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Cholesky Decomposition Method for Solving Fully Fuzzy Linear System of Equations with Trapezoidal Fuzzy Number

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Abstract. In this paper Cholesky decomposition method is used to solve a fully fuzzy linear system of equation with trapezoidal fuzzy numbers as entries. A numerical example is also dealt with.

Keywords: Trapezoidal fuzzy number, Cholesky decomposition method

AMS Mathematics Subject Classification (2010): 03E72

1. Introduction

The concepts of fuzzy numbers and fuzzy arithmetic were introduced by Zadeh. Fuzzy systems are used to solve fuzzy metric spaces, fuzzy differential equations, fuzzy linear and non linear system etc. A major application of fuzzy number arithmetic is to solve fully fuzzy linear systems. Problems under Physics, Economics and Engineering should be represented by fuzzy rather than crisp numbers. We develop numerical procedures that would treat fully fuzzy linear system and solve them. Solving fuzzy n × n linear system whose coefficient matrix is crisp and the right hand side column is an arbitrary fuzzy number vector was introduced by Friedman et al [6]. Mosleh [8] introduced LU decomposition method for solving fuzzy linear systems. Muzziloi [9] developed fully fuzzy linear system of the form $A_1x + b_1 = A_2x + b_2$ where A_1, A_2 are square matrices of fuzzy coefficients, b_1, b_2 are fuzzy numbers. Dehgan et al. [2] considered fully fuzzy linear system of the form Ax = b where A is a fuzzy matrix, x is a fuzzy vector, and the constant b are vectors.

The formation of this paper is organized as follows. In section 2, preliminary concepts of trapezoidal fuzzy number matrices have been discussed. In section 3, a new algorithm to solve fuzzy linear system in the form of trapezoidal fuzzy number matrices is proposed. In section 4 numerical example have been discussed to solve a solution using Cholesky decomposition method. In section 5, conclusion about the results is established.

2. Preliminaries

Definition 2.1. The characteristic function $\mu_{\tilde{A}}$ of a crisp set A of X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$: X → [0,1] . The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A}=\{(x, \mu_{\tilde{A}}(x)); x \in X\}$ is called fuzzy set.

Definition 2.2. A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & if a \le x \le b \\ 1 & if b \le x \le c \\ \frac{d-x}{d-c}, & if c \le x \le d \end{cases}$$

Definition 2.3. Two fuzzy numbers $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, \mathrm{h})$ are said to be equal if and if $a = e$, $b = f, c = g$ and $d = h$.

2.4. Arithmetic operation on trapezoidal numbers

Let $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, \mathrm{h})$ are two trapezoidal fuzzy numbers then

(i) $\tilde{A} \oplus \tilde{B} = (a, b, c, d) \oplus (e, f, g, \mathrm{h})$
$=(a + e, b + f, c + g, d + h)$

(ii) $-\tilde{A} = -(a, b, c, d) = (-b, -a, d, c)$

(iii) If $\tilde{A} \ge 0$ and $\tilde{B} \ge 0$then$\tilde{A} \otimes \tilde{B} = (a, b, c, d) \otimes (e, f, g, \mathrm{h})$
$= (ae, bf, ag + ce, bh + df)$

Definition 2.5. A matrix $\tilde{A} = (a_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is afuzzy number. A fuzzy matrix $\tilde{A}$ is positive denoted by $\tilde{A}$>0 if each element of $\tilde{A}$ be positive. To represent $n \times n$ fuzzy matrix $\tilde{A} = (a_{ij})_{nxn}$ such that matrix $\tilde{a}_{ij}= (a_{ij}, b_{ij}, m_{ij}, n_{ij})$ with the new notation $\tilde{A}$ = (*A*, *B*, *M*,*N*) where *A* = (a_{ij}), *B* = (b_{ij}), *M* = (m_{ij}), *N* = (n_{ij}) arefour $n \times n$ crisp matrices.

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Definition 2.6. Consider the $n \times n$ fuzzy linear systems of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$
$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix of the above equation is $\tilde{A} \otimes \tilde{x} = \tilde{b}$where the coefficientmatrix $\tilde{A} = (a_{ij})$, $1 \le i, j \le n$ is a $n \times n$ fuzzy matrix and $\tilde{x}, \tilde{b} \in F(R)$.This system is called fully fuzzy linear system (FFLS).

