Theorems 5.4 and 5.5 , Kronecker product of these FNSMs A and B be fuzzy neutrosophic soft block matrix which is reflexive of type-4.

Theorem 5.7. If FNSMs A and B be reflexive of type-1 then Kronecker sum of these FNSMs is also reflexive of type-1.

Proof: Let FNSMs A and B of order $m \times m$ and $n \times n$ respectively be reflexive of type -1, then $\langle a_{ii}^{T}, a_{ii}^{I}, a_{ii}^{F} \rangle = \langle 1,1,0 \rangle$ and $\langle b_{ii}^{T}, b_{ii}^{I}, b_{ii}^{F} \rangle = \langle 1,1,0 \rangle$. Then the Kronecker sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix

$S = A \oslash B = A \otimes (I_1)_n + (I_1)_m \otimes B$

where $(I_1)_n$ and $(I_1)_m$ are the fuzzy neutrosophic soft identity matrices.Now fuzzy neutrosophic soft identity matrices $(I_1)_n)$ and $(I_1)_m$ are reflexive FNSMs of type-1.Therefore, by the Theorem 5.2 , direct sum of reflexive FNSMs of type-1, is again reflexive of type-1.

Hence Kronecker sum of FNSMs reflexive of type-1 is also reflexive of type-1.

Example. Let

$$A = \begin{bmatrix} \langle 0.7, 0.6, 0.3 \rangle & \langle 0.6, 0.5, 0.5 \rangle \\ \langle 0.6, 0.5, 0.4 \rangle & \langle 0.8, 0.6, 0.1 \rangle \end{bmatrix} B = \begin{bmatrix} \langle 0.8, 0.6, 0.4 \rangle & \langle 0.5, 0.4, 0.6 \rangle & \langle 0.3, 0.2, 0.4 \rangle \\ \langle 0.4, 0.3, 0.4 \rangle & \langle 0.9, 0.4, 0.3 \rangle & \langle 0.6, 0.3, 0.5 \rangle \\ \langle 0.4, 0.3, 0.5 \rangle & \langle 0.5, 0.2, 0.4 \rangle & \langle 0.8, 0.7, 0.4 \rangle \end{bmatrix}$$

be two reflexive FNSMs of order 2 and 3 respectively of type-2,type-3 or type-4.

Now the Kronecker sum of these FNSMs A and B be fuzzy neutrosophic soft block matrix S where

S= $A \oslash B = A \otimes (I_1)_3 + (I_1)_2 \otimes B =$

$$\begin{bmatrix} \langle 0.8,0.6,0.3 \rangle & \langle 0.5,0.4,0.6 \rangle & \langle 0.3,0.2,0.4 \rangle & \langle 0.6,0.5,0.5 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle \\ \langle 0.4,0.3,0.4 \rangle & \langle 0.9,0.6,0.3 \rangle & \langle 0.6,0.2,0.5 \rangle & \langle 0,0,1 \rangle & \langle 0.6,0.5,0.5 \rangle & \langle 0,0,1 \rangle \\ \langle 0.4,0.3,0.5 \rangle & \langle 0.5,0.2,0.4 \rangle & \langle 0.8,0.7,0.3 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0.6,0.5,0.5 \rangle \\ \langle 0.6,0.5,0.4 \rangle & \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0.8,0.6,0.1 \rangle & \langle 0.5,0.4,0.6 \rangle & \langle 0.3,0.2,0.4 \rangle \\ \langle 0,0,1 \rangle & \langle 0.6,0.5,0.4 \rangle & \langle 0,0,1 \rangle & \langle 0.4,0.3,0.4 \rangle & \langle 0.9,0.6,0.1 \rangle & \langle 0.6,0.3,0.5 \rangle \\ \langle 0,0,1 \rangle & \langle 0,0,1 \rangle & \langle 0.6,0.5,0.4 \rangle & \langle 0.4,0.3,0.5 \rangle & \langle 0.5,0.2,0.4 \rangle & \langle 0.8,0.7,0.4 \rangle \end{bmatrix}$$

Here $(s_{ii}^{F} \vee s_{jj}^{F}) \le s_{ij}^{F}$ and $(s_{ii}^{T} \wedge s_{jj}^{T}) \ge s_{ij}^{T}, (s_{ii}^{I} \wedge s_{jj}^{I}) \ge s_{ij}^{I}$, for all i=1,2,...,6 and j=1,2,...,6.

Hence, fuzzy neutrosophic soft block matrix S is reflexive of type-2, type-3 or type-4.

6. Conclusion

In this article partitioning of FNSM in to an FNSBM. Authors has been might focused on different types of fuzzy neutrosophic soft sub matrix, direct sum, Kronecker sum, Kronecker product of FNSM, relational operation on FNSM and FNSBM

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