3. Proposed method

Consider the crisp linear system of equation $\tilde{A} \otimes \tilde{x} = \tilde{b}$

Where $\tilde{A} = (A, B, M, N)$ and $\tilde{x} = (x, y, z, w) \geq 0$and $\tilde{b} = (b, g, \mathrm{h}, k) \geq 0$
Then $(A, B, M, N) \otimes (x, y, z, w) = (b, g, \mathrm{h}, k)$

Using (2.4)

$$(Ax, By, Az + Mx, Bw + Ny) = (b, g, h, k)$$

Again using (2.3)

$$Ax = b$$
$$By = g$$
$$Az + Mx = h$$
$$Bw + Ny = k$$

Assuming that A and B are non-singular matrices we have

$$x = A^{-1}b$$
$$y = B^{-1}g$$
$$z = A^{-1}(h - Mx)$$
$$w = B^{-1}(k - Ny)$$

3.1. Cholesky decomposition method

Given any fuzzy linear system of equations in the form of trapezoidal fuzzy matrices that can be decomposed into the form such that $\tilde{A} = LL^T$, where A is the symmetric and positive definite and L is lower triangular matrix.

Assume that $\tilde{A}$ = (A, B,M,N) where A and B are the full rank crisp matrices.

Then if we let $LL^T = \tilde{A}$

$$(\mathrm{L}_1, \mathrm{L}_2, \mathrm{L}_3, \mathrm{L}_4) \otimes ({L_1}^T, {L_2}^T, {L_3}^T, {L_4}^T) = (A, B, M, N)$$

Using (2.4)

$$(\mathrm{L}_1 {L_1}^T, \mathrm{L}_2 {L_2}^T, \mathrm{L}_1 {L_3}^T + \mathrm{L}_3 {L_1}^T, \mathrm{L}_2 {L_4}^T + \mathrm{L}_4 {L_2}^T) = (A, B, M, N)$$

Consider the fully fuzzy linear systems.$\tilde{A} \otimes \tilde{x} = \tilde{b}$
Where $\tilde{A} = (A, B, M, N)$,$\tilde{x} = (x, y, z, w) \geq 0$ $and \tilde{b} = (b, g, \mathrm{h}, k) \geq 0$.

$$(A, B, M, N) \otimes (x, y, z, w) = (b, g, \mathrm{h}, k)$$
$$(\mathrm{L}_1 {L_1}^T, \mathrm{L}_2 {L_2}^T, \mathrm{L}_1 {L_3}^T + \mathrm{L}_3 {L_1}^T, \mathrm{L}_2 {L_4}^T + \mathrm{L}_4 {L_2}^T) \otimes (x, y, z, w) = (b, g, \mathrm{h}, k)$$

Using (2.4)

$$(\mathrm{L}_1 {L_1}^T x, \mathrm{L}_2 {L_2}^T \mathrm{y}, \mathrm{L}_1 {L_1}^T \mathrm{z} + (\mathrm{L}_1 {L_3}^T + \mathrm{L}_3 {L_1}^T) x, \mathrm{L}_2 {L_2}^T \mathrm{w} + (\mathrm{L}_2 {L_4}^T + \mathrm{L}_4 {L_2}^T) \mathrm{y}) = (b, g, \mathrm{h}, k)$$

The current system by use of definition (2.3) is rewritten as

$$\mathrm{L}_1 {L_1}^T x = b$$
$$\mathrm{L}_2 {L_2}^T \mathrm{y} = \mathrm{g}$$
$$\mathrm{L}_1 {L_1}^T \mathrm{z} + (\mathrm{L}_1 {L_3}^T + \mathrm{L}_3 {L_1}^T) x = h$$
$$\mathrm{L}_2 {L_2}^T \mathrm{w} + (\mathrm{L}_2 {L_4}^T + \mathrm{L}_4 {L_2}^T) \mathrm{y} = \mathrm{k}$$

Therefore

$$x = {L_1^T}^{-1} L_1^{-1} b$$
$$\mathrm{y} = {L_2^T}^{-1} L_2^{-1} \mathrm{g}$$

$$z = L_1^{T^{-1}} L_1^{-1}[h - (L_1 L_3{}^T + L_3 L_1{}^T)x]$$
$$z = L_1^{T^{-1}} L_1^{-1}[h - Mx]$$
$$w = L_2^{T^{-1}} L_2^{-1}[k - (L_2 L_4{}^T + L_4 L_2{}^T)y]$$
$$w = L_2^{T^{-1}} L_2^{-1}[k - Ny]$$

3.2. Algorithm

Step 1:

Assume that $\tilde{A}$ = (*A*,*B*,*M*,*N*) where A and B are the full rank crisp matrices.
Compute by Choesky decomposition $A = L_1 L_1{}^T$ and $B = L_2 L_2{}^T$.

Step 2:

Compute M = $L_1 L_3{}^T + L_3 L_1{}^T$ and N = $L_2 L_4{}^T + L_4 L_2{}^T$.

Step 3:

Compute the solution of the fully fuzzy system $\tilde{A} = (A, B, M, N)$ and $\tilde{x} = (x, y, z, w) \geq 0$
And $\widetilde{b} = (b, g, h, k)$ as follows

$$x = L_1^{T^{-1}} L_1^{-1} b$$
$$y = L_2^{T^{-1}} L_2^{-1} g$$
$$z = L_1^{T^{-1}} L_1^{-1}[h - Mx]$$
$$w = L_2^{T^{-1}} L_2^{-1}[k - Ny]$$

4. Numerical example

Consider the following fully fuzzy linear system

$$(16,6,2,2) \otimes (x_1, y_1, z_1, w_1) \oplus (4,6,1,2) \otimes (x_2, y_2, z_2, w_2) = (27,66,26,58)$$

$$(4,5,1,1) \otimes (x_1, y_1, z_1, w_1) \oplus (6,8,1,2) \otimes (x_2, y_2, z_2, w_2) = (35,70,25,55)$$

with entries as trapezoidal fuzzy numbers.

Solution: The given fully fuzzy linear system can be written as

$$\begin{bmatrix} (16,6,2,2) & (4,6,1,2) \\ (4,5,1,1) & (6,8,1,2) \end{bmatrix} \begin{bmatrix} (x_1, y_1, z_1, w_1) & (x_2, y_2, z_2, w_2) \\ (x_1, y_1, z_1, w_1) & (x_2, y_2, z_2, w_2) \end{bmatrix} = \begin{bmatrix} (27,66,26,58) \\ (35,70,25,55) \end{bmatrix}$$

where

$A = \begin{bmatrix} 16 & 4 \\ 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 6 \\ 5 & 8 \end{bmatrix}$, $M = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$,$N = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$

$b = \begin{bmatrix} 27 \\ 35 \end{bmatrix}$,$g = \begin{bmatrix} 66 \\ 70 \end{bmatrix}$,$h = \begin{bmatrix} 26 \\ 25 \end{bmatrix}$,$k = \begin{bmatrix} 58 \\ 55 \end{bmatrix}$

By applying the above algorithm usingCholesky decomposition,

$$A = L_1 L_1{}^T = \begin{bmatrix} 4 & 0 \\ 1 & \sqrt{5} \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 0 & \sqrt{5} \end{bmatrix}$$
$$B = L_2 L_2{}^T = \begin{bmatrix} \sqrt{6} & 0 \\ \sqrt{6} & \sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{6} & \sqrt{6} \\ 0 & \sqrt{2} \end{bmatrix}$$
$$x = L_1^{T^{-1}} L_1^{-1} b$$
$$x = \frac{1}{4\sqrt{5}} \begin{bmatrix} \sqrt{5} & -1 \\ 0 & 4 \end{bmatrix} . \frac{1}{4\sqrt{5}} \begin{bmatrix} \sqrt{5} & 0 \\ -1 & 4 \end{bmatrix} . \begin{bmatrix} 27 \\ 35 \end{bmatrix}$$
$$x = \begin{bmatrix} 0.28 \\ 5.65 \end{bmatrix}$$
$$y = L_2^{T^{-1}} L_2^{-1} g$$

$$y = \frac{1}{\sqrt{12}}\begin{bmatrix}\sqrt{2} & -\sqrt{6}\\ 0 & \sqrt{6}\end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix}\sqrt{2} & 0\\ -\sqrt{6} & \sqrt{6}\end{bmatrix}. \begin{bmatrix}66\\ 70\end{bmatrix}$$

$$y = \begin{bmatrix}9\\ 2\end{bmatrix}$$

$$\mathrm{z} = {L_1^T}^{-1} L_1^{-1}[h - \mathrm{M}x]$$

$$z = \begin{bmatrix}0.53\\ 2.82\end{bmatrix}$$

$$\mathrm{w} = {L_2^T}^{-1} L_2^{-1}[\mathrm{k} - \mathrm{Ny}]$$

$$w = \begin{bmatrix}3\\ 3\end{bmatrix}$$

Therefore the solution to the given system is

$$\tilde{x}_1 = (0.28, 9, 0.53, 3) \text{ and } \tilde{x}_2 = (5.65, 2, 2.82, 3).$$

5. Conclusion

In this article, a new methodology is applied to find the solution of fully fuzzy linear systems in the form of trapezoidal fuzzy matrices.

